D^0-$\overline{D}^0$ mixing parameter y in the factorization-assisted topological-amplitude approach*

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Abstract: We calculate the D^0-$\overline{D}^0$ mixing parameter y in the factorization-assisted topological-amplitude (FAT) approach, considering contributions from $D^0 \to$ PP, PV, and VV modes, where P (V) stands for a pseudoscalar (vector) meson. The $D^0\to$PP and PV decay amplitudes are extracted in the FAT approach, and the $D^0\to$VV decay amplitudes with final states in the longitudinal polarization are estimated via the parameter set for $D^0\to$PV. It is found that the VV contribution to y, being of order of 10^{-4}, is negligible, and that the PP and PV contributions amount only up to $y_{\rm PP+PV}=(0.21\pm0.07)\%$, a prediction more precise than those previously obtained in the literature, and much lower than the experimental data $y_{\rm exp}=(0.61\pm0.08)\%$. We conclude that D^0 meson decays into other two-body and multi-particle final states are relevant to the evaluation of y, so it is difficult to understand it fully in an exclusive approach.

Keywords: mixing of charmed mesons, strong interactions, weak interactions, decays of charmed mesons

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1 Introduction

Studies of neutral meson mixings have marked glorious progress in particle physics: kaon mixing led to the first CP violation observed in the $K_L\to\pi\pi$ decays [1]; the masses of the charm quark [2] and top quark [3, 4] were, before their discoveries, estimated through the GIM mechanism involved in kaon and B_d meson mixings, respectively. The neutral meson mixings are still a potential regime for searching for new physics nowadays, because the relevant flavor-changing amplitudes are loop-suppressed in the Standard Model. To get closer to this goal, it is crucial to understand the mixing dynamics to high precision. The $B_{d(s)}$ meson mixing is well described in the heavy quark effective theory [5, 6], indicating that both the power expansion parameter $1/m_b$ and the strong coupling $\alpha_s(m_b)$ at the scale of the bottom quark mass m_b are small enough to justify a perturbative analysis. However, understanding D^0-$\overline{D}^0$ mixing has remained a challenge since its first observation [7–9]. It is suspected that $1/m_c$ and $\alpha_s(m_c)$, with m_c being the charm quark mass, may be too large to allow perturbative expansion.

The products $V_{ib}V_{id}^*$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, i=u, c, and t, which appear in the box diagram responsible for the B_d meson mixing, are of the same order. In the B_s meson mixing, $V_{tb}V_{ts}^*$ and $V_{cb}V_{cs}^*$ are of the same order, and both much larger than $V_{ub}V_{us}^*$. Hence, an intermediate top quark with a much higher mass moderates the GIM can-

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cellation, giving a dominant contribution to the bottom mixing. In the D^0-$\overline{D}^0$ mixing an intermediate bottom quark does not play an important role due to the tiny product $V_{cb}V_{ub}^*$. The charm mixing is then governed by the difference between the other two intermediate quarks s and d, namely, by $SU(3)$ symmetry breaking effects, to which the nonperturbative contribution is expected to be significant.

The current world averages of the charm mixing parameters are given by [10]

$$x=(0.46^{+0.14}_{-0.15})\%, \qquad y=(0.62\pm0.08)\%, \tag{1}$$

assuming no CP violation in charm decays[1)]. There are two approaches in the literature, inclusive and exclusive, for the evaluation of the charm mixing parameters. The former, with short-distance contributions calculated based on the heavy quark expansion, leads to values of x and y two or three orders of magnitude lower than the data, even after the operators of dimension nine [12, 13] or both α_s and subleading $1/m_c$ corrections [12] are taken into account. Obviously, the mass difference between the s and d quarks cannot collect all $SU(3)$ breaking effects in charm decays, which may instead originate mainly from hadronic final states [14]. This speculation is supported by the argument [15] that a modest quark-hadron duality violation of about 20% explains the discrepancy between inclusive predictions and the data.

Contributions to the charm mixing from individual intermediate hadronic channels are summed up in an exclusive approach. It was noticed [16, 17] that the $SU(3)$ breaking effects only from the phase space naturally induce x and y at the order of one percent, but are hard to predict quantitatively. In a qualitative analysis based on U-spin and its breaking [18], it was found that contributions from two-body decays might be small, and four-body decays may lead to y at the measured level. The only quantitative study in the literature was given in the topological diagrammatic approach [19], showing that the $D\to PP$ and PV decays contribute to y at the order of 10^{-3}: $y_{PP}=(0.86\pm0.41)\times10^{-3}$, $y_{PV}=(2.69\pm2.53)\times10^{-3}$ $(A,A1)$ and $y_{PV}=(1.52\pm2.20)\times10^{-3}$ $(S,S1)$ from two different solutions. The uncertainties of the predictions are too large to give a definite conclusion. With abundant data collected on two-body D meson decays [20], it is now likely that a better control on $SU(3)$ breaking effects can be obtained [21, 22], and that the mixing parameter y can be analyzed precisely in an exclusive way.

In this paper we will address this issue in the factorization-assisted topological-amplitude (FAT) approach [21, 22], which provides a more precise treatment of the $SU(3)$ breaking effects from two-body hadronic D meson decays, as indicated by the improved global fit to the measured branching ratios compared to Ref. [19]. Distinct from the traditional diagrammatic approach based on the $SU(3)$ symmetry [19, 23, 24], the $SU(3)$ breaking effects in phase space, decay constants, form factors, and strong phases associated with various final states are captured in the FAT approach. It is well known that the $SU(3)$ breaking effects in the singly Cabibbo-suppressed modes are significant. For instance, the ratio of the $D^0\to K^+K^-$ and $\pi^+\pi^-$ branching fractions should be unity in the limit of $SU(3)$ symmetry, but is measured to be about 2.8. This approach has been successfully applied to studies of the $D\to PP$ [21] and $D\to PV$ [22, 25] decays, including all the Cabibbo-favored, singly Cabibbo-suppressed, and doubly Cabibbo-suppressed modes, as well as the charmed [26] and charmless [27, 28] B meson decays. In particular, the predicted difference of the direct CP asymmetries $\Delta a_{CP}^{dir}\equiv a_{CP}^{dir}(K^+K^-)-a_{CP}^{dir}(\pi^+\pi^-)=(-0.6\sim-1.9)\times10^{-3}$ was later confirmed by the LHCb data, $\Delta a_{CP}^{dir}=(-0.61\pm0.76)\times10^{-3}$ [29].

It is expected that the contributions from two-body D meson decays to the $D^0-\overline{D}^0$ mixing can be properly addressed in the FAT approach. The $D^0\to PP$ and PV decay amplitudes required for the evaluation of the mixing parameter y are extracted in the FAT approach. The $D^0\to VV$ decay amplitudes with final states in the longitudinal polarization are estimated via the parameter set for $D^0\to PV$, which does yield corresponding branching ratios in agreement with data. We will show that the $D^0\to PP$, PV and VV channels contribute $y_{PP}=(0.10\pm0.02)\%$, $y_{PV}=(0.11\pm0.07)\%$, and $y_{VV}\sim10^{-4}$, respectively, to the mixing parameter, with small uncertainties. Namely, the above two-body channels alone, which take up about 50% of the total D^0 meson decay rate, cannot explain the D^0-$\overline{D}^0$ mixing in an exclusive approach. Therefore, other two-body and multi-particle hadronic D meson decays are relevant to the calculation of y. These are, however, extremely difficult to analyze in an exclusive approach at the current stage. A new strategy to understand charm mixing dynamics is necessary.

In Section 2 we update the determination of the $D^0\to PP$ and PV amplitudes by performing a global fit to the latest data of the branching ratios in the FAT approach. Their contributions to the charm mixing parameter y are then obtained. The $D^0\to VV$ amplitudes for the longitudinal polarization are estimated via the

1) As CP violation is allowed, the mixing parameters turn into [10, 11]

$$x=(0.32\pm0.14)\%, \qquad y=(0.69^{+0.06}_{-0.07})\%. \tag{2}$$

parameter set for the PV modes in Section 3, and found to give a small contribution to y. Section 4 contains the summary.

2 $y_{\rm PP}$ and $y_{\rm PV}$

The $\rm D^0$-$\rm \overline{D}^0$ mixing parameter y is defined by

$$y \equiv \frac{\Gamma_1 - \Gamma_2}{2\Gamma}, \tag{3}$$

where $\Gamma_{1,2}$ represent the widths of the mass eigenstates $\rm D_{1,2}$, and $\Gamma = (\Gamma_1 + \Gamma_2)/2$. In the assumption of CP conservation, the mass eigenstates are identical to the CP eigenstates, *i.e.*, $|\rm D_1\rangle = |D_+\rangle$ and $|\rm D_2\rangle = |D_-\rangle$, with $|\rm D_\pm\rangle = (|D^0\rangle \pm |\overline{D}^0\rangle)/\sqrt{2}$. Here we adopt the convention of $\mathcal{CP}|\rm D^0\rangle = +|\overline{D}^0\rangle$. The parameter y can be computed via the formula

$$\begin{aligned} y &= \frac{1}{2\Gamma}\sum_n \rho_n (|\mathcal{A}(\rm D_+ \to n)|^2 - |\mathcal{A}(D_- \to n)|^2) \\ &= \frac{1}{\Gamma}\sum_n \eta_{\rm CP}(n)\rho_n \mathcal{R}e[\mathcal{A}(\rm D^0 \to n)\mathcal{A}^*(D^0 \to \bar{n})], \end{aligned} \tag{4}$$

in which ρ_n is the phase-space factor for the $\rm D^0/\overline{D}^0$ decay into the final state n, and the transformation $\mathcal{CP}|n\rangle = \eta_{\rm CP}|\bar{n}\rangle$ has been applied. For the PP and PV modes, $\eta_{\rm CP} = +1$, and for the VV modes, $\eta_{\rm CP} = (-1)^L$, with L denoting the orbital angular momentum of the final state. The following expression is also employed in the literature [19],

$$y = \sum_n \eta_{\rm CKM}(n)\eta_{\rm CP}(n)\cos\delta_n \sqrt{\mathcal{B}(\rm D^0 \to n)\mathcal{B}(D^0 \to \bar{n})}, \tag{5}$$

where δ_n is the relative strong phase between the $\rm D^0 \to n$ and $\rm D^0 \to \bar{n}$ amplitudes, and $\eta_{\rm CKM} = (-1)^{n_s}$, with n_s being the number of s or $\bar{\rm s}$ quarks in the final state.

The FAT approach is based on the factorization of short-distance and long-distance dynamics in the topological amplitudes for D meson decays into Wilson coefficients and hadronic matrix elements of effective operators, respectively. The relevant tree-level topological amplitudes include the color-favored tree-emission diagram T, the color-suppressed tree-emission diagram C, the W-exchange diagram E, and the W-annihilation diagram A. The hadronic matrix elements are partly computed in the naive factorization with nonfactorizable contributions being parameterized into strong parameters. A D meson decay amplitude is then decomposed into these topological diagrams, each of which further takes into account channel-dependent $SU(3)$ symmetry breaking effects. Through a global fit to the abundant decay-rate data, the strong parameters are determined and can be used to make predictions for unmeasured branching ratios and CP asymmetries. The resultant channel-dependent phases will be employed for the evaluation of y here. It is noticed that the W-exchange diagram E appears only in $\rm D^0$ meson decays, while the W-annihilation diagram A contributes only to $\rm D^+$ and $\rm D_s^+$ meson decays. For the study of $\rm D^0$-$\rm \overline{D}^0$ mixing, we focus on the $\rm D^0$ meson decay modes, so that the irrelevant strong parameters associated with the amplitudes A can be removed from the global fits.

For the explicit parametrizations of the $\rm D \to PP$ and PV amplitudes in the FAT approach, we refer to Refs. [21] and [22], respectively. Below we update the sets of strong parameters determined by the latest data:

$$\begin{aligned} &\chi^C = -0.81 \pm 0.01, \quad \phi^C = 0.22 \pm 0.14, \quad S_\pi = -0.92 \pm 0.07, \\ &\chi^E_{\rm q} = 0.056 \pm 0.002, \quad \phi^E_{\rm q} = 5.03 \pm 0.06, \quad \chi^E_{\rm s} = 0.130 \pm 0.008, \\ &\phi^E_{\rm s} = 4.37 \pm 0.10, \end{aligned} \tag{6}$$

for the $\rm D^0 \to PP$ decays, and

$$\begin{aligned} &S_\pi = -1.88 \pm 0.12, \quad \chi^C_P = 0.63 \pm 0.03, \quad \phi^C_P = 1.57 \pm 0.11, \\ &\chi^C_V = 0.71 \pm 0.03, \quad \phi^C_V = 2.77 \pm 0.10, \quad \chi^E_{\rm q} = 0.49 \pm 0.03, \\ &\phi^E_{\rm q} = 1.61 \pm 0.07, \quad \chi^E_{\rm s} = 0.54 \pm 0.03, \quad \phi^E_{\rm s} = 2.23 \pm 0.08, \end{aligned} \tag{7}$$

for the $\rm D^0 \to PV$ decays. In both the PP and PV modes, the parameter Λ related to the soft scale in D meson decays is fixed to be 0.5 GeV. The decay constants of the vector mesons are from Ref. [30], and other theoretical inputs are the same as in Refs. [21, 22]. The minimal χ^2 per degree of freedom is 1.1 for the data of 13 PP modes, and 1.8 for the data of 19 PV modes. The $\rm D^0 \to PP$ and PV branching fractions predicted in the FAT approach are given in Tables 1 and 2, and agree well with the data. The cosines of the relative strong phases, $\cos\delta_n$, in Eq. (5), listed in the rows of the $\rm D^0 \to n$ and $\rm D^0 \to \bar{n}$ decays, reveal the channel dependence and the $SU(3)$ symmetry breaking effects. Those shown as 1 are for the modes with CP eigenstates, i.e., $n = \bar{n}$. Those shown as 1.0 ± 0.0 are for the modes, in which the relative strong phases vanish with tiny uncertainties in the FAT approach. The expression 1.0 ± 0.0 means that those strong phases can deviate from zero in principle, but turn out to vanish with tiny uncertainties in the FAT approach. The values of $\cos\delta_n$ can never be greater than unity. It is observed that the $\rm D^0 \to K^\pm \rho^\mp$ and $\rm D^0 \to K^\pm K^{*\mp}$ decays exhibit nonvanishing relative strong phases around 10 degrees, different from the approximation $\cos\delta_n = 1$ assumed in Ref. [19]. To confirm that the values of $\cos\delta_n$ are close to unity in the $\rm D^0 \to PP$ decays, we have allowed the W-exchange diagrams E in the Cabibbo-favored and doubly Cabibbo-suppressed modes to carry different strong phases, which may lead to nonvanishing δ_n. The associated global fit indeed indicates that the results in Table 1 remain unaltered.

Table 1. Branching ratios in units of 10^{-3} and cosines of the relative strong phases for the $D^0 \to PP$ decays. Predictions $\mathcal{B}$(FAT) in the FAT approach are compared with the experimental data $\mathcal{B}$(exp) [20]. Topological parametrizations are also given with $\lambda_{ij} = V_{ci}^* V_{uj}$, in which each topological amplitude, including the $SU(3)$ symmetry breaking effects, is actually mode-dependent.

modes	parametrization	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	$\cos\delta_n$
$\pi^0\overline{K}^0$	$\frac{1}{\sqrt{2}}\lambda_{sd}(C-E)$	24.0±0.8	24.2±0.8	1.0±0.0
π^+K^-	$\lambda_{sd}(T+E)$	39.3±0.4	39.2±0.4	0.99999±0.00001
$\eta\overline{K}^0$	$\lambda_{sd}[\frac{1}{\sqrt{2}}(C+E)\cos\phi-E\sin\phi]$	9.70±0.6	9.6±0.6	1.0±0.0
$\eta'\overline{K}^0$	$\lambda_{sd}[\frac{1}{\sqrt{2}}(C+E)\sin\phi+E\cos\phi]$	19.0±1.0	19.5±1.0	1.0±0.0
$\pi^+\pi^-$	$\lambda_{dd}(T+E)$	1.421±0.025	1.44±0.02	1
K^+K^-	$\lambda_{dd}(T+E)$	4.01±0.07	4.05±0.07	1
$K^0\overline{K}^0$	$\lambda_{dd}E+\lambda_{ss}E$	0.36±0.08	0.29±0.07	1
$\pi^0\eta$	$-\lambda_{dd}E\cos\phi-\frac{1}{\sqrt{2}}\lambda_{ss}C\sin\phi$	0.69±0.07	0.74±0.03	1
$\pi^0\eta'$	$-\lambda_{dd}E\sin\phi+\frac{1}{\sqrt{2}}\lambda_{ss}C\cos\phi$	0.91±0.14	1.08±0.05	1
$\eta\eta$	$\frac{1}{\sqrt{2}}\lambda_{dd}(C+E)\cos^2\phi+\lambda_{ss}(2E\sin^2\phi-\frac{1}{\sqrt{2}}C\sin2\phi)$	1.70±0.20	1.86±0.06	1
$\eta\eta'$	$\frac{1}{\sqrt{2}}\lambda_{dd}(C+E)\sin2\phi+\lambda_{ss}(E\sin2\phi-\frac{1}{\sqrt{2}}C\cos2\phi)$	1.07±0.26	1.05±0.08	1
$\pi^0\pi^0$	$\frac{1}{\sqrt{2}}\lambda_{dd}(C-E)$	0.826±0.035	0.78±0.03	1
π^0K^0	$\frac{1}{\sqrt{2}}\lambda_{ds}(C-E)$		0.069±0.002	1.0±0.0
π^-K^+	$\lambda_{ds}(T+E)$	0.133±0.009	0.133±0.001	0.99999±0.00001
ηK^0	$\lambda_{ds}[\frac{1}{\sqrt{2}}(C+E)\cos\phi-E\sin\phi]$		0.027±0.002	1.0±0.0
$\eta'K^0$	$\lambda_{ds}[\frac{1}{\sqrt{2}}(C+E)\sin\phi+E\cos\phi]$		0.056±0.003	1.0±0.0

Table 2. Branching ratios in units of 10^{-3} and cosines of the relative strong phases for the $D^0 \to PV$ decays. Predictions $\mathcal{B}$(FAT) in the FAT approach are compared with the experimental data $\mathcal{B}$(exp) [20]. Topological parametrizations are also given with $\lambda_{ij} = V_{ci}^* V_{uj}$, in which each topological amplitude, including the $SU(3)$ symmetry breaking effects, is actually mode-dependent.

modes	parametrization	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	$\cos\delta_n$
$\pi^0\overline{K}^{*0}$	$\frac{1}{\sqrt{2}}\lambda_{sd}(C_P-E_P)$	37.5±2.9	35.9±2.2	1.0±0.0
$\overline{K}^0\rho^0$	$\frac{1}{\sqrt{2}}\lambda_{sd}(C_V-E_V)$	$12.8^{+1.4}_{-1.6}$	13.5±1.4	1.0±0.0
π^+K^{*-}	$\lambda_{sd}(T_V+E_P)$	54.3±4.4	62.5±2.7	0.9994±0.0006
$K^-\rho^+$	$\lambda_{sd}(T_P+E_V)$	111.0±9.0	105.0±5.2	0.983±0.002
$\eta\overline{K}^{*0}$	$\lambda_{sd}(\frac{1}{\sqrt{2}}(C_P+E_P)\cos\phi-E_V\sin\phi)$	9.6±3.0	6.1±1.0	1.0±0.0
$\eta'\overline{K}^{*0}$	$\lambda_{sd}(\frac{1}{\sqrt{2}}(C_P+E_P)\sin\phi+E_V\cos\phi)$	<1.10	0.19±0.01	1.0±0.0
$\overline{K}^0\omega$	$\frac{1}{\sqrt{2}}\lambda_{sd}(C_V+E_V)$	22.2±1.2	22.3±1.1	1.0±0.0
$\overline{K}^0\phi$	$\lambda_{sd}E_P$	$8.47^{+0.66}_{-0.34}$	8.2±0.6	1.0±0.0
$\pi^+\rho^-$	$\lambda_{dd}(T_V+E_P)$	5.09±0.34	4.5±0.2	0.9995±0.0005
$\pi^-\rho^+$	$\lambda_{dd}(T_P+E_V)$	10.0±0.6	9.2±0.3	0.9995±0.0005
K^+K^{*-}	$\lambda_{ss}(T_V+E_P)$	1.62±0.15	1.8±0.1	0.977±0.003
K^-K^{*+}	$\lambda_{ss}(T_P+E_V)$	4.50±0.30	4.3±0.2	0.977±0.003
$K^0\overline{K}^{*0}$	$\lambda_{ss}E_P+\lambda_{dd}E_V$	0.18±0.04	0.19±0.03	1.0±0.0
$\overline{K}^0K^{*0}$	$\lambda_{ss}E_V+\lambda_{dd}E_P$	0.21±0.04	0.19±0.03	1.0±0.0
$\eta\rho^0$	$\frac{1}{2}\lambda_{dd}(C_V-C_P-E_P-E_V)\cos\phi-\frac{1}{\sqrt{2}}\lambda_{ss}C_V\sin\phi$		1.4±0.2	1
$\eta'\rho^0$	$\frac{1}{2}\lambda_{dd}(C_V-C_P-E_P-E_V)\sin\phi+\frac{1}{\sqrt{2}}\lambda_{ss}C_V\cos\phi$		0.25±0.01	1
$\pi^0\rho^0$	$-\frac{1}{2}\lambda_{dd}(C_P+C_V-E_P-E_V)$	3.82±0.29	4.1±0.2	1
$\pi^0\omega$	$-\frac{1}{2}\lambda_{dd}(C_V-C_P+E_P+E_V)$	0.117±0.035	0.10±0.03	1
$\pi^0\phi$	$\frac{1}{\sqrt{2}}\lambda_{ss}C_P$	1.35±0.10	1.4±0.1	1
$\eta\omega$	$\frac{1}{2}\lambda_{dd}(C_V+C_P+E_P+E_V)\cos\phi-\frac{1}{\sqrt{2}}\lambda_{ss}C_V\sin\phi$	2.21±0.23	2.0±0.1	1
$\eta'\omega$	$\frac{1}{2}\lambda_{dd}(C_V+C_P+E_P+E_V)\sin\phi+\frac{1}{\sqrt{2}}\lambda_{ss}C_V\cos\phi$		0.044±0.004	1
$\eta\phi$	$\lambda_{ss}(\frac{1}{\sqrt{2}}C_P\cos\phi-(E_P+E_V)\sin\phi)$	0.14±0.05	0.18±0.04	1

Continued on next page

Table 2. – continued from previous page

$\pi^0 K^{*0}$	$\frac{1}{\sqrt{2}}\lambda_{\rm ds}(C_{\rm P}-E_{\rm V})$		0.103±0.006	1.0±0.0
$K^0\rho^0$	$\frac{1}{\sqrt{2}}\lambda_{\rm ds}(C_{\rm V}-E_{\rm P})$		0.039±0.004	1.0±0.0
$\pi^- K^{*+}$	$\lambda_{\rm ds}(T_{\rm P}+E_{\rm V})$	$0.345^{+0.180}_{-0.102}$	0.40±0.02	0.9994±0.0006
$K^+\rho^-$	$\lambda_{\rm ds}(T_{\rm V}+E_{\rm P})$		0.144±0.009	0.983±0.002
ηK^{*0}	$\lambda_{\rm ds}(\frac{1}{\sqrt{2}}(C_{\rm P}+E_{\rm V})\cos\phi-E_{\rm P}\sin\phi)$		0.017±0.003	1.0±0.0
$\eta' K^{*0}$	$\lambda_{\rm ds}(\frac{1}{\sqrt{2}}(C_{\rm P}+E_{\rm V})\sin\phi+E_{\rm P}\cos\phi)$		0.00055±0.00004	1.0±0.0
$K^0\omega$	$\frac{1}{\sqrt{2}}\lambda_{\rm ds}(C_{\rm V}+E_{\rm P})$		0.064±0.003	1.0±0.0
$K^0\phi$	$\frac{1}{\sqrt{2}}\lambda_{\rm ds}E_{\rm V}$		0.024±0.002	1.0±0.0

Based on Eqs. (6) and (7), we calculate the $D\to PP$ and $D\to PV$ contributions to y by means of Eq. (4), deriving

$$y_{\rm PP} = (1.00\pm0.19)\times10^{-3}, \tag{8}$$

$$y_{\rm PV} = (1.12\pm0.72)\times10^{-3}, \tag{9}$$

respectively. Our results are consistent with those in Ref. [19]: $y_{\rm PP}=(0.86\pm0.41)\times10^{-3}$, $y_{\rm PV}=(2.69\pm2.53)\times10^{-3}$ $(A,A1)$ and $y_{\rm PV}=(1.52\pm2.20)\times10^{-3}$ $(S,S1)$ from two different solutions, but with much smaller uncertainties. We stress that the predictions for $y_{\rm PP}$ and $y_{\rm PV}$ presented in this work are the most precise to date. The uncertainties of the parameters in Eqs. (6) and (7) are basically controlled by those most precisely measured channels, explaining why $y_{\rm PP}$, with the more precise PP data, is more certain than $y_{\rm PV}$. It is also the reason why the fit results for the most precisely measured branching ratios like $\mathcal{B}(\pi^0\overline{K}^0)$ and $\mathcal{B}(\overline{K}^0\omega)$ have uncertainties similar to those of the data, while the fit results for the less precisely measured ones like $\mathcal{B}(\eta\eta)$ and $\mathcal{B}(\pi^- K^{*+})$ have considerably smaller uncertainties. Besides, the branching ratios are correlated to each other by the strong parameters in the FAT approach, so the uncertainties are greatly reduced. Since the $SU(3)$ symmetry is assumed in the topological diagrammatic approach [19], the charm mixing parameter y cannot be extracted in principle. Instead, the data of the branching ratios were directly input into Eq. (5) by taking $\cos\delta_n=1$ [19] as mentioned before, such that the uncertainties of the data are summed up in the evaluation of y. Some other efforts have been devoted to global fits of the PP or PV data recently [31–33]. However, it is unlikely that a precise prediction for y can be made without thorough exploration of the $SU(3)$ breaking effects in the relevant D meson decays.

3 $y_{\rm VV}$

There exist three different polarizations in the final state of a $D\to VV$ channel, whose corresponding amplitudes can be expressed in the transversity basis (A_0, $A_{||}$, $A_\perp$), or equivalently in the partial-wave basis (S, P, D). The decay amplitudes for different polarizations are independent, and should be described by different sets of strong parameters in the FAT approach. At least six strong parameters are required for the longitudinal amplitude A_0 alone, but only one channel has been observed, with the longitudinal branching ratio $\mathcal{B}_0(D^0\to\rho^0\rho^0)=(1.25\pm0.10)\times10^{-3}$ [20]. The situation for the transverse amplitudes is even worse. It is impossible to extract all the $D\to VV$ amplitudes in the FAT approach due to the lack of experimental data at present.

As a bold attempt, we estimate the $D\to VV$ longitudinal amplitudes by means of the strong parameters in Eq. (7) extracted from the PV data. In detail, the factorizable part in an emission-type amplitude is treated in the naive factorization hypothesis, and the associated nonfactorizable amplitude $\chi_{\rm V}^C e^{i\phi_{\rm V}^C}$ is assumed to be identical to that of the corresponding PV amplitude. We adopt the definition of the vector meson decay constant $f_{\rm V}$ via

$$\langle V(q)|\bar{q}\gamma_\mu(1-\gamma_5)q'|0\rangle = f_{\rm V}m_{\rm V}\varepsilon^*_\mu(q), \tag{10}$$

and the definition of the $D\to V$ transition form factors $V^{\rm DV}$, $A_1^{\rm DV}$, $A_2^{\rm DV}$, and $A_0^{\rm DV}$ via

$$\begin{aligned}
&\langle V(k)|\bar{q}\gamma_\mu(1-\gamma_5)c|D(p)\rangle\\
=&\frac{2}{m_{\rm D}+m_{\rm V}}\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho k^\sigma V^{\rm DV}(q^2)\\
&-i\left(\varepsilon^*_\mu-\frac{\varepsilon^*\cdot q}{q^2}q_\mu\right)(m_{\rm D}+m_{\rm V})A_1^{\rm DV}(q^2)\\
&+i\left((p+k)_\mu-\frac{m_{\rm D}^2-m_{\rm V}^2}{q^2}q_\mu\right)\frac{\varepsilon^*\cdot q}{m_{\rm D}+m_{\rm V}}A_2^{\rm DV}(q^2)\\
&-i\frac{2m_{\rm V}(\varepsilon^*\cdot q)}{q^2}q_\mu A_0^{\rm DV}(q^2),
\end{aligned} \tag{11}$$

where ε is the polarization vector, the m's are the meson masses, and the momentum $q=p-k$. The emission-type amplitudes are then expressed as

$$\begin{aligned}
T(C) =& \frac{G_{\rm F}}{\sqrt{2}}V_{\rm CKM}a_1(\mu)\left(a_2^C(\mu)\right)f_{V_1}m_1\\
&\times\Bigg[-ix(m_{\rm D}+m_2)A_1^{{\rm DV}_2}(m_1^2)\\
&+i\frac{2m_{\rm D}^2p_{\rm c}^2}{(m_{\rm D}+m_2)m_1m_2}A_2^{{\rm DV}_2}(m_1^2)\Bigg],
\end{aligned} \tag{12}$$

in which the Wilson coefficients and the kinetic quantities are given by

$$a_1(\mu) = \frac{C_1(\mu)}{N_c}+C_2(\mu),$$
$$a_2^C(\mu) = C_1(\mu)+C_2(\mu)\left(\frac{1}{N_c}+\chi_V^C e^{i\phi_V^C}\right), \quad (13)$$

$$x=\frac{m_D^2-m_1^2-m_2^2}{2m_1m_2}, \qquad p_c^2=\frac{m_1^2m_2^2(x^2-1)}{m_D^2}, \quad (14)$$

respectively. The values of the form factors $A_{1,2}^{DV}$ are input from Ref. [34]. The annihilation-type amplitudes are taken directly from the PV modes with the replacement of the meson masses and decay constants, explicitly written as

$$E=-i\frac{G_F}{\sqrt{2}}V_{CKM}C_2(\mu)\chi_{q(s)}^E e^{i\phi_{q(s)}^E} f_D\frac{f_{V_1}f_{V_2}}{f_\rho^2}m_D|p_c|. \quad (15)$$

After estimating the $D \to VV$ longitudinal amplitudes, we can derive the corresponding branching ratios straightforwardly. The comparison of our predictions with the data will tell whether the PV-inspired amplitudes are reasonable. The $D^0 \to VV$ longitudinal branching ratios in the FAT approach are listed in Table 3, and compared with the data of the total and longitudinal branching fractions. A general consistency with the data is seen, especially for the single observed longitudinal branching ratio $\mathcal{B}_{long}(D^0\to\rho^0\rho^0)$. For those channels with only measured total branching ratios, most of our predictions for the longitudinal branching ratios do not exceed the data, after considering the uncertainties. Our result for the $D^0\to\overline{K}^{*0}\omega$ mode is larger than the data, but the measurement of this mode was performed in 1992 [35], and should be updated. It is thus a fair claim that our simple estimates for the $D^0\to VV$ longitudinal amplitudes are satisfactory. Certainly, more experimental effort toward improved understanding of the $D \to VV$ decays into final states with different polarizations is encouraged.

A longitudinal amplitude A_0 is a linear combination of the partial waves S and D, namely, of the $L=0$ and 2 final states, leading to $\eta_{CP}(n)=+1$ in Eq. (4). Inserting the amplitudes estimated above into Eq. (4), we obtain the longitudinal VV contribution

$$y_{VV}=(-0.42\pm0.34)\times10^{-3}. \quad (16)$$

The central value of y_{VV} is lower than those of y_{PP} and y_{PV} in Eqs. (8) and (9), because the $SU(3)$ breaking effects are much smaller in the VV modes. Even though Eq. (16) contains a relatively large uncertainty in our approach, and the contributions from the transverse polarizations have not yet been included, it is reasonable to postulate that y_{VV} represents a minor contribution to y.

In summary, our predictions for the mixing parameter y agree well with the postulation in Ref. [18]: y is generated only at the second order in U-spin symmetry breaking effects, so the contribution to y from two-body modes, for which the U-spin symmetry works better, might be small. Multi-particle decays, for which the U-spin breaking effects are expected to be more significant, should be the major source of y. We stress that we do not attempt a full understanding of y here, and our results for y_{PP}, y_{PV} and y_{VV} are consistent with the fact that y is generated at second order in the $SU(3)$ symmetry breaking.

Table 3. Branching ratios for the $D^0\to VV$ decays in units of 10^{-3}. Estimations of the longitudinal branching ratios in the FAT approach are compared with the data for the total and longitudinal branching ratios [20].

modes	$\mathcal{B}_{tot}$(exp)	$\mathcal{B}_{long}$(exp)	$\mathcal{B}_{long}$(FAT)
$\rho^0\overline{K}^{*0}$	15.9±3.5		13.2±1.3
ρ^+K^{*-}	65.0±25.0		34.7±1.4
$\overline{K}^{*0}\omega$	11.0±5.0		34.9±2.7
$\rho^+\rho^-$			3.2±0.1
$K^{*+}K^{*-}$			1.1±0.05
$K^{*0}\overline{K}^{*0}$			0.010±0.002
$\rho^0\rho^0$	1.83±0.13	1.25±0.13	1.1±0.1
$\rho^0\omega$			0.95±0.07
$\rho^0\phi$			0.65±0.04
$\omega\omega$			0.47±0.07
$\omega\phi$			1.41±0.09
ρ^0K^{*0}			0.038±0.004
ρ^-K^{*+}			0.123±0.005
$K^{*0}\omega$			0.100±0.008

4 Summary

In this paper we have calculated the D^0-$\overline{D}^0$ mixing parameter y in the FAT approach, considering the $D^0\to PP$, PV, and VV channels. The $D^0\to PP$ and PV decay amplitudes were extracted from the latest data using the FAT approach, and the $D^0\to VV$ decay amplitudes for the longitudinal polarization were estimated via the parameter set for the PV modes. It has been confirmed that the PV-inspired amplitudes work well for explaining the observed $D^0 \to VV$ branching ratios. We then derived the contribution from the PP and PV modes as

$$y_{PP+PV}=(0.21\pm0.07)\%, \quad (17)$$

which is much more precise than previous predictions in the literature, and far below the data $y_{exp}=(0.61\pm0.08)\%$. It has been also found that the contribution from the longitudinal VV modes, being of order 10^{-4}, is negligible. This observation is consistent with the fact that y is generated at second order in $SU(3)$ symmetry breaking. We conjecture that considering the above two-body D meson decays alone in an exclusive approach cannot account for the charm mixing, and that hadronic channels to other two-body and multi-particle final states are relevant to the evaluation of y. However, it is currently very difficult, if not impossible, to gain full control of the $SU(3)$ symmetry breaking effects in all these modes in an exclusive approach. As stated in the Introduction, the inclusive approach leads to values of x and y two or three orders of magnitude lower than the data. Therefore, a new strategy has to be proposed for complete understanding of the charm mixing dynamics in the Standard Model. We will leave this subject to a future project.

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