Gauging universe expansion via scalar fields

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Abstract: In this study, we investigate the expansion of the FRLW universe in the open, closed, and flat geometries. The universe is dominated by a scalar field (spatially homogeneous) as a source of dark energy. We consider the three different classes of scalar fields – quintessence, tachyonic, and phantom field – for our analysis. A mathematical analysis is carried out by considering these three scalar fields with exponential and power-law potentials. Both potentials give exponential expansion in the open, closed, and flat FRLW universes. It is found that quintessence, tachyonic, and phantom scalar fields are indistinguishable under the slow roll approximation.

Keywords: quientessence, tachyonic scalar field, phantom field, dark energy

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I. INTRODUCTION

Astrophysical and cosmological observations [1–6] indicate accelerated expansion of the universe. Recent Planck data [7–10] indicate that nearly 70% of the total contents in the universe are exotic, showing repulsive gravity dubbed as dark energy. The observed accelerated expansion of the universe at the present epoch can be explained by dark energy as modified matter in the energy-momentum tensor. Apart from the dark energy model theory of modified geometry in Einstein's field equations, there are alternatives to accelerated expansion. Two approaches to dark energy are popular in cosmology. In one, dark energy is constant, and the cosmological constant is one of the simplest candidates for it. The cosmological constant, having an equation of state $w=-1$, does not evolve with time and suffers from a serious issue called the cosmological constant problem. The decaying cosmological constant [11–16] is one step toward the resolution of this issue. In the other approach, dark energy is not constant. This is commonly known as the dynamical dark energy model, which has been studied by several authors [17–21]. A cosmic real scalar field is a candidate for dynamical dark energy. A detailed study of the cosmological behavior of the scalar field is available in literature [22–29].

The exact form of the scale factor of expansion has yet to be constrained by observations. It may be possible that we have encountered another inflationary era apart from the early inflationary phase of the universe. In this article, we obtain the cosmic expansion in the dynamical dark energy model by considering three classes of scalar fields. In section II, we consider quintessence and obtain the expansion factor for two specific potentials: exponential and power law form. In sections III and IV, we consider the same forms of potential for the tachyonic scalar field and phantom, respectively.

The non-canonical Lagrangian of scalar field ϕ is given as [30–36]

$$\mathcal{L}=X\left(\frac{X}{M^4}\right)^{\alpha-1}-V(\phi), \tag{1}$$

where M is a constant with the dimension of mass, α is a dimensionless parameter, and $X=\frac{1}{2}\dot{\phi}^2=\frac{1}{2}(\frac{\mathrm{d}\phi}{\mathrm{d}t})^2$. The Friedmann–Robertson–Walker (FRW) metric is given as

$$\mathrm{d}s^2=\mathrm{d}t^2-a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2}+r^2\mathrm{d}\Theta^2+r^2\sin^2\Theta\mathrm{d}\Phi^2\right], \tag{2}$$

where $a(t)$ is the scale factor for expansion, and k is the Gaussian curvature parameter with values $-1,0,+1$ assigned for the open, flat, and closed universes. The energy-momentum stress tensor takes the following form:

$$T^{\mu\nu}=\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\partial^\nu\phi-g^{\mu\nu}\mathcal{L}. \tag{3}$$

One can obtain the pressure and energy density of the

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scalar field from Eq. (3) as follows:

$$\rho_\phi = \left(\frac{\partial \mathcal{L}}{\partial X}\right)(2X) - \mathcal{L}, \tag{4a}$$

$$P_\phi = \mathcal{L}. \tag{4b}$$

Using the Lagrangian (Eq. (1)) in Eqs. (4a) and (4b), one can obtain the following expression for the pressure and energy density of a non-canonical scalar field:

$$P_\phi = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi), \tag{5a}$$

$$\rho_\phi = (2\alpha-1)X\left(\frac{X}{M^4}\right)^{\alpha-1} + V(\phi). \tag{5b}$$

The Friedman equations without the cosmological constant Λ are

$$\frac{\dot{a}^2+k}{a^2} = \frac{1}{3m_p^2}\left((2\alpha-1)X\left(\frac{X}{M^4}\right)^{\alpha-1} + V(\phi)\right), \tag{6a}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{3m_p^2}\left((2\alpha-1)X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi)\right), \tag{6b}$$

where $m_p^2 = \dfrac{1}{8\pi G}$ and $c=1$.

The action for the scalar field is given as

$$\mathcal{A} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R + X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi) + \mathcal{L}_m\right], \tag{7}$$

where R is the Ricci scalar and $\mathcal{L}_m$ is the Lagrangian of the matter field. The equation of motion can be obtained from the variation of action (Eq. (7)) as

$$\ddot{\phi} + \frac{3H\dot{\phi}}{2\alpha-1} + \left(\frac{2M^4}{\dot{\phi}^2}\right)^{\alpha-1}\left(\frac{V'(\phi)}{\alpha(2\alpha-1)}\right) = 0, \tag{8}$$

where $V'(\phi) = \dfrac{\mathrm{d}V}{\mathrm{d}\phi}$ and $H(t) = \dfrac{\dot{a}(t)}{a(t)}$ is the Hubble parameter.

II. QUINTESSENCE WITH EXPONENTIAL AND POWER LAW POTENTIAL

The non-canonical Lagrangian (Eq. (1)) leads to its canonical (quintessence) form for $\alpha = 1$; thus, from Eqs. (5a) and (5b), we obtain the following form of the energy density and pressure for a spatially homogeneous quintessence field:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{9}$$

The Friedman equations (6a) and (6b) with $\alpha = 1$ and Gaussian curvature parameter k are expressed as follows:

$$\frac{\dot{a}^2+k}{a^2(t)} = \frac{1}{3m_p^2}\rho_\phi, \tag{10}$$

$$\frac{\dot{a}^2+k}{a^2(t)} = \frac{1}{3m_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{11}$$

Under the slow roll approximation, the potential term of quintessence dominates over the kinetic term ($\dot{\phi}^2 \ll 1$, $\rho_\phi \sim V(\phi)$). Thus, we have

$$\frac{\dot{a}^2+k}{a^2(t)} = \frac{1}{3m_p^2}V(\phi), \tag{12}$$

and the Hubble parameter H is given by

$$H^2 = \frac{1}{3m_p^2}V(\phi) - \frac{k}{a^2(t)}. \tag{13}$$

The energy conservation equation is given as

$$\dot{\rho}_\phi + 3H(1+\omega_\phi)\rho_\phi = 0, \tag{14}$$

and the equation of motion for the scalar field (Eq. (8)) with $\alpha = 1$ is

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0. \tag{15}$$

Becuase the slow roll approximation demands $\ddot{\phi} \approx 0$, the time derivative of quintessence is

$$\dot{\phi} = -\frac{V'(\phi)}{\sqrt{\dfrac{3}{m_p^2}V(\phi) - \dfrac{9k}{a^2(t)}}}. \tag{16}$$

A. Cosmic expansion with exponential potential

We consider the exponential form of the potential

$$V(\phi) = V_0\exp(-\beta\phi), \tag{17}$$

so that

$$V'(\phi) = -\beta V(\phi). \tag{18}$$

The scaling solution of ρ_ϕ can be obtained from the continuity equation (Eq. (14)) by considering the constant equation of state ω_ϕ as

$$\rho_\phi = \rho_\phi^0 \left(\frac{a}{a_0}\right)^{-3(1+\omega_\phi)}, \tag{19}$$

where ρ_ϕ^0 and a_0 are the present value of the energy density and scale factor. Under the slow roll approximation, the scaling solution (Eq. (19)) is reduced to the following form:

$$a^2(t) \sim E^2 (V(\phi))^{-\frac{2}{3(1+\omega_\phi)}}, \tag{20}$$

where $E = a_0(\rho_\phi^0)^{\frac{1}{3(1+\omega_\phi)}}$. From Eqs. (16), (18), and (20), we have the following form of the kinetic term for spatially homogeneous quintessence:

$$\dot{\phi}^2 = \frac{\beta^2}{\dfrac{3}{m_p^2 V(\phi)} - \dfrac{9k}{E^2 \quad V(\phi)^{2-n}}}, \tag{21}$$

where $n = \dfrac{2}{3(1+\omega_\phi)}$. With the exponential potential (Eq. (17)), Eq. (21) is reduced to the following form:

$$\beta \mathrm{d}t = \sqrt{\frac{3}{m_p^2 V_0}\exp(\beta\phi) - \frac{9kV_0^{n-2}}{E^2}\exp[-(n-2)\beta\phi]}\,\mathrm{d}\phi. \tag{22}$$

The general solution of the above equation for $n \neq 2$ can be obtained as

$$\frac{2}{\beta(n-2)}\left[\sqrt{T\mathrm{e}^{\phi_0\beta} - U\mathrm{e}^{-\phi_0\beta(n-2)}}\left((n-1)\,{}_2F_1\left(1,\frac{1}{2(n-1)};\frac{n}{2(n-1)};\frac{T\mathrm{e}^{\phi_0(n-1)\beta}}{U}\right) - n + 2\right)\right.$$
$$\left. + \sqrt{T\mathrm{e}^{\beta\phi} - U\mathrm{e}^{-(\beta(n-2)\phi)}}\left(-(n-1)\,{}_2F_1\left(1,\frac{1}{2(n-1)};\frac{n}{2(n-1)};\frac{T\mathrm{e}^{(n-1)\beta\phi}}{U}\right) + n - 2\right)\right] = \beta t, \tag{23}$$

and for $n = 2$, the solution is

$$\frac{2}{\beta}\left[\sqrt{T\mathrm{e}^{\beta\phi} - U} - \sqrt{T\mathrm{e}^{\beta\phi_0} - U} + \sqrt{U}\left(\tan^{-1}\left(\frac{\sqrt{T\mathrm{e}^{\beta\phi_0} - U}}{\sqrt{U}}\right) - \tan^{-1}\left(\frac{\sqrt{T\mathrm{e}^{\beta\phi} - U}}{\sqrt{U}}\right)\right)\right] = \beta t. \tag{24}$$

Here $T = \dfrac{3}{m_p^2 V_0}$, $U = \dfrac{9kV_0^{n-2}}{E^2}$, and ${}_2F_1$ is the hypergeometric functional series [37] defined as

$${}_2F_1(p,q,r,s) = \sum_{m=0}^{\infty} \frac{(p)_m \ (q)_m \ s^m}{r_m \ m!}, \tag{25}$$

where $(p_m) = 1$ for $m = 0$, and $(p_m) = p(p+1)...(p+m-1)$ for $m > 0$. For small value of ϕ, the Binomial expansion in Eq. (22) gives

$$\int_{\phi_0}^{\phi(t)} \left(\sqrt{\frac{3}{m_p^2 V_0}}\exp(\beta\phi/2) - \frac{3k\sqrt{3m_p^2}V_0^{n-3/2}}{2E^2}\right.$$
$$\left.\exp(-(2n-3)\beta\phi/2)\right)\mathrm{d}\phi = \beta t, \tag{26}$$

where $\phi(t) = \phi_0$ at $t = 0$. After the integration, the last equation becomes

$$\frac{2}{\beta}\exp(\beta\phi(t)/2) + \frac{2Q}{(2n-3)\beta}\exp(-(2n-3)\beta\phi(t)/2)$$
$$= \frac{\beta}{A}t + \frac{2}{\beta}\exp(\beta\phi_0/2) + \frac{2Q}{(2n-3)\beta}\exp(-(2n-3)\beta\phi_0/2), \tag{27}$$

where $Q = \dfrac{3km_p^2V_0^{n-1}}{2E^2}$ and $A = \dfrac{\sqrt{3}}{m_p\sqrt{V_0}}$.

Power series approximation of the exponential in Eq. (27) provides the following expression for $\phi(t)$

$$\phi(t) = \frac{1}{1-Q}\left\{\frac{\beta}{A}t - \frac{2}{\beta}\Big(1 - \exp(\beta\phi_0/2)\Big)\right.$$
$$\left. - \frac{2Q}{(2n-3)\beta}\Big[1 - \exp[-(2n-3)\beta\phi_0/2]\Big]\right\}. \tag{28}$$

Using the Friedman equations (6a) and (6b) with $\alpha = 1$, the Gaussian curvature parameter k, and Eq. (28), one can obtain the Hubble parameter:

$$H(t)=\frac{\dot{a}}{a(t)}=\sqrt{\frac{V_0}{3m_p^2}\exp(-\beta\phi)-\frac{kV_0^n}{E^2}\exp(-n\beta\phi)}. \tag{29}$$

Under the Binomial approximation, Eq. (29) can be simplified as

$$\frac{\dot{a}}{a(t)}\sim\sqrt{\frac{V_0}{3m_p^2}}\exp(-\beta\phi/2)\left[1-\frac{kV_0^{n-1}3m_p^2}{2E^2}\exp(-(n-1)\beta\phi)\right]. \tag{30}$$

By defining

$$\Delta=-\frac{2}{\beta}\Big(1-\exp(\beta\phi_0/2)\Big)-\frac{2Q}{(2n-3)\beta}\Big(1-\exp(-(2n-3)\beta\phi_0/2)\Big), \tag{31}$$

the functional form of $\phi(t)$ (Eq. (28)) can be written compactly as

$$\phi(t)=\frac{1}{1-Q}\left(\frac{\beta}{A}t+\Delta\right), \tag{32}$$

where Δ is a constant.

Thus, Eq. (30), together with Eqs. (31) and (32), provides the following analytical form of the scale factor:

$$a(t)=a_0\exp\left\{\frac{\sqrt{\frac{V_0}{3m_p^2}}\left\{\exp\left[\frac{-\beta}{2(1-Q)}\left(\frac{\beta}{A}t+\Delta\right)\right]-\exp\left[\frac{-\beta\Delta}{2(1-Q)}\right]\right\}}{\frac{-\beta^2}{2A(1-Q)}}-\frac{\frac{kV_0^{n-1/2}\sqrt{3m_p^2}}{2E^2}\left\{\exp\left[-(2n-1)\frac{\beta}{2(1-Q)}\left(\frac{\beta}{A}t+\Delta\right)\right]-\exp\left[-(2n-1)\frac{\beta\Delta}{2(1-Q)}\right]\right\}}{\frac{-(2n-1)\beta^2}{2A(1-Q)}}\right\}. \tag{33}$$

For the condition $n=\frac{1}{2}$, we have $\omega_\phi=\frac{1}{3}$, and the quintessence mimics the radiation. The scale factor $a(t)$ and constant Δ are reduced to

$$a(t)=a_0\exp\left\{\frac{\sqrt{\frac{V_0}{3m_p^2}}\left\{\exp\left[\frac{-\beta}{2(1-Q)}\left(\frac{\beta}{A}t+\Delta\right)\right]-\exp\left[\frac{-\beta\Delta}{2(1-Q)}\right]\right\}}{\frac{-\beta^2}{2A(1-Q)}}-\frac{k\sqrt{3m_p^2}}{2E^2}t\right\}, \tag{34}$$

$$\Delta=-\frac{2}{\beta}\Big[1-\exp(\beta\phi_0/2)\Big]+\frac{Q}{\beta}\Big[1-\exp(\beta\phi_0/2)\Big]. \tag{35}$$

B. Cosmic expansion with power law potential

In the previous section, we showed that the quintessence field with exponential potential leads to the exponential expansion of the universe. In this section, we consider the power law form of potential, which several authors have used for different reasons.

$$V=V_0\phi^x, \tag{36}$$

Here, x is a real number. Using Eqs. (16), (20), and (36), we obtain

$$\dot{\phi}=-\frac{x}{\phi\sqrt{\frac{3}{m_p^2V_0}\phi^{-x}-\frac{9kV_0^{n-2}}{E^2}\phi^{x(n-2)}}}. \tag{37}$$

The general solution of Eq. (37) is obtained in the form of curvature parameter k

$$\frac{2}{T(x-4)}\left[\phi_0^{x-1}\left(\phi_0^{2-x}\left(T-U\phi_0^{(n-1)x}\right)\right)^{3/2}{}_2F_1\left(1,-\frac{3nx-4x+4}{2x-2nx};-\frac{2nx-3x+4}{2x-2nx};\frac{U\phi_0^{(n-1)x}}{T}\right)\right.$$

$$-\phi^{x-1}\left(\phi^{2-x}\left(T-U\phi^{(n-1)x}\right)\right)^{3/2}{}_2F_1\left(1,-\frac{3nx-4x+4}{2x-2nx};-\frac{2nx-3x+4}{2x-2nx};\frac{U\phi^{(n-1)x}}{T}\right)\Bigg]=-xt, \tag{38}$$

where $T=\dfrac{3}{m_p^2V_0}$ and $U=\dfrac{9kV_0^{n-2}}{E^2}$.

Using the Friedman equations (6a) and (6b) with $\alpha=1$, the Gaussian curvature parameter k, and the slow roll approximation of the field, the scale factor can be obtained as

$$a(t)=a_0\exp\left[\int_0^t\sqrt{\frac{V_0}{3m_p^2}\phi^x-\frac{kV_0^n}{E^2}\phi^{nx}}\mathrm{d}t\right]. \tag{39}$$

The form of the scale factor again becomes exponential. The quintessence field for both forms of potential (exponential and power law) leads to the exponential form of the scale factor, indicating the exponential expansion of the universe.

III. TACHYON WITH EXPONENTIAL AND POWER LAW POTENTIAL

The Lagrangian for the tachyonic scalar field comes from string theory [38–40], whose action is given as

$$\mathcal{A}=\int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{16\pi G}-V(\phi)\sqrt{1-\partial^i\phi\partial_i\phi}\right). \tag{40}$$

The Lagrangian for the tachyonic scalar field is defined as $\mathcal{L}=-V(\phi)\sqrt{1-2X}$; thus, the energy density and pressure of the field are given as follows:

$$\rho_\phi=-\frac{V(\phi)}{\sqrt{1-\dot\phi^2}},\quad P_\phi=-V(\phi)\sqrt{1-\dot\phi^2}. \tag{41}$$

The equation of motion of the tachyonic field is

$$\ddot\phi=-(1-\dot\phi^2)\left[3H\dot\phi+\frac{1}{V(\phi)}\frac{\mathrm{d}V}{\mathrm{d}\phi}\right], \tag{42}$$

and under the slow roll condition $\dot\phi^2\ll 1$, $\ddot\phi\sim 0$, it is reduced to

$$3H\dot\phi+\frac{1}{V(\phi)}\frac{\mathrm{d}V}{\mathrm{d}\phi}=0. \tag{43}$$

Therefore,

$$\dot\phi=-\frac{V'(\phi)}{3HV(\phi)}. \tag{44}$$

A. Cosmic expansion with exponential potential

Using the exponential form of potential defined by Eqs. (17) and (44), the kinetic term of the tachyonic scalar field is expressed as

$$\dot\phi=\frac{\beta}{3\sqrt{\dfrac{1}{3m_p^2}V(\phi)-\dfrac{k}{a^2(t)}}}. \tag{45}$$

Using Eq. (20), we obtain

$$\dot\phi=\frac{\beta}{\sqrt{\dfrac{3}{m_p^2}V(\phi)-\dfrac{9k}{E^2}V(\phi)^n}}, \tag{46}$$

where $n=\dfrac{2}{3(1+\omega_\phi)}$.

Substituting the exponential form of potential (Eq. (17)), the last equation simplifies to

$$\sqrt{\frac{3V_0}{m_p^2}\exp(-\beta\phi)-\frac{9kV_0^n}{E^2}\exp(-n\beta\phi)}\,\mathrm{d}\phi=\beta\mathrm{d}t. \tag{47}$$

The solution of Eq. (47) is the function of the hypergeometric series

$$\frac{2}{\beta Sn}\Bigg[\left(S-Re^{\beta(n-1)\phi_0}\right)\sqrt{Re^{-\beta\phi_0}-Se^{\beta(-n)\phi_0}}\,{}_2F_1\left(1,1+\frac{1}{2-2n};\frac{n-2}{2(n-1)};\frac{Re^{(n-1)\beta\phi_0}}{S}\right)$$
$$-\left(S-Re^{\beta(n-1)\phi}\right)\sqrt{Re^{-\beta\phi}-Se^{\beta(-n)\phi}}\,{}_2F_1\left(1,1+\frac{1}{2-2n};\frac{n-2}{2(n-1)};\frac{Re^{(n-1)\beta\phi}}{S}\right)\Bigg]=\beta t, \tag{48}$$

where $R=\dfrac{3V_0}{m_p^2}$ and $S=\dfrac{9kV_0^n}{E^2}$.

Using the Binomial expansion approximation in Eq. (47), we obtain

$$\int_{\phi_0}^{\phi(t)}\left[\sqrt{\frac{3V_0}{m_p^2}}\exp(-\beta\phi/2)-\frac{3\sqrt{3m_p^2}kV_0^{n-1/2}}{2E^2}\times\exp(-(2n-1)\beta\phi/2)\right]. \tag{49}$$

The functional form of $\phi(t)$ can be obtained for $n\neq 1/2$ by considering the substitutions $A=\sqrt{\dfrac{3V_0}{m_p^2}}$, $B=\dfrac{3\sqrt{3m_p^2}kV_0^{n-1/2}}{2E^2}$, and $Q=\dfrac{B}{A}$, as follows:

$$\frac{-2}{\beta}\exp(-\beta\phi(t)/2)+\frac{2}{\beta}\exp(-\beta\phi_\phi/2)+\frac{2Q}{(2n-1)\beta}\exp(-(2n-1)\beta\phi(t)/2)-\frac{2Q}{(2n-1)\beta}\exp(-(2n-1)\beta\phi_0/2)=\frac{\beta}{A}t. \tag{50}$$

Power series expansion of $\exp(-\beta\phi)$ in Eq. (50) gives

$$\phi(t)=\frac{1}{1-Q}\left[\frac{\beta}{A}t+\Gamma\right], \tag{51}$$

where Γ is defined as

$$\Gamma=\frac{2}{\beta}\Big(1-\exp(-\beta\phi_\phi/2)\Big)-\frac{2Q}{(2n-1)\beta}\Big(1-\exp(-(2n-1)\beta\phi_0/2)\tag{52}$$

Recalling the definition of the Hubble parameter (Eq. (13)),

$$\frac{\dot a}{a(t)}=\sqrt{\frac{V(\phi)}{3m_p^2}-\frac{kV(\phi)^n}{E^2}}. \tag{53}$$

For the exponential form of the potential Eq. (17), we have

$$\frac{\dot a}{a(t)}=\sqrt{\frac{V_0}{3m_p^2}}\exp(-\beta\phi/2)\times\left(1-\frac{kV_0^{n-1}3m_p^2}{E^2}\exp(-(n-1)\beta\phi)\right)^{\frac{1}{2}}. \tag{54}$$

Using the Binomial approximation in Eq. (54), we obtain

$$\frac{\dot a}{a(t)}=\left\{\sqrt{\frac{V_0}{3m_p^2}}\exp(-\beta\phi/2)-\frac{kV_0^{n-1/2}\sqrt{3m_p^2}}{2E^2}\exp[-(n-1/2)\beta\phi]\right\}. \tag{55}$$

For $n\neq 1/2$ or $\omega_\phi\neq 1/3$, the scale factor can be obtained using Eqs. (51) and (55) as

$$a(t)=a_0\exp\left\{\frac{\sqrt{\frac{V_0}{3m_p^2}}\left\{\exp\left[\frac{-\beta}{2(1-Q)}\left(\frac{\beta}{A}t+\Gamma\right)\right]-\exp\left[\frac{-\beta\Gamma}{2(1-Q)}\right]\right\}}{\frac{-\beta^2}{2A(1-Q)}}-\frac{\frac{kV_0^{n-1/2}\sqrt{3m_p^2}}{2E^2}\left\{\exp\left[-(2n-1)\frac{\beta}{2(1-Q)}\left(\frac{\beta}{A}t+\Gamma\right)\right]-\exp\left[-(2n-1)\frac{\beta\Gamma}{2(1-Q)}\right]\right\}}{\frac{-(2n-1)\beta^2}{2A(1-Q)}}\right\}. \tag{56}$$

For $n=1/2$, we have $\omega_\phi=1/3$, implying that the scalar field mimics the equation of state for radiation. In this case, Eq. (49) leads to the following form of expansion factor $a(t)$

$$a(t)=a_0\exp\left\{\frac{\sqrt{\frac{V_0}{3m_p^2}}\left\{\exp\left[\frac{-\beta}{2(1-Q)}\left(\frac{\beta}{A}t+\Gamma\right)\right]-\exp\left[\frac{-\beta\Gamma}{2(1-Q)}\right]\right\}}{\frac{-\beta^2}{2A(1-Q)}}-\frac{k\sqrt{3m_p^2}}{2E^2}t\right\}, \tag{57}$$

and the constant Γ becomes

$$\Gamma = \frac{2}{\beta}\left(1-\exp(\beta\phi_0/2)\right) - Q\phi_0. \tag{58}$$

B. Cosmic expansion for power law potential

We use the power law potential defined in Eq. (36). The functional form of $\phi(t)$ can be obtained for this potential using Eqs. (44), (20), and (13) as

$$\frac{2}{R(x+4)(-2\text{nx}+x-4)}\left[\phi^{1-x}\sqrt{\phi^2(R\phi^x-S\phi^{\text{nx}})}\left(R(-2\text{nx}+x-4)\phi^x - S(\text{nx}-x)\phi^{\text{nx}}\,{}_2F_1\left(1,\frac{3\text{nx}-2x+4}{2\text{nx}-2x};\frac{4\text{nx}-3x+4}{2\text{nx}-2x};\frac{S\phi^{\text{nx}-x}}{R}\right)\right)\right.$$
$$\left.\phi_0^{1-x}\sqrt{R\phi_0^{x+2}-S\phi_0^{\text{nx}+2}}\left(-S(\text{nx}-x)\phi_0^{\text{nx}}\,{}_2F_1\left(1,\frac{3\text{nx}-2x+4}{2\text{nx}-2x};\frac{4\text{nx}-3x+4}{2\text{nx}-2x};\frac{S\phi_0^{\text{nx}-x}}{R}\right)-R(2\text{nx}-x+4)\phi_0^x\right)\right] = -xt, \tag{59}$$

where k is the curvature parameter, $R=\frac{3V_0}{m_p^2}$, and $S=\frac{9kV_0^n}{E^2}$.

Under the slow roll approximation, the Friedman equation gives

$$a(t) = a_0\exp\left[\int_0^t\sqrt{\frac{V_0}{3m_p^2}\phi^x - \frac{kV_0^n}{E^2}\phi^{nx}\mathrm{d}t}\right]. \tag{60}$$

The analytical form of the scale factor can be obtained by substituting the field's expression from Eq. (59). The noteworthy part is the scale factor's exponential form, which indicates the universe's exponential expansion under the slow roll approximation for the tachyonic scalar field with power-law potential.

IV. PHANTOM WITH EXPONENTIAL AND POWER LAW POTENTIALS

The energy density and pressure for the phantom (negative kinetic term in quintessence) are given by

$$\rho_\phi = -\frac{1}{2}\dot\phi^2 + V(\phi), \quad p_\phi = -\frac{1}{2}\dot\phi^2 - V(\phi), \tag{61}$$

and the equation of motion for the phantom field is

$$\ddot\phi + 3H\dot\phi - V' = 0. \tag{62}$$

Under the slow roll approximation, we have

$$\dot\phi = \frac{V'(\phi)}{\sqrt{\frac{3}{m_p^2}V(\phi) - \frac{9k}{a^2(t)}}}. \tag{63}$$

A. Cosmic expansion with exponential potential

We consider the exponential potential defined in Eq. (17) for the phantom field in this section. Under the slow roll approximation, the phantom field $\phi(t)$ can be obtained as

$$\frac{2}{\beta(n-2)}\left[\sqrt{T\mathrm{e}^{\phi_0\beta}-U\mathrm{e}^{-\phi_0\beta(n-2)}}\left((n-1)\,{}_2F_1\left(1,\frac{1}{2(n-1)};\frac{n}{2(n-1)};\frac{T\mathrm{e}^{\phi_0(n-1)\beta}}{U}\right)-n+2\right)\right.$$
$$\left.+\sqrt{T\mathrm{e}^{\beta\phi}-U\mathrm{e}^{-(\beta(n-2)\phi)}}\left(-(n-1)\,{}_2F_1\left(1,\frac{1}{2(n-1)};\frac{n}{2(n-1)};\frac{T\mathrm{e}^{(n-1)\beta\phi}}{U}\right)+n-2\right)\right] = -\beta t, \tag{64}$$

where $T=\frac{3}{m_p^2V_0}$ and $U=\frac{9kV_0^{n-2}}{E^2}$.

For very small values of the phantom field, i.e., $\phi(t)\ll 1$, the solution of Eq. (64) can be approximated as

$$\sqrt{\frac{3}{m_p^2V_0}}\exp(\beta\phi/2)\sqrt{1-\frac{9kV_0^{n-2}}{E^2}\left(\frac{m_p^2V_0}{3}\right)\exp[-(n-1)\beta\phi]}\,\mathrm{d}\phi = -\beta\mathrm{d}t. \tag{65}$$

Using the Binomial expansion and slow roll approximation, the above equation takes the following form:

$$\left\{\sqrt{\frac{3}{m_p^2V_0}}\exp(\beta\phi/2)-\frac{3k\sqrt{3m_p^2}V_0^{n-3/2}}{2E^2}\exp[-(n-3/2)\beta\phi]\right\}\mathrm{d}\phi=-\beta\mathrm{d}t. \tag{66}$$

For the case of $n\neq\frac{3}{2}$, we have

$$\frac{2}{\beta}\exp(\beta\phi(t)/2)+\frac{2Q}{(2n-3)\beta}\exp[-(2n-3)\beta\phi(t)/2]=-\frac{\beta}{A}t+\frac{2}{\beta}\exp(\beta\phi_0/2)+\frac{2Q}{(2n-3)\beta}\exp[-(2n-3)\beta\phi_0/2], \tag{67}$$

where $A=\sqrt{\frac{3}{m_p^2V_0}}$, $B=\frac{3k\sqrt{3m_p^2}V_0^{n-3/2}}{2E^2}$, $Q=\frac{B}{A}$, and ϕ_0 is the value of ϕ at the epoch $t=0$.

Power series expansion of the exponential terms (with $\phi\ll 1$) provides the following form of $\phi(t)$

$$\phi(t)=\frac{1}{1-Q}\left\{-\frac{\beta}{A}t-\frac{2}{\beta}\left[1-\exp(\beta\phi_0/2)\right]-\frac{2Q}{(2n-3)\beta}\left(1-\exp[-(2n-3)\beta\phi_0/2]\right)\right\}. \tag{68}$$

Using the definition of the Hubble parameter (Eq. (13)) along with Eq. (20), we obtain

$$\frac{\dot a}{a(t)}=\left\{\sqrt{\frac{V_0}{3m_p^2}}\exp(-\beta\phi/2)-\frac{kV_0^{n-1/2}\sqrt{3m_p^2}}{2E^2}\exp(-(n-1/2)\beta\phi)\right\}. \tag{69}$$

for the phantom field with exponential potential. Using the expression of the phantom field (Eq. (68)), the last equation is rewritten as

$$\frac{\dot a}{a(t)}=\left\{\sqrt{\frac{V_0}{3m_p^2}}\exp\left[\frac{-\beta}{2(1-Q)}\left(-\frac{\beta}{A}t+\Delta\right)\right]-\frac{kV_0^{n-1/2}\sqrt{3m_p^2}}{2E^2}\exp\left[-(2n-1)\frac{\beta}{2(1-Q)}\left(-\frac{\beta}{A}t+\Delta\right)\right]\right\}, \tag{70}$$

where $\Delta=-\frac{2}{\beta}\left(1-\exp(\beta\phi_0/2)\right)-\frac{2Q}{(2n-3)\beta}\left(1-\exp(-(2n-3)\beta\phi_0/2)\right)$.

Therefore, for $n\neq 3/2$ or $\omega_\phi\neq 1/3$, Eq. (70) leads to the following form of the expansion factor:

$$a(t)=a_0\exp\left\{\frac{\sqrt{\frac{V_0}{3m_p^2}}\left\{\exp\left[\frac{-\beta}{2(1-Q)}\left(-\frac{\beta}{A}t+\Delta\right)\right]-\exp\left[\frac{-\beta\Delta}{2(1-Q)}\right]\right\}}{\frac{\beta^2}{2A(1-Q)}}\right.$$

$$\left.-\frac{\frac{kV_0^{n-1/2}\sqrt{3m_p^2}}{2E^2}\left\{\exp\left[-(2n-1)\frac{\beta}{2(1-Q)}\left(-\frac{\beta}{A}t+\Delta\right)\right]-\exp\left[-(2n-1)\frac{\beta\Delta}{2(1-Q)}\right]\right\}}{\frac{(2n-1)\beta^2}{2A(1-Q)}}\right\}. \tag{71}$$

When $n = 1/2$, we have $\omega_\phi = 1/3$, where the phantom field mimics the equation of state for radiation, and Eq. (70) gives the exponential expansion

$$a(t) = a_0 \exp\left\{ \frac{\sqrt{\frac{V_0}{3m_p^2}}\left\{\exp\left[\frac{-\beta}{2(1-Q)}\left(-\frac{\beta}{A}t+\Delta\right)\right] - \exp\left[\frac{-\beta\Delta}{2(1-Q)}\right]\right\}}{\frac{\beta^2}{2A(1-Q)}} - \frac{k\sqrt{3m_p^2}}{2E^2}t \right\}, \tag{72}$$

where Δ is a constant defined as

$$\Delta = -\frac{2}{\beta}\Big(1-\exp(\beta\phi_0/2)\Big) + \frac{Q}{\beta}\Big(1-\exp(\beta\phi_0/2)\Big). \tag{73}$$

B. Cosmic expansion with power law potential

We consider the power law form of potential in this section. Following the same procedure used in the previous section, the general time-dependent expression for the phantom field under the slow roll approximation can be obtained as

$$\frac{2}{T(x-4)}\left[\phi_0^{x-1}\left(\phi_0^{2-x}\left(T-U\phi_0^{(n-1)x}\right)\right)^{3/2} {}_2F_1\left(1,-\frac{3nx-4x+4}{2x-2nx};-\frac{2nx-3x+4}{2x-2nx};\frac{U\phi_0^{(n-1)x}}{T}\right)\right.$$
$$\left. -\phi^{x-1}\left(\phi^{2-x}\left(T-U\phi^{(n-1)x}\right)\right)^{3/2} {}_2F_1\left(1,-\frac{3nx-4x+4}{2x-2nx};-\frac{2nx-3x+4}{2x-2nx};\frac{U\phi^{(n-1)x}}{T}\right)\right] = xt, \tag{74}$$

where k is the curvature parameter, $T = \dfrac{3}{m_p^2 V_0}$, and $U = \dfrac{9kV_0^{n-2}}{E^2}$.

The expansion factor due to the phantom field with power law potential is obtained as

$$a(t) = a_0 \exp\left[\int_0^t \sqrt{\frac{V_0}{3m_p^2}\phi^x - \frac{kV_0^n}{E^2}\phi^{nx}}\mathrm{d}t\right]. \tag{75}$$

The time dependence of the scale factor can be obtained by eliminating $\phi(t)$ from Eqs. (75) and (74).

For the phantom scalar field, the equation of state is less than −1. Thus, the universe dominated by phantom fields always shows inflation. We obtained the scale factor under the slow roll approximation for the phantom field. However, this field gives the exponential expansion even without the slow roll condition owing to its equation of state being less than −1. If the phantom field interacts with background matter (baryonic and dark matter), the scale factor may change by the factor of interaction coupling strength. Furthermore, in the absence of a fundamental theory of the dark sector, it will be challenging to constrain the interaction coupling strength.

V. CONCLUSION

A mathematical analysis was performed, and the implications of three classes of the scalar field, i.e., quintessence, tachyonic, and phantom, under the slow roll approximation were investigated. We considered the two forms of potential: the exponential function given by Eq. (17) and the real power, i.e., x of the scalar field, as the power law potential given by Eq. (36). The expansion factors in the cases of the exponential and power-law potentials for quintessence are given by Eqs. (33) and (39), respectively. For these potentials, Eqs. (54) and (60) give the expansion factors of the universe dominated by a spatially homogeneous tachyonic scalar field, and Eqs. (70) and (75) give the phantom field dominated universe expansion factors. We found that the quintessence and tachyonic scalar field give the exponential expansion for both potentials. The exponential nature of the scale factor for both fields is identical to that in the case of the phantom field. Therefore, in this regard, the three scalar fields are indistinguishable under the slow roll approximation.

In reference [41], it was shown that the form of the scale factor for the known potential always emerges as exponential in the case of the spatially flat universe. Generalizingtheworktosuittheuniversewithdifferentgeometriccurvatures, we showed through our calculations that, in both the open and closed geometries, the expansion factor of the universe emerges to be of exponential form. The slow roll condition is bound to make the evolution of the universe exponential in nature, as evident from the calculations.

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