Lepton mass correction in partial wave analyses of charmed meson semi-leptonic decays*

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Abstract: We derive a parameterization formula for the partial wave analyses of charmed meson semi-leptonic decays while considering the effects of lepton mass. Because the proposed super-tau-charm factory will reach a significantly enhanced luminosity and BESIII is collecting new $\psi(3770)\to D\bar{D}$ data, our results will help improve the measurement precision of future partial wave analyses of charmed meson semi-muonic decays.

Keywords: partial wave analysis, charmed meson, semi-leptonic decay

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I. INTRODUCTION

The standard model, as a theory of elementary particle interactions, has achieved great success in describing experimental data with various energies. Nevertheless, the ~ 2 GeV masses of charmed mesons place them in the region where perturbative QCD is not applicable, which raises the challenges faced by both theory and experiments. In recent years, standard model testing with high precision measurements has become one of the hottest topics in the charm sector.

The semi-leptonic decays of charmed mesons, in which hadronic and weak currents can be well separated, provide a clean platform for studying the mechanism of the c quark to $d(s)$ quark transition and play an important role in understanding strong and weak interactions. Their partial decay width accesses the product of the hadronic form factor, which describes the strong-interaction in the hadronic current connecting initial and final hadrons, and the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cs}|$ or $|V_{cd}|$, which parameterizes the weak interaction between different quark flavors. Partial wave analyses of the four-body semi-leptonic decays of charmed mesons allow us to extract the form factors in the $D\to V\ell^+\nu_\ell$ and $D\to S\ell^+\nu_\ell$ transitions (where $\ell=e,\mu$, V and S denote vector and scalar mesons, respectively). $K^*(892)^{-(0)}$ resonance has been studied in the $D^{0(+)}\to K^{0(-)}\pi^+e^+\nu_e$ decay by the CLEO, BABAR, and BESIII collaborations [1–4]. BESIII has also studied ρ^- resonance in $D^0\to\pi^-\pi^0e^+\nu_e$ [5], and ρ^0, ω, and $f_0(500)$ resonances in $D^+\to\pi^+\pi^-e^+\nu_e$ [5]. In the near future, more partial wave analyses are expected to be performed with high-statistics datasets, including $D_{(s)}$ semi-muonic decays[1)].

However, lepton mass is neglected in the parameterization formula used for the amplitude analyses of charmed meson semi-leptonic decays. This should be manageable in the case of semi-electronic decays, but will cause bias in semi-muonic decays, which downgrades the advantage of high-statistics from the new data and conflicts with the purpose of precision measurement. In this study, we derive the formula while considering the mass of leptons based on Refs. [6, 7]. The results are presented in the format used in experimental analyses; therefore, researchers can easily adopt our results to their experimental analyses. Charge-conjugated decay modes are implied throughout this paper.

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1) At present, there is no partial wave analyses of $D_{(s)}$ semi-muonic decays reported yet because low statistics and high-level background caused by μ-π misidentification.

II. FORMALISM

First, we define the kinematic variables and discuss their relations. The four-body semi-leptonic decay $D\to M_1M_2\ell^+\nu_\ell$ is considered, where D is the parent meson, $M_{1(2)}$ is the product meson, and $\ell=e,\mu$. The momentum four-vectors and invariant masses are denoted by p and m, respectively. For convenience, the independent four-vector combinations are defined as

$$P^\mu=p^\mu_{M_1}+p^\mu_{M_2},\quad Q^\mu=p^\mu_{M_1}-p^\mu_{M_2},\\ L^\mu=p^\mu_\ell+p^\mu_\nu,\quad N^\mu=p^\mu_\ell-p^\mu_\nu. \tag{1}$$

A four-body decay can be uniquely described via kinematic parameterization with five variables (besides spin). The squared masses of the hadronic system M_1M_2 and leptonic system $\ell^+\nu_\ell$ are chosen as two of the five variables,

$$s_M=P^2,\quad s_L=L^2, \tag{2}$$

and the following relations can be easily derived:

$$Q^2=2m^2_{M_1}+2m^2_{M_2}-s_M,\quad N^2=2m^2_\ell+2m^2_\nu-s_L, \tag{3}$$

$$L\cdot P=\frac{m_D^2-s_M-s_L}{2},\quad L\cdot N=m^2_\ell-m^2_\nu,\quad P\cdot Q=m^2_{M_1}-m^2_{M_2}. \tag{4}$$

The other three variables are chosen as the angle between the M_2 three-momentum and the D direction in the M_1M_2 rest frame (θ_M), the angle between ν_ℓ and the D direction in the $\ell^+\nu_\ell$ rest frame (θ_L), and the angle between the two decay planes (ϕ)[1]. The angles θ_M, θ_L, and ϕ are illustrated in Fig. 1. The various relationships between scalar-product invariants can be written as

$$\begin{aligned}
L\cdot Q=&L\cdot P\frac{m^2_{M_1}-m^2_{M_2}}{s_M}+X\beta_M\cos\theta_M,\\
N\cdot P=&L\cdot P\frac{m^2_\ell-m^2_\nu}{s_L}+X\beta_L\cos\theta_L,\\
N\cdot Q=&L\cdot P\beta_M\beta_L\cos\theta_M\cos\theta_L+\frac{m^2_\ell-m^2_\nu}{s_L}X\beta_M\cos\theta_M\\
&+\frac{m^2_{M_1}-m^2_{M_2}}{s_M}X\beta_L\cos\theta_L+L\cdot P\frac{m^2_{M_1}-m^2_{M_2}}{s_M}\frac{m^2_\ell-m^2_\nu}{s_L}\\
&-\sqrt{s_M}\sqrt{s_L}\beta_M\beta_L\sin\theta_M\sin\theta_L\cos\phi,\\
\epsilon_{\mu\nu\rho\sigma}L^\mu N^\nu P^\rho Q^\sigma=&X\sqrt{s_M}\sqrt{s_L}\beta_M\beta_L\sin\theta_M\sin\theta_L\sin\phi,
\end{aligned} \tag{5}$$

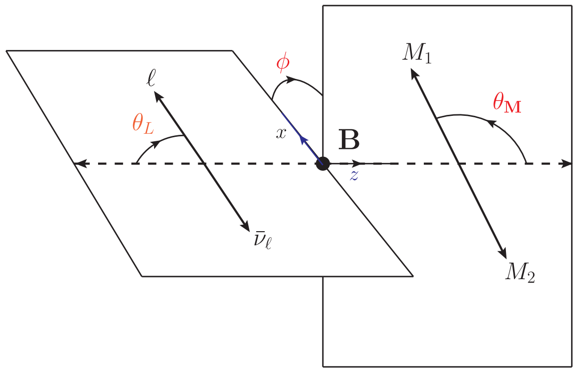

Fig. 1. (color online) Definition of angles θ_L, θ_M, and ϕ in the cascade decay $D\to M_1M_2\ell^+\nu_\ell$.

where β_M is the three-momentum modulus of the meson in the center-of-mass frame of the meson-meson system, β_L the three-momentum modulus of the lepton in the center-of-mass frame of the lepton-neutrino system, and X an element of phase space,

$$\begin{aligned}
\beta_M&=\sqrt{(s_M-m^2_+)(s_M-m^2_-)}/s_M,\\
\beta_L&=\sqrt{(s_L-m^2_{L^+})(s_L-m^2_{L^-})}/s_L,\\
X&=\sqrt{m_D^4+s_L^2+s_M^2-2s_Dm_L^2-2s_Mm_D^2-2s_Ms_L}/2,
\end{aligned} \tag{6}$$

with

$$m_{L^+}=m_\ell+m_\nu;\quad m_{L^-}=m_\ell-m_\nu;\\ m_+=m_{M_1}+m_{M_2};\quad m_-=m_{M_1}-m_{M_2}. \tag{7}$$

Compared to Refs. [6, 7], we do not neglect the lepton mass.

Next, from the effective Hamiltonian at the quark level for $D\to M_1M_2\ell^+\nu_\ell$, the decay amplitude is given by

$$\begin{aligned}
&\mathcal{A}(D\to M_1M_2\ell^+\nu_\ell)\\
&=\frac{G_F}{\sqrt2}V_{q_1q_2}\langle M_2M_1|\bar q_1\gamma_\mu(1-\gamma_5)q_2|D\rangle\bar u(p_\ell)\gamma^\mu(1-\gamma_5)v(p_\nu),
\end{aligned} \tag{8}$$

where G_F is the Fermi constant, and $V_{q_1q_2}$ is the element of the Cabibbo-Kobayashi-Maskawa matrix. The hadronic matrix element can be written in terms of four form factors, $w_\pm$, r, and h, which are defined by

$$\begin{aligned}
&\langle M_2M_1|\bar q_1\gamma_\mu(1-\gamma_5)q_2|D\rangle\\
&=h\epsilon^{\mu\nu\alpha\beta}p^\nu_DP^\alpha Q^\beta+\mathrm{i}rL^\mu+\mathrm{i}w_+P^\mu+\mathrm{i}w_-Q^\mu,
\end{aligned} \tag{9}$$

1) ϕ in this paper is defined as $-\phi$ in [6, 7]

where the form factors $w_\pm$, r, and h are functions of s_M, s_L, and $\cos\theta_M$, and $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita symbol.

The differential decay rate takes the form

$$\begin{aligned}\mathrm{d}\Gamma =&\frac{G_{\rm F}^2|V_{q_1q_2}|^2}{(4\pi)^6 m_D^3}X\beta_M I(s_M,s_L,\theta_M,\theta_L,\phi)\\ &\times \mathrm{d}s_M\mathrm{d}s_L\mathrm{d}\cos\theta_M\mathrm{d}\cos\theta_L\mathrm{d}\phi.\end{aligned}\tag{10}$$

To study the structure of the hadron system, that is, the form factors of the M_1M_2 system, the decay intensity I is decomposed with respect to θ_L and ϕ, which is written as

$$\begin{aligned}I =&I_1+I_2\cos2\theta_L+I_3\sin^2\theta_L\cos2\phi+I_4\sin2\theta_L\cos\phi\\ &+I_5\sin\theta_L\cos\phi+I_6\cos\theta_L+I_7\sin\theta_L\sin\phi\\ &+I_8\sin2\theta_L\sin\phi+I_9\sin^2\theta_L\sin2\phi,\end{aligned}\tag{11}$$

where $I_{1,2,...,9}$ depend only on s_M, s_L, and ϕ. We can further express $I_{1,2,...,9}$ in terms of form factors,

$$\begin{aligned}I_1 =&\frac{1}{4}(2-\beta_L)\beta_L|F_1|^2+\left(\frac{\beta_L}{2}-\frac{\beta_L^2}{8}\right)\sin^2\theta_M(|F_2|^2+|F_3|^2)\\ &+\frac{1}{2}(1-\beta_L)\beta_L|F_4|^2,\\ I_2 =&-\frac{\beta_L^2}{4}\left[|F_1|^2-\frac{1}{2}\sin^2\theta_M(|F_2|^2+|F_3|^2)\right],\\ I_3 =&-\frac{\beta_L^2}{4}\left[\sin^2\theta_M(|F_2|^2-|F_3|^2)\right],\\ I_4 =&\frac{\beta_L^2}{2}\sin\theta_M\mathrm{Re}(F_1F_2^\star),\\ I_5 =&-\beta_L\sin\theta_M\left\{\mathrm{Re}(F_1F_3^\star)-(1-\beta_L)\mathrm{Re}(F_2F_4^\star)\right\},\\ I_6 =&-\beta_L\sin^2\theta_M\mathrm{Re}(F_2F_3^\star)-\beta_L(1-\beta_L)\mathrm{Re}(F_1F_4^\star),\\ I_7 =&\beta_L\sin\theta_M\left\{\mathrm{Im}(F_1F_2^\star)+(1-\beta_L)\mathrm{Im}(F_3F_4^\star)\right\},\\ I_8 =&-\frac{\beta_L^2}{2}\sin\theta_M\mathrm{Im}(F_1F_3^\star),\\ I_9 =&\frac{\beta_L^2}{2}\sin^2\theta_M\mathrm{Im}(F_2F_3^\star),\end{aligned}\tag{12}$$

where $F_{1,2,3,4}$ are the form factors

$$\begin{aligned}F_1 =&Xw_+ + (\beta_M P\cdot L\cos\theta_M+\frac{m_+m_-}{s_M}X)w_-,\\ F_2 =&\beta_M\sqrt{s_M}\sqrt{s_L}w_-,\\ F_3 =&X\beta_M\sqrt{s_M}\sqrt{s_L}h,\\ F_4 =&s_Lr+P\cdot Lw_+ + (X\beta_M\cos\theta_M+\frac{m_+m_-}{s_M}P\cdot L)w_-.\end{aligned}\tag{13}$$

For the purpose of discussing the angular momentum of M_1M_2, for example, S- and P-waves, the partial wave expansions in spherical harmonics for the form factors $F_{1,2,3,4}$ are written as

$$\begin{aligned}F_1(s_M,s_L,\cos\theta_M)=&\sum_{l=0}^{\infty}F_{1l}(s_M,s_L)P_l(\cos\theta_M),\\ F_2(s_M,s_L,\cos\theta_M)=&\sum_{l=1}^{\infty}\frac{1}{\sqrt{l(l+1)}}F_{2l}(s_M,s_L)\frac{\mathrm{d}P_l(\cos\theta_M)}{\mathrm{d}\cos\theta_M},\\ F_3(s_M,s_L,\cos\theta_M)=&\sum_{l=1}^{\infty}\frac{1}{\sqrt{l(l+1)}}F_{3l}(s_M,s_L)\frac{\mathrm{d}P_l(\cos\theta_M)}{\mathrm{d}\cos\theta_M},\\ F_4(s_M,s_L,\cos\theta_M)=&\sum_{l=0}^{\infty}F_{4l}(s_M,s_L)P_l(\cos\theta_M).\end{aligned}\tag{14}$$

Moreover, the decay $D\to M_1M_2\ell^+\nu_\ell$ may occur via intermediate states, such as vector or scalar mesons, which provides information about the intermediate resonances [8, 9]. The amplitudes of $D\to(S\to M_1M_2)\ell^+\nu_\ell$ and $D\to(V\to M_1M_2)\ell^+\nu_\ell$ can be given by

$$\begin{aligned}&\mathcal{A}(D\to(V\to M_1M_2)\ell^+\nu_\ell)\\ =&\langle V|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|D\rangle\epsilon\cdot Qg_{VM_1M_2}D_{FV}\bar{u}(p_\ell)\gamma^\mu(1-\gamma_5)v(p_\nu),\\ &\mathcal{A}(D\to(S\to M_1M_2)\ell^+\nu_\ell)\\ =&\langle S|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|D\rangle g_{SM_1M_2}D_{FS}\bar{u}(p_\ell)\gamma^\mu(1-\gamma_5)v(p_\nu),\end{aligned}\tag{15}$$

where

$$\begin{aligned}&\langle V|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|D\rangle=\\ &-\epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu*}P_D^\alpha P^\beta\frac{2V_0(s_L)}{m_V+m_D}-\mathrm{i}\left(\epsilon_\mu^*-\frac{\epsilon^*\cdot L}{L^2}L_\mu\right)(m_V+m_D)A_1(s_L)\\ &+\mathrm{i}\left(P_{D\mu}+P_\mu-\frac{m_D^2-m_V^2}{L^2}L_\mu\right)\epsilon^*\cdot L\frac{A_2(s_L)}{m_V+m_D}\\ &-\mathrm{i}\frac{2m_V\epsilon^*\cdot L}{L^2}L_\mu A_0(s_L),\\ &\langle S|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|D\rangle=\mathrm{i}(f^+(s_L)P_\mu+f^-(s_L)L_\mu),\end{aligned}\tag{16}$$

with

$$\begin{aligned}&\sum\epsilon^{\mu*}\epsilon^\nu=-g^{\mu\nu}+\frac{P^\mu P^\nu}{P^2},\\ &\mathcal{A}(D\to M_1M_2\ell^+\nu_\ell)=\mathcal{A}(D\to(V\to M_1M_2)\ell^+\nu_\ell)\\ &\qquad+\mathcal{A}(D\to(S\to M_1M_2)\ell^+\nu_\ell).\end{aligned}\tag{17}$$

Here, $f^\pm(s_L)$ is the $D\to S$ form factor, $A_{0,1,2}$ are the

$D\to V$ axial-vector form factors, and V_0 is the $D\to V$ vector form factor [10, 11]. Finally, we can obtain F_{1-4} in the helicity basis (for *S*- and *P*-waves only),

$$\begin{aligned}F_1(s_M,s_L,\cos\theta_M)=&Xf^+(s_L)g_{SM_1M_2}D_{FS}\\&+\cos\theta_M\beta_M g_{VM_1M_2}D_{FV}\sqrt{s_Ls_M}H_0(s_L),\\F_2(s_M,s_L,\cos\theta_M)=&\frac{1}{2}\beta_M g_{VM_1M_2}D_{FV}\sqrt{s_Ls_M}\\&\times(H_+(s_L)+H_-(s_L)),\\F_3(s_M,s_L,\cos\theta_M)=&\frac{1}{2}\beta_M g_{VM_1M_2}D_{FV}\sqrt{s_Ls_M}\\&\times(H_+(s_L)-H_-(s_L)),\\F_4(s_M,s_L,\cos\theta_M)=&s_Lf^-(s_L)g_{SM_1M_2}D_{FS}\\&+2\cos\theta_M\beta_M g_{VM_1M_2}D_{FV}\sqrt{s_Ls_M}H_t(s_L),\end{aligned}\tag{18}$$

with

$$\begin{aligned}H_0(s_L)=&\frac{1}{\sqrt{s_Ls_M}}\left[P\cdot L(m_V+m_D)A_1(s_L)-2\frac{X^2}{m_V+m_D}A_2(s_L)\right]\\H_\pm(s_L)=&(m_V+m_D)A_1(s_L)\mp\frac{2X}{m_V+m_D}V_0(s_L),\\H_t(s_L)=&\frac{X}{\sqrt{s_L}}[A_0(s_L)+A_1(s_L)+A_2(s_L)],\end{aligned}\tag{19}$$

where $g_{SM_1M_2}(g_{VM_1M_2})$ is the coupling constant, $D_{FS}(D_{FV})$ is derived from the propagator for $S(V)$. In the case of Breit-Wigner lineshapes, $D_{FS}=1/(s_M-m_S^2+\mathrm{i}m_S\Gamma_S)$ and $D_{FV}=1/(s_M-m_V^2+\mathrm{i}m_V\Gamma_V)$, or $D_{FS}=1/(s_M-m_S^2+\mathrm{i}\frac{s_M}{m_S}\Gamma_S)$ and $D_{FV}=1/(s_M-m_V^2+\mathrm{i}\frac{s_M}{m_V}\Gamma_V)$ if decay width $\Gamma_S(\Gamma_V)$ is not negligible [12]. For the scalar meson $f_0(980)$, it couples to the channels $K\bar{K}$ and $\pi\pi$ strongly. Likewise, $a_0(980)$ couples to $K\bar{K}$ and $\pi\eta$ strongly. One needs to replace the usual Breit-Wigner lineshape by the Flatté formula [13] to take into account the coupled channel effect. For the broad $\rho(770)$ resonance, it is customary to adapt the Gounaris-Sakurai model [14] to account for the $\pi\pi$ rescattering.

III. DISCUSSION AND CONCLUSION

In this study, we derive the parameterization formula of a four-body semi-leptonic decay while considering the effects of lepton mass and express the differential decay width in the format used in partial wave analyses. We can obtain the parameterization formula used in Refs. [3–5], which neglects lepton mass (or set $\beta_L=1$); however, while this does not significantly influence semi-electronic decays, neglecting the lepton mass can result in a bias of up to ~ 1 % in partial wave analyses for charmed meson semi-muonic decays.

BESIII is accumulating data samples with an integrated luminosity of 20 fb^{-1} at a center-of-mass energy of 3.773 GeV (for D^0 and $D^\pm$ mesons) and has collected 7.33 fb^{-1} data samples at 4.128 – 4.226 GeV (for D_s^+ mesons) [15]. In addition, the proposed super-tau-charm factory will be able to reach significantly enhanced luminosities. These aim at testing the standard model with high precision in the charm sector; however, precise theoretical parameterization is also needed. With the correction for lepton mass presented in this paper, partial wave analyses can be performed to study the form factors in the $D_{(s)}\to V\mu^+\nu_\mu$ and $D_{(s)}\to S\mu^+\nu_\mu$ decays more precisely.

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