Cosmic acceleration caused by the extra-dimensional evolution in a generalized Randall-Sundrum model*

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Abstract: We investigate an $(n+1)$-dimensional generalized Randall-Sundrum model with an anisotropic metric which has three different scale factors. One obtains a positive effective cosmological constant $\Omega_{\rm eff}\sim 10^{-124}$ (in Planck units), which only needs a solution $kr\simeq 50-80$ without fine tuning. Both the visible and hidden brane tensions are positive, which renders the two branes stable. Then, we find that the Hubble parameter is close to a constant in a large region near its minimum, thus causing the acceleration of the universe. Meanwhile, the scale of extra dimensions is smaller than the observed scale but greater than the Planck length. This may suggest that the observed present acceleration of the universe is caused by the extra-dimensional evolution.

Keywords: Hubble parameter, extra dimensions, anisotropic

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1 Introduction

The current cosmic acceleration is an unexpected picture of the universe, revealed by the data sets of the last two decades from astrophysics and cosmology [1-15]. These data, which come from the cosmic microwave background radiation, supernovae surveys, baryon acoustic oscillations, etc, indicate that the universe consists of 5% ordinary baryonic matter, 27% dark matter, and 68% dark energy [6-16]. Dark energy not only has an unknown form of energy but also has not been detected directly. Additionally, dark energy is very similar to the cosmological constant which was proposed by Einstein. In Planck units, the observed value of the cosmological constant is an extravagantly tiny positive value of order 10^{-124}. This is the well-known cosmological constant fine tuning problem [17, 18]. There have been numerous attempts in order to solve this problem, such as quintessence, the anthropic principle, the $f(R)$ model, etc. [19-26]. But none of these theories are problem-free. In astrophysics and cosmology, it is still an open question.

Another perspective for resolving the problem described above, which seems to be more radical, is the following: must the dimensions of our universe be four? Are there any extra dimensions which are too small to be observed? Does the evolution of these extra dimensions contribute to the current cosmic acceleration? If so, would this help in solving the cosmological constant fine tuning problem? Therefore, we have investigated some higher-dimensional theories [27-40]. Among them, the Randall-Sundrum (RS) two-brane model [30], which has a natural solution to the hierarchy problem with warped extra dimension, has attracted our attention. The hierarchy problem is essentially a fine tuning problem that can be described as: why is there such a large discrepancy between the electroweak scale/Higgs mass $M_{EW}\sim 1$

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TeV and the Planck mass $M_{pl} \sim 10^{16}$ TeV? In the RS two-brane scenario, our universe is described by a five dimensional line element [30]

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 dy^2, \tag{1}$$

where y is the extra dimensional coordinate, r_c is the extra dimensional compactification radius, $e^{-2\sigma}$ is the well-known warp factor with the term $\sigma = kr_c|\phi|$, $k = \sqrt{-\Lambda/24M^3}$, and M is the five dimensional Planck mass. Then a large hierarchy is generated by the warp factor $e^{-2kr_c\pi}$, meanwhile one requires only $kr \approx 10$. The cosmological constant fine tuning problem is similar to the hierarchy problem. In the RS model, the visible brane is unstable, which is caused by the negative brane tension. Furthermore, the cosmological constant on the visible brane is zero, which is not consistent with our data sets of the last two decades [31, 41].

The above problems can be solved in a generalized RS braneworld scenario in which $g_{\mu\nu}$ replaces $\eta_{\mu\nu}$ in the RS model [31]. In this scenario, the tension of the visible brane and the hidden brane can both be positive with a negative induced cosmological constant. It is very interesting because both branes are stable [42-47]. In order to be consistent with the current constraints, the negative induced cosmological constant Ω should be transformed into the positive effective cosmological constant $\Omega_{\rm eff}$. This positive $\Omega_{\rm eff}$ can be obtained in an $(n+1)$-dimensional (-d) generalized RS model with two $(n-1)$-branes instead of two 3-branes [48]. In this model, adopting an anisotropic metric ansatz with two different scale factors, one obtains the positive effective cosmological constant $\Omega_{\rm eff} \sim 10^{-124}$ (in Planck units), which only needs a solution $kr \simeq 50-80$ without fine tuning. The cosmological constant fine tuning problem can be solved quite well [48].

But there is no reason to exclude the possibility of the anisotropic metric ansatz with the form of scale factors more than two. In this paper, we investigate an $(n+1)$-dimensional generalized Randall-Sundrum model with an anisotropic metric that has three different scale factors. We obtain that H_1 has a lower bound, $H_{1\min}$. Near this minimum value, the Hubble parameter is seen to be a constant in a large region, thus causing the acceleration of the universe. Meanwhile, the scale of extra dimension is smaller than the observed scale but greater than the Planck length. This may suggest that the observed present acceleration of the universe is caused by the extra-dimensional evolution rather than by dark energy. Our work is organized as follows: In Sec. 2, by considering the two $(n-1)$-branes with the matter field Lagrangian in the $(n+1)$-d generalized RS model, the n-d Einstein field equations are obtained. In Sec. 3, we focus on the evolution of a $(n+1)$-brane solved from the above field equation with an anisotropic metric ansatz that has three different scale factors. Finally, the summary and conclusion are presented in Sec. 4.

2 $(n+1)$-d generalized Randall-Sundrum model

We consider an $(n+1)$-d generalized RS braneworld model that is consistent with Ref. [48]. The action S_{n+1} is:

$$S_{n+1} = S_{\rm bulk} + S_{\rm vis} + S_{\rm hid}, \tag{2}$$

where $S_{\rm bulk}$ is the bulk action and $S_{\rm vis}$ and $S_{\rm hid}$ are the $(n-1)$-brane visible action and hidden action, respectively:

$$S_{\rm bulk} = \int d^n x dy \sqrt{-G}(M_{n+1}^{n-1}R - \Lambda), \tag{3}$$

$$S_{\rm vis} = \int d^n x \sqrt{-g_{\rm vis}}(\mathcal{L}_{\rm vis} - V_{\rm vis}), \tag{4}$$

$$S_{\rm hid} = \int d^n x \sqrt{-g_{\rm hid}}(\mathcal{L}_{\rm hid} - V_{\rm hid}), \tag{5}$$

where Λ denotes a bulk cosmological constant, M_{n+1} is the $(n+1)$-d fundamental mass scale, G_{AB} and R are the $(n+1)$-d metric tensor and Ricci scalar, respectively, $\mathcal{L}_i$ is the matter field Lagrangian of the visible and hidden branes, and V_i is the tension of the visible and hidden branes, here with $i = $ hid or vis. In this $(n+1)$-d generalized RS scenario, the metric takes the form:

$$ds^2 = G_{AB}dx^A dx^B = e^{-2A(y)}g_{ab}dx^a dx^b + r^2 dy^2, \tag{6}$$

where $e^{-2A(y)}$ is known as the warp factor, capital letter $A, B, ...$ indices run over all spacetime coordinate labels, y is the extra dimensional coordinate of length r, lowercase letter $a, b = 0, 1, 2, \cdots, n-1$ does not include the coordinate y, and g_{ab} is the n-d metric tensor. Variation with respect to the metric G_{AB} and after some easy manipulations then modulo surface terms, one obtains

$$R_{AB} - \frac{1}{2}G_{AB}R = \frac{1}{2M_{n+1}^{n-1}}\Big\{-G_{AB}\Lambda + \sum_i [T^i_{AB}\delta(y-y_i) - G_{ab}\delta^a_A\delta^b_B V_i\delta(y-y_i)]\Big\}, \tag{7}$$

where R_{AB} is the $(n+1)$ -d Ricci tensor and T^i_{AB} is the $(n+1)$-d energy-momentum tensors. Note that here the energy-momentum tensor is given by $T^{ia}_b = {\rm diag}[-c_i, c_i, \cdots, c_i]$ [47, 48]. A solution to Eq. (7) with the metric tensor in Eq. (6) has been derived in Ref. [48], and reads

$$A = -\ln[\omega\cosh(k|y| + c_-)], \tag{8}$$

where the constant $k \equiv \sqrt{-\Lambda/[M_{n+1}^{n-1}n(n-1)]} \simeq$ Planck mass. ω is given by

$$\omega \equiv \sqrt{\frac{-2\Omega}{(n-1)(n-2)k^2}}, \tag{9}$$

and the term c_- takes the form

$$c_- \equiv \ln \frac{1-\sqrt{1-\omega^2}}{\omega}. \tag{10}$$

Meanwhile, an n-d Einstein field equation can be obtained:

$$\widetilde{R}_{ab} - \frac{1}{2} g_{ab} \widetilde{R} = -\Omega g_{ab}, \tag{11}$$

where Ω is the induced cosmological constant on the visible brane, and $\widetilde{R}$ and $\widetilde{R}_{ab}$ are the n-d Ricci scalar and Ricci tensor, respectively,

Note that the solution derived above has the negative induced cosmological constant Ω. Here we do not consider the situation where $\Omega > 0$, since the tension on the visible brane is negative, which results in instability [31, 41, 47, 48]. Thus, an anisotropic metric is assumed to be of the following form [47-49]:

$$g_{ab} = \mathrm{diag}[-1, a_1^2(t), a_2^2(t), a_3^2(t), \cdots, a_{n-1}^2(t)], \tag{12}$$

where a_i is the scale factor. The case where the scale factors on the visible brane evolve with two different rates has been studied recently [48]. In this case, we can obtain the positive effective cosmological constant $\Omega_{\rm eff} \simeq 10^{-124}$ and only requiring $kr \simeq 50-80$, where for convenience, the Planck mass has been set to unity. Thus, the cosmological constant fine tuning problem can be solved quite well. Furthermore, the three dimensional (3D) Hubble parameter $H(z)$ is consistent with the cosmic chronometers dataset extracted from [6-15]. The observed 3D universe naturally shifts from deceleration expansion to accelerated expansion. This shows that the accelerated expansion of the observed universe is intrinsically an extra-dimensional phenomenon. But there is no reason to make the scale factor evolve with only two kinds of rates. Therefore, we investigate the case where the scale factors on the visible brane evolve with three different rates.

3 Anisotropic evolution of $(n-1)$-brane

For the anisotropic metric Eq. (12) with three kinds of scale factors and the negative induced cosmological constant $\Omega \sim -10^{-124}$, the field equations (11) can be written:

$$\sum_i n_i(n_i-1)H_i^2 + \sum_{i\neq j} n_i n_j H_i H_j = 2\Omega, \tag{13}$$

$$\sum_i n_i \dot{H}_i - \dot{H}_1 + \left(\sum_i n_i H_i\right)^2 - H_1 \sum_i n_i H_i = 2\Omega, \tag{14}$$

$$\sum_i n_i \dot{H}_i - \dot{H}_2 + \left(\sum_i n_i H_i\right)^2 - H_2 \sum_i n_i H_i = 2\Omega, \tag{15}$$

$$\sum_i n_i \dot{H}_i - \dot{H}_3 + \left(\sum_i n_i H_i\right)^2 - H_3 \sum_i n_i H_i = 2\Omega, \tag{16}$$

where $i = 1,2,3$, and the terms n_1, n_2, and n_3 are the number of dimensions which evolve with three kinds of rates, respectively, the Hubble parameter $H \equiv \dot{a}/a$, and $\dot{H}_i$ is the first time derivative of H_i. Computing the sum of Eqs. (14), (15), and (16) yields a simplified expression for $\sum_i n_i H_i$:

$$\sum_i n_i H_i = -\chi_1 \tan\beta, \tag{17}$$

where the term $\beta = \chi_1 t + \theta_0$, θ_0 is the initial phase angle which is determined by the scale of the formation of the brane, and the term χ_1 takes the form

$$\chi_1 = \sqrt{\frac{-2(n-1)\Omega}{n-2}}. \tag{18}$$

It is convenient to redefine the sum of the Hubble parameters in the following manner:

$$\sum_i n_i H_i \equiv f. \tag{19}$$

Replacing Eqs. (17) and (19) into Eq. (14), then performing some manipulations, one obtains

$$\dot{H}_1 + H_1 f = \dot{f} + f^2 - 2\Omega. \tag{20}$$

The solution of the above equation is

$$H_1 = \mathrm{e}^{-\int f \mathrm{d}t} \left[\int (\dot{f} + f^2 - 2\Omega) \mathrm{e}^{\int f \mathrm{d}t} \mathrm{d}t + c \right], \tag{21}$$

where c is an integration constant. Combining Eqs. (17) and (19), H_1 is given by

$$H_1 = -\frac{\chi_1}{n-1} \tan\beta + c \sec\beta. \tag{22}$$

Using Eqs. (19) and (20), Eq. (13) can be rewritten as

$$n_2 H_2^2 + n_3 H_3^2 = -n_1 H_1^2 + f^2 - 2\Omega. \tag{23}$$

Eqs. (22) and (23) could be combined to give the following equation, eliminating H_2 completely:

$$\left(n_3 + \frac{n_3^2}{n_2}\right) H_3^2 + \frac{2n_3}{n_2}(n_1 H_1 - f) H_3 + \left[\left(\frac{1}{n_2} - 1\right) f^2 + \frac{n_1^2}{n_2} H_1^2 - \frac{2n_1}{n_2} f H_1 + n_1 H_1^2 + 2\Omega\right] = 0. \tag{24}$$

Then the Hubble parameter H_3 can be obtained:

$$H_3 = \frac{f - n_1 H_1}{n_2 + n_3} - \frac{1}{n_2 + n_3}\left[\frac{n_2}{n_3}(n_2 + n_3 - 1)f^2 - \frac{n_1 n_2}{n_3}(n-1)H_1^2 + \frac{2n_1 n_2}{n_3} f H_1 - \frac{2n_2}{n_3}(n_2 + n_3)\Omega\right]^{1/2}. \tag{25}$$

Combining Eqs. (17), (19), and (22), H_3 is given by

$$H_3 = -\frac{\chi_1}{n-1}\tan\beta - \frac{\chi_3 + n_1 c}{n_2 + n_3}\sec\beta, \tag{26}$$

where the term χ_3 is

$$\chi_3 = \sqrt{-2\frac{n_2}{n_3}(n_2 + n_3)\Omega - n_1\frac{n_2}{n_3}(n-1)c^2}. \tag{27}$$

Note here that we have chosen H_2 to always be greater than H_3. Finally, using Eqs. (17) and (26), one can also obtain the Hubble parameter H_2 :

$$H_2 = -\frac{\chi_1}{n-1}\tan\beta + \frac{\chi_2 - n_1 c}{n_2 + n_3}\sec\beta, \tag{28}$$

where the term χ_2 is

$$\chi_2 = \sqrt{-2\frac{n_3}{n_2}(n_2 + n_3)\Omega - n_1\frac{n_3}{n_2}(n-1)c^2}. \tag{29}$$

It is easy to see that the integration constant c is constrained by Eqs. (27) and (29). The constant c must be set to a value less than $c_{\max}$ to guarantee that the value in the root is greater than zero, which yields

$$c \leqslant \sqrt{\frac{-2\Omega(n_2 + n_3)}{n_1(n-1)}} \equiv c_{\max}, \tag{30}$$

where $c_{\max}$ is the maximum value of c. It is evident from Eqs. (26), (27), (28), and (29) that, if $c = c_{\max}$, H_2 is equal to H_3. For convenience, we define a parameter d satisfying

$$c = d c_{\max}, \tag{31}$$

where the value of d is between 0 and 1. First, we investigate the effect of parameter d on the Hubble parameters H. We choose $n_1 = 3$, which is most in line with the presently observed three dimensional (3D) space. Further setting $n_2 = 1$ and $n_3 = 1$, we plot the Hubble parameters H_1, H_2, and H_3 as a function of β in Fig. 1 with $d = 0$, $d = 0.5$, and $d = 1$ respectively. In Fig. 1(a)-(c), the three curves having same type (color) correspond to three different values of d, respectively. In Fig. 1(a), we plot the Hubble parameter H_1 as a function of β. When the parameter is $d = 0$ or $d = 0.5$, the Hubble parameter H_1 monotonically decreases with β. And when

$$d > \sqrt{n_1/[(n_2 + n_3)(n-2)]} \equiv d_{\min} = \sqrt{6}/4 \approx 0.612$$

for $n_1 = 3$, $n_2 = 1$, and $n_3 = 1$, the Hubble parameter H_1 has a minimum

$$H_{1\min} = \frac{\sqrt{c^2(n-1)^2 - \chi_1^2}}{n-1}, \tag{32}$$

when the term β in Eq. (22) takes the form

$$\beta_{\min} = \arcsin\left[\frac{\chi_1}{c(n-1)}\right]. \tag{33}$$

If $\beta \leq \beta_{\min}$, the Hubble parameter H_1 is a monotonic function decreasing with time, which is not in accordance with cosmological observations and experiments. As can be seen easily from Fig. 1(a), the the Hubble parameter H_1 has a minimum when $d = d_{\min} \approx 0.612$. This case may be consistent with the present observations because H_1

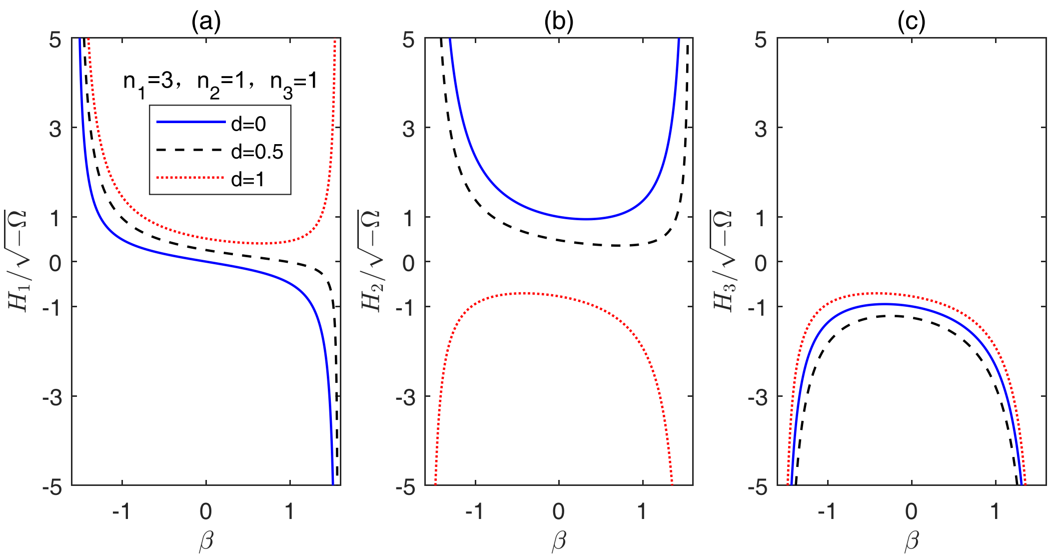

Fig. 1. (color online) The Hubble parameters H_1 ($n_1 = 3$), H_2 ($n_2 = 1$), and H_3 ($n_3 = 1$), with three different parameter values d. (a) The Hubble parameters H_1 with $d = 0, 0.5, 1$ correspond to the solid (blue) curve, the dashed (black) curve, and the dotted curve (red) respectively ($n_2 = 1$). (b)-(c) The case for the Hubble parameters H_2 and H_3 , respectively.

tends to a constant near the minimum, which can lead to accelerated expansion without the contribution of dark energy (or an inflaton field).

Combining Eqs. (18), (22), (28) and (29), we obtain $H_1 = H_2$ if c satisfies

$$c = \sqrt{\frac{-2\Omega n_3}{(n-1)(n_1+n_2)}} \equiv c_{eq}. \tag{34}$$

It is shown that H_2 tends toward H_1 when $c \to c_{eq}$. In the case $c = c_{eq}$, the parameter d_{eq} is defined as:

$$d_{eq} = \frac{c_{eq}}{c_{\max}} = \sqrt{\frac{n_1 n_3}{(n_1+n_2)(n_2+n_3)}}. \tag{35}$$

For $n_1 = 3$, $n_2 = 1$, and $n_3 = 1$, $d_{eq} = \sqrt{6}/4 \approx 0.612$. Since the extra dimension in our universe is not observed presently, it cannot be too large, and the current scale of extra dimensions is still outside the observable range. When d is equal to d_{eq}, the extra dimension Hubble parameter H_2 is converted to H_1. This case is inconsistent with the presently observed 3D space. The Hubble parameter H_2 of the extra dimensions is plotted in Fig. 1(b) for $d = 0, 0.5, 1$. It can be easily seen that the Hubble parameter H_2 has a minimum with $d = 0$ and 0.5, which can lead to accelerated expansion of the extra dimensions. We are not interested in this situation because it is not in line with observation. However, the Hubble parameter H_2 is always negative with $d = 1$, which ensures that the extra dimensions always exceed the observable range. Note here that the ratio of $d_{\min}$ to d_{eq} is

$$\frac{d_{\min}}{d_{eq}} = \sqrt{\frac{(n_1+n_2)}{n_3(n_1+n_2+n_3-1)}} \leqslant 1, \tag{36}$$

where $d_{\min}/d_{eq} = 1$ if and only if $n_3 = 1$. So we only consider the case $d \gg d_{eq}$ because then we obtain $d \geqslant d_{\min}$ when $d \geqslant d_{eq}$.

In Fig. 2, we have plotted the parameter d_{eq} versus the number of extra dimensions n_2 and n_3, respectively. The figure on the left is the curve of the parameter d_{eq} with n_3 when $n_2 = 1$, 5, and 30. It is shown that d_{eq}

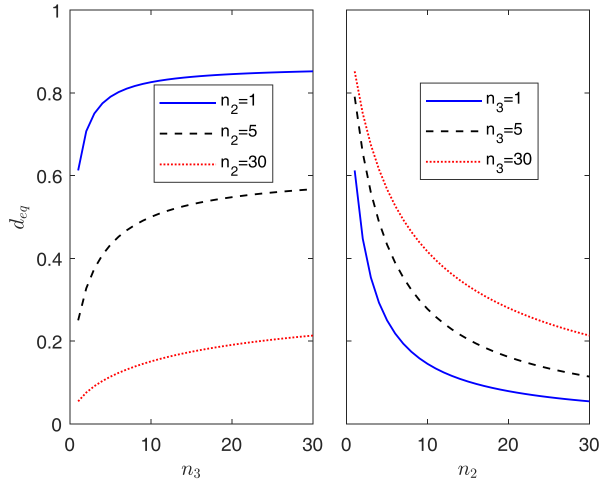

Fig. 2. (color online) The parameter d_{eq} versus the number of extra dimensions n_2 and n_3, respectively. The figure on the left is the curve of the parameter d_{eq} with n_3 when $n_2 = 1, 5, 30$, and the figure on the right depicts the evolution of d_{eq} with n_2 when $n_3 = 1, 5, 30$.

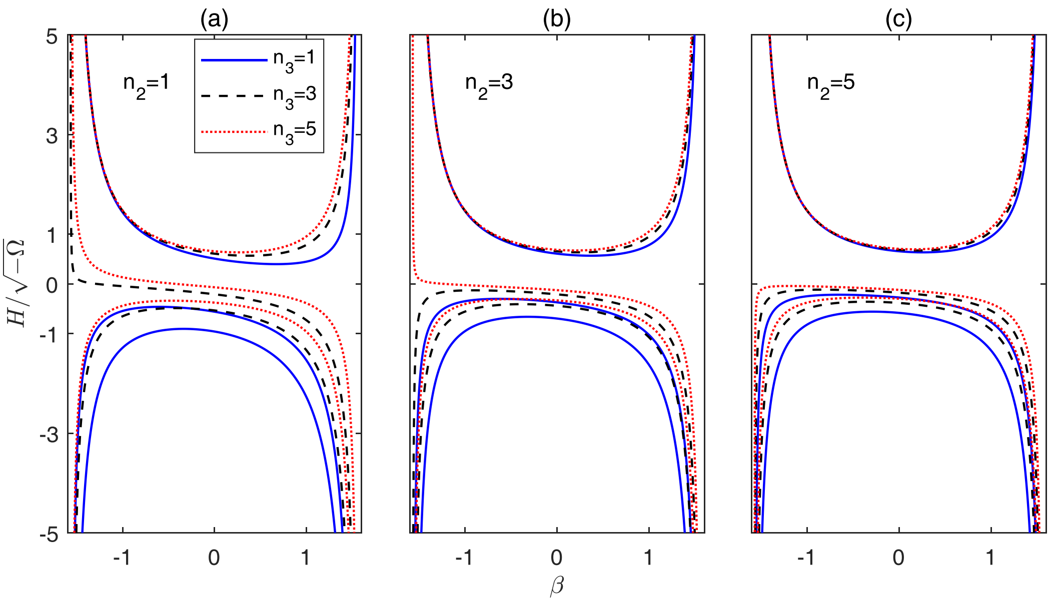

Fig. 3. (color online) The Hubble parameters H_1 ($n_1 = 3$), H_2, and H_3 when $d = 0.98$. (a) The Hubble parameters with $n_3 = 1, 3, 5$ correspond to the solid (blue) curve, the dashed (black) curve and the dotted curve (red) respectively ($n_2 = 1$). (b)-(c) The cases for $n_2 = 3$ and $n_2 = 5$, respectively.

monotonically increases with n_3. The parameter d_{eq} tends toward $\sqrt{n_1/(n_1+n_2)}$ in the $n_3\to\infty$ limit. In particular, $d_{eq}\to\sqrt{3}/2\approx 0.866$ for $n_2=1$. The figure on the right depicts the evolution of d_{eq} with n_2 when $n_3=1$, 5, and 30. In this case, d_{eq} monotonically decreases with n_2. The parameter $d_{eq}\to 0$ in the $n_2\to\infty$ limit. To be consistent with observation, we should set d to be greater than 0.866 and closer to 1. Furthermore, we set the constant $d=0.98$ in the following.

In Fig. 3(a)-(c), we plot the Hubble parameters H_1, H_2 and H_3 as a function of β with $n_1=3$ when $d=0.98$. From top to bottom, the three curves having same type (color) corresponds to H_1, H_2, and H_3, respectively, for fixed values of n_2 and n_3. In Fig. 3(a), we plot the Hubble parameter as a function of β at $n_2=1$. From top to bottom, the three solid curves correspond to H_1, H_2, and H_3 at $n_3=1$. The three dashed curves ($n_3=3$) and the three dotted curves ($n_3=5$) are similar to the above case. The Hubble parameter of the extra dimensions is always negative at $n_3=1$. With the increase of n_3, the coordinate of the minimum value of H_1 tends toward $\beta=0$. Meanwhile, H_2 changes from positive to negative in the region near $\beta\sim -1.5$ and H_3 is closer to zero. In Fig. 3(b) and (c), the Hubble parameters as a function of β are shown with $n_2=3$ and $n_2=5$. H_2 is always negative with $n_2=5$, which is similar to the situation in Fig. (1) of Ref. [48]. H_1 has a lower bound $H_{1\min}=\sqrt{0.98^2c_{\max}^2-\chi_1^2/(n-1)^2}$ when $\beta_{\min}=\arcsin\{\eta_1/[0.98(n-1)c_{\max}]\}$. In the region near $\beta_{\min}$, we find that $\Omega_{\rm eff}>0$ is of order $-\Omega$. This situation is similar to the case with two different scale factors, and the negative induced cosmological constant Ω can be transformed into the positive effective cosmological constant $\Omega_{\rm eff}$. This tells us that the observed current cosmic acceleration is intrinsically an extra-dimensional phenomenon rather than dark energy. The cosmological constant fine tuning problem can be solved by this extra-dimensional evolution.

From Eqs. (22), (26), and (28) we can get the scale factors a_1, a_2, and a_3 of the form

$$a_1=a_{10}|\cos\beta|^{\frac{1}{n-1}}|\sec\beta+\tan\beta|^{\frac{c}{\chi_1}},\tag{37}$$

$$a_2=a_{20}|\cos\beta|^{\frac{1}{n-1}}|\sec\beta+\tan\beta|^{\frac{\chi_2-n_1c}{\chi_1(n_2+n_3)}},\tag{38}$$

$$a_2=a_{30}|\cos\beta|^{\frac{1}{n-1}}|\sec\beta+\tan\beta|^{-\frac{\chi_3+n_1c}{\chi_1(n_2+n_3)}},\tag{39}$$

where a_{10}, a_{20}, and a_{30} are the scale factors when the brane forms. Further, since the volume of the visible brane is obtained by

$$V_b=a_1^{n_1}a_2^{n_2}a_3^{n_3}=a_{10}^{n_1}a_{20}^{n_2}a_{30}^{n_3}\cos\beta,\tag{40}$$

when the brane is just forming, there is no particular reason to make the scale factor different, so we choose $a_{10}=a_{20}=a_{30}$. If the initial scale of the brane is of order 10^{35} in Planck units, and considering the presently observed scale of our universe (of approximate order 10^{61}), we obtain the scale of extra dimension which is at least of order 10^{22} with $n_2=n_3=3$. It can be shown that the scale of the extra dimension should be much larger than the Planck length. This ensures that physics is still valid in the evolution of the visible brane. The θ_0 we obtain is close to $-\pi/2$ if one wants to have a sufficiently small initial scale. So in the region of $\theta_0+\pi/2\ll\eta_1 t\ll\pi/2$, the Hubble parameter H_1 is of the form:

$$\begin{aligned}H_1&\simeq\left[\frac{c}{\chi_1}+\frac{1}{n-1}\right]\frac{1}{t}\\&=\frac{1}{n-1}\left[1+d\sqrt{\frac{(n_2+n_3)(n-2)}{n_1}}\right]\frac{1}{t}.\end{aligned}\tag{41}$$

When $n_2=n_3=1$ and $d=0.98$, H_1 is about $0.52t$, which is as similar to the radiation dominant era. In the limit $n_2\to\infty$ (or $n_3\to\infty$), $H_1\simeq\sqrt{3}d/3t$.

4 Summary and conclusion

In conclusion, we investigate an $(n+1)$-d generalized Randall-Sundrum model with an anisotropic metric that has three different scale factors. In this model, we obtain the positive effective cosmological constant $\Omega_{\rm eff}\sim 10^{-124}$ (in Planck unit), which only needs a solution $kr\simeq 50-80$ without fine tuning. This is consistent with the case of two different scale factors.

In this model, the Hubble parameter H_2 tends toward H_1 when the integration constant d tends toward d_{eq}. This indicates that the Hubble parameter of observable dimensions is related to the value of the integral constant d. For convenience, here we have selected the Hubble parameter H_1 to show the observable dimensions. To be consistent with observation, we should set d to be greater than 0.866 and closer to 1. Further setting the constant $d=0.98$, we obtain that H_1 has a lower bound $H_{1\min}=\sqrt{0.98^2c_{\max}^2-\chi_1^2/(n-1)^2}$ when $\beta_{\min}=\arcsin\{\eta_1/[0.98(n-1)c_{\max}]\}$. Meanwhile, the scale of extra dimension is smaller than the observed scale but greater than the Planck length. This may suggest that the observed current cosmic acceleration is caused by extra-dimensional evolution rather than dark energy (or an inflaton field). Of course, there are still many problems to be solved in this model. This includes the question of how the quantum fluctuations contribute to the amount of the expected value of the cosmological constant.

References

1 A. G. Riess *et al*., Astron. J., **116**: 1009 (1998)
2 S. Perlmutter *et al*., Astrophys. J., **517**: 565 (1999)
3 C. L. Bennett *et al*., Astrophys. J. Suppl. Ser., **148**: 1 (2013)
4 C. B. Netterfield *et al*., Astrophys. J., **571**: 604 (2002)
5 N. W. Halverson *et al*., Astrophys. J., **568**: 38 (2002)
6 J. Magaña, M. H. Amante, M. A. Garcia-Aspeitia *et al*., Monthly Notices of the Royal Astronomical Society, **476**(1): 1036-1049 (2018)
7 A. Gómez-Valent and L. Amendolab, JCAP, **04**: 051 (2018)
8 C. Zhang, H. Zhang, S. Yuan *et al*., Res. Astron. Astrophys., **14**: 1221 (2014)
9 R. Jiménez, L. Verde, T. Treu *et al*., Astrophys. J., **593**: 622 (2003)
10 J. Simon, L. Verde, and R. Jiménez, Phys. Rev. D, **71**: 123001 (2005)
11 M. Moresco *et al*., JCAP, **08**: 006 (2012)
12 M. Moresco *et al*., JCAP, **05**: 014 (2016)
13 A.L. Ratsimbazafy *et al*., Mon. Not. Roy. Astron. Soc., **467**: 3239 (2017)
14 D. Stern, R. Jiménez, L. Verde *et al*., JCAP, **02**: 008 (2010)
15 M. Moresco, Mon. Not. Roy. Astron. Soc., **450**: L16 (2015)
16 P. J. E. Peebles and B. Ratra, Rev. Mod. Phys., **75**: 559 (2003)
17 S. Weinberg, Rev. Mod. Phys., **61**: 1 (1989)
18 L. Amendola and S. Tsujikawa, *Dark energy: theory and observations*, (Cambridge University Press, 2010)
19 Peebles, P. J. E., and B. Ratra, Astrophys. J., **325**: L17 (1988)
20 C. Wetterich, Nucl. Phys. B, **302**: 668 (1988)
21 I. Zlatev, L. M. Wang, and P. J. Steinhardt, Phys. Rev. Lett., **82**: 896 (1999)
22 R. R. Caldwell, Phys. Lett. B, **545**: 23 (2002)
23 B. Feng, X. L. Wang, and X. M. Zhang, Phys. Lett. B, **607**: 35 (2005)
24 T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys., **82**: 451 (2010)
25 G. R. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B, **485**: 208 (2000)
26 R. Bousso and J. Polchinski, J. High Energy Phys., **0006**: 006 (2000)
27 Th. Kaluza, Sitzungseber. Press. Akad. Wiss. Phys. Math. Klasse 996 (1921); O. Klein, Z. Phys., **37**: 895 (1926); Nature (London), **118:** 516 (1926)
28 N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B, **429**: 263 (1998)
29 N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D, **59**: 086004 (1999)
30 L. Randall and R. Sundrum, Phys. Rev. Lett., **83**: 3370 (1999)
31 S. Das, D. Maity, and S. Sengupta, J. High Energy Phys., **05**: 042 (2008)
32 I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos *et al*., Phys. Lett. B, **436**: 257 (1998)
33 A. Das, D. Maity, T. Paul *et al*., Eur. Phys. J. C, **77**: 813 (2017)
34 R. Sundrum, Phys. Rev. D, **59**: 085009 (1999)
35 J. Lykken and L. Randall, J. High Energy Phys., **06**: 014 (2000)
36 I. Antoniadis, Phys. Lett. B, **246**: 377 (1990)
37 L. Visinelli, N. Bolis, and S. Vagnozzi, Phys. Rev. D, **97**: 064039 (2018)
38 S. Vagnozzi and L. Visinelli, arXiv: 1905.12421
39 T. Paul and S. SenGupta, arXiv: 1808.00172v1
40 J. Polchinski, String Theory. Vol. 2: Superstring theory and beyond, (Cambridge University Press, 1998)
41 R.Koley, J. Mitra, and S. SenGupta, Phys. Rev. D, **79**: 041902(R) (2009)
42 J. Mitra, T. Paul, and S. SenGupta, Eur. Phys. J. C, **77**: 833 (2017)
43 S. Chakraborty and S. SenGupta, Eur. Phys. J. C, **75**(11): 538 (2015)
44 I. Banerjee, S. Chakraborty, and S. SenGupta, Phys. Rev. D, **99**: 023515 (2019)
45 S. Chakraborty and S. SenGupta, Phys. Rev. D, **92**: 024059 (2015)
46 S. Chakraborty, and S. SenGupta, Eur. Phys. J. C, **76**: 552 (2016)
47 G.-Z Kang, D.-S. Zhang, L. Du *et al*., Chin. Phys. C, **43**(9): 095101 (2019)
48 G.-Z Kang, D.-S. Zhang, H.-S. Zong *et al*., arXiv: 1906.05442
49 C. A. Middleton and E. Stanley, Phys. Rev. D, **84**: 085013 (2011)