$D^*\bar{D}_1(2420)$ and $D\bar{D}'^*(2600)$ interactions and the charged charmonium-like state $Z(4430)$ *

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Abstract: The $D^*\bar{D}_1(2420)$ and $D\bar{D}'^*(2600)$ interactions are studied in a one-boson-exchange model. Isovector bound state solutions with spin parity $J^P=1^+$ are found from the $D^*\bar{D}_1(2420)$ interaction, which may be related to the observed charged charmonium-like state $Z(4430)$. There is no bound state solution found from the $D\bar{D}'^*(2600)$ interaction.

Keywords: exotic state, charmed meson interaction, one-boson-exchange model, Bethe-Salpeter equation

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1 Introduction

A resonant structure near 4.43 GeV in the $\pi^\pm\psi'$ invariant mass distribution was first observed by the Belle Collaboration [1], and is the first evidence of the existence of charged charmonium-like states. The mass $M = 4433 \pm 4(\text{stat}) \pm 2(\text{syst})$ MeV and width $\Gamma = 45^{+18}_{-13}(\text{stat})^{+30}_{-13}(\text{syst})$ MeV were extracted by using a Breit-Wigner resonance shape. A higher mass $M=4485^{+22+28}_{-22-11}$ MeV and a larger width $\Gamma=200^{+41+26}_{-46-35}$ MeV were reported by the Belle Collaboration through a full amplitude analysis of $B^0\to\psi'K^+\pi^-$ decay and a spin parity of $J^P=1^+$ was favored over other hypotheses [2]. Recently, the LHCb Collaboration released their new result on the $B^0\to\psi'\pi^-K^+$ decay, which confirmed the existence of the 1^+ resonant structure $Z(4430)$ with a mass $4475\pm7^{+15}_{-25}$ MeV and a width $172\pm13^{+37}_{-34}$ MeV [3].

The $Z(4430)$ was observed in the $\psi'\pi$ invariant mass spectrum, which suggests that it should be an exotic state beyond the conventional $c\bar{c}$ picture, which has a neutral charge. Many theoretical efforts have been made to understand the internal structure of the $Z(4430)$ and a number of explanations have been offered. Since the $Z(4430)$ carries charge, the hybrid interpretation is excluded [4]. It is natural to explain the charge carrier $Z(4430)$ as a multiquark system in which, as well as $c\bar{c}$, there exist other light quarks. The first type of multiquark explanation is the excited tetraquark [5–10] where four quarks are in a color singlet. Another type of multiquark explanation is a loosely bound state composed of two charmed mesons [11, 12], or charmed baryons[13]. There also exist several nonresonant explanations, such as the threshold cusp effect [14] and a cusp in the $D^*\bar{D}_1(2420)$ channel [15].

The $Z(4430)$ mass measured by the Belle Collaboration [1], $M=4433\pm4(\text{stat})\pm2(\text{syst})$ MeV, is close to the $D^*\bar{D}_1(2420)$ threshold, so it has been popular to explain the $Z(4430)$ as a S-wave $D^*\bar{D}_1(2420)$ molecular state with $J^P=0^-$ in the one-boson-exchange (OBE) model [16, 17]. A calculation in the context of the QCD sum rule also favors the $D^*\bar{D}_1(2420)$ bound state explanation with spin-parity 0^- [18]. The new Belle and LHCb results suggest the spin-parity of $Z(4430)$ is 1^+, however. With such an assignment of spin parity, a new calculation by Barnes et al. suggests that the $Z(4430)$ is either a $D^*\bar{D}_1$ state dominated by long-range π exchange, or a $D\bar{D}^*(1S,2S)$ state with short-range components [19].

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It has also been suggested that the $Z(4430)$ may be from S-wave $D\bar{D}_1'^*(2600)$ interaction because the $Z(4430)$ mass is very close to the $D\bar{D}_1'^*(2600)$ threshold [20].

In this paper, the $D^*\bar{D}_1(2420)$ and $D\bar{D}'^*(2600)$ interactions will be studied by solving the Bethe-Salpeter equation combined with the one-boson-exchange model. The $Z(4430)$ mass is close to the threshold of four configurations, $D^*\ \bar{D}_1'(2430)$, $D^*\bar{D}_1(2420)$, $D\bar{D}'^*(2600)$, and $D^*\bar{D}'^*(2550)$. The large width of the $D_1'(2430)$, however, $\Gamma=384^{+130}_{-110}$ MeV [21], which means a very short lifetime, makes it difficult to bind it and the D^* together to form a bound state with width about 170 MeV. The configuration $D^*\bar{D}'^*(2550)$ has also been related to the $Z(4430)$ in the literature. However, its threshold is about 100 MeV higher than the $Z(4430)$ mass. In this work, the constituents will be treated as stable particles as in the OBE model [16, 17]. However, the physical widths of $D_1(2420)$ and $D'^*(2600)$ are about 27 MeV and 93 MeV, respectively. Form factors will be introduced to compensate the self energy effects. The non-zero width will also introduce the three-body effect, which is not included in the current work considering that the thresholds of the three-body channels, such as $DD\pi$, $D^*D^*\pi$ and $DD^*\pi$, are much lower than the mass of the $Z(4430)$. It is also the reason why the configuration $D^*\bar{D}'^*$ (2500) is excluded. Since only loosely bound states are considered, only two configurations, $D^*\bar{D}_1(2420)$ and $D\bar{D}_1'^*(2600)$, will be included in the current calculation.

The paper is organized as follows. In the next section a theoretical framework will be developed to study the $D^*\bar{D}_1$ and $D\bar{D}'^*$ interactions (we omit the numbers for the masses, 2420 and 2600 respectively, here and hereafter) by solving the Bethe-Salpeter equation. In Section 3, the potential is derived with the help of effective Lagrangians from the heavy quark effective theory. The numerical results are given in Section 4. A summary is given in the last section.

2 Bethe-Salpeter equation for vertices

The Bethe-Salpeter equation is a powerful tool to study bound state problems such as the deuteron [22]. A Bethe-Salpeter formalism was developed and applied to study the $Y(4274)$ and its decay pattern [23,24], the $\Sigma_c(3250)$ as D_0^* (2400)N system [25] and the $N(1875)$ as $\Sigma(1385)K$ system [26]. In Refs. [27–29], the $B\bar{B}^*/D\bar{D}^*$ system was also studied by solving the Bethe-Salpeter equation with boson exchange mechanism to explore the possible relationship between the recently observed $Z_b(10610)/Z_c(3900)$ and the $B\bar{B}^*/D\bar{D}^*$ interaction. We start from the Bethe-Salpeter equation for vertex $|\Gamma\rangle$,

$$|\Gamma\rangle=\mathcal{V}G|\Gamma\rangle, \tag{1}$$

where $\mathcal{V}$ and G are the potential kernel and the propagator for the two constituents of the system. The vertex function of the system with two configurations can be written as

$$|\Gamma\rangle=\Gamma^{D^*\bar{D}_1}|D^*\bar{D}_1\rangle+\Gamma^{D\bar{D}'^*}|D\bar{D}'^*\rangle, \tag{2}$$

where $\Gamma_{D^*\bar{D}_1}$ and $\Gamma_{D\bar{D}'^*}$ are the vertex functions after separating out the flavor parts $|D^*\bar{D}_1\rangle$ and $|D\bar{D}'^*\rangle$. In this paper $SU(2)$ symmetry is considered, so the same vertex function is used for both configurations.

The explicit flavor structures for isovectors (T) or isoscalars (S) $|D^*\bar{D}_1\rangle$ are [17]

$$\begin{aligned}
|D^*\bar{D}_1\rangle_T^+&=\frac{1}{\sqrt{2}}\big(|D^{*+}\bar{D}_1^0\rangle+c|D_1^+\bar{D}^{*0}\rangle\big),\\
|D^*\bar{D}_1\rangle_T^-&=\frac{1}{\sqrt{2}}\big(|D^{*-}\bar{D}_1^0\rangle+c|D_1^-\bar{D}^{*0}\rangle\big),\\
|D^*\bar{D}_1\rangle_T^0&=\frac{1}{2}\Big[\big(|D^{*+}D_1^-\rangle-|D^{*0}\bar{D}_1^0\rangle\big)\\
&\qquad+c\big(|D_1^+D^{*-}\rangle-|D_1^0\bar{D}^{*0}\rangle\big)\Big],\\
|D^*\bar{D}_1\rangle_S^0&=\frac{1}{2}\Big[\big(|D^{*+}D_1^-\rangle+|D^{*0}\bar{D}_1^0\rangle\big)\\
&\qquad+c\big(|D_1^+D^{*-}\rangle+|D_1^0\bar{D}^{*0}\rangle\big)\Big],
\end{aligned} \tag{3}$$

where $c=\pm$ corresponds to C-parity $C=\mp$. The flavor structure for $D\bar{D}'^*$ configuration is analogous to that of the $D^*\bar{D}_1$ configuration.

The vertex function is rewritten as

$$|\Gamma\rangle=\sum_{i=1}^{N}\Gamma^i\sum_{a=1}^{n}\delta^{i,a}|i,a\rangle, \tag{4}$$

with $i=1$ or 2 for configuration D^*D_1 or DD'^*, and a stands for the different components in a configuration. $\delta^{i,a}$ is the factor for $|i,a\rangle$ in Eq. (3) structure). After multiplying $\langle j,b|$ on both sides, the Bethe-Salpeter equation becomes

$$\Gamma^j=\sum_i\tilde{\mathcal{V}}^{ji}G^i\Gamma^i,\text{with}\tilde{\mathcal{V}}^{ji}=\sum_{b,a}\delta^{j,b}\delta^{i,a}\langle j,b|V|i,a\rangle. \tag{5}$$

The above equation is a coupled-channel equation for the two channels $D^*\bar{D}_1$ and $D\bar{D}'^*$ involved.

The Bethe-Salpeter equation is a 4-dimensional integral equation. It is popular to reduce it to a 3-dimensional equation by a quasipotential approximation, and in principle there exist infinite choices to make the quasipotential approximation. As in Ref. [27], we adopt the covariant spectator theory [30, 31] to make the 3-dimensional reduction. With the help of the onshellness of the heavier constituent 2, D_1/D'^*, the numerator of the propagator $P_2^{\mu\nu}=\sum_{\lambda_2}\epsilon^{\mu}_{2\lambda_2}\epsilon^{\nu\dagger}_{2\lambda_2}$ with $\epsilon^{\mu}_{2\lambda_2}$ being the polarization vector with helicity λ_2. Different from Ref. [27], where the off-shell constituent is a pseudoscalar particle D, constituent 1 here is a vector meson D^*. So we will make an approximation $P_1^{\mu\nu}=\sum_{\lambda_1}\epsilon^{\mu}_{1\lambda_1}\epsilon^{\nu\dagger}_{1\lambda_1}$ with polarization $\epsilon^{\mu}_{1\lambda_1}$ on shell. Such an approximation will introduce an uncertainty of about several percent in the numerator of the propagator, which will be further smeared by the introduction of form factors which will be given in the next section. Now, the equation for the vertex is of a form

$$|\Gamma^i_{\lambda_1\lambda_2}\rangle=\sum_{j,\ \lambda'_1\lambda'_2}\tilde{\mathcal{V}}^{ij}_{\lambda_1\lambda_2\lambda'_1\lambda'_2}\ G^j_0\ |\Gamma^j_{\lambda'_1\lambda'_2}\rangle. \quad (6)$$

Written down in the center of mass frame where $P=(W,0)$, the propagator is

$$\begin{aligned}G_0&=2\pi\mathrm{i}\frac{\delta^+(k_2{}^2-m_2^2)}{k_1{}^2-m_1^2}\\&=2\pi\mathrm{i}\frac{\delta^+(k^0_2-E_2(\boldsymbol{k}))}{2E_2(\boldsymbol{k})[(W-E_2(\boldsymbol{k}))^2-E_1^2(\boldsymbol{k})]},\end{aligned} \quad (7)$$

where $k_1=(k^0_1,\boldsymbol{k})=(E_1(\boldsymbol{k}),\boldsymbol{k}),\ k_2=(k^0_2,-\boldsymbol{k})=(W-E_1(\boldsymbol{k}),-\boldsymbol{k})$ with $E_{1,2}(\boldsymbol{k})=\sqrt{m^2_{1,2}+|\boldsymbol{k}|^2}$.

The integral equation can be written explicitly as

$$\begin{aligned}&(W-E^i_1(\boldsymbol{k})-E^i_2(\boldsymbol{k}))\phi^i_{\lambda_1\lambda_2}(\boldsymbol{k})\\&=\sum_{j,\ \lambda'_1\lambda'_2}\int\frac{d\boldsymbol{k}'}{(2\pi)^3}V^{ij}_{\lambda_1\lambda_2\lambda'_1\lambda'_2}(\boldsymbol{k},\boldsymbol{k}',W)\phi^{j,j}_{\lambda'_1\lambda'_2}(\boldsymbol{k}'),\end{aligned} \quad (8)$$

with

$$\begin{aligned}&V^{ij}_{\lambda_1\lambda_2\lambda'_1\lambda_2}(\boldsymbol{k},\boldsymbol{k}',W)\\&=\frac{i\ \bar{\mathcal{V}}^{ij}_{\lambda_1\lambda_2\lambda'_1\lambda'_2}(\boldsymbol{k},\boldsymbol{k}',W)}{\sqrt{2E^i_1(\boldsymbol{k})2E^i_2(\boldsymbol{k})2E'^j_1(\boldsymbol{k}')2E'^j_2(\boldsymbol{k}')}},\end{aligned} \quad (9)$$

where the reduced potential kernel

$$\bar{\mathcal{V}}^{ij}_{\lambda_1\lambda_2\lambda'_1\lambda'_2}(\boldsymbol{k},\boldsymbol{k}',W)=F^i(\boldsymbol{k})\tilde{\mathcal{V}}^{ij}_{\lambda_1\lambda_2\lambda'_1\lambda'_2}(\boldsymbol{k},\boldsymbol{k}',W)F^j(\boldsymbol{k}'), \quad (10)$$

with a factor as $F^i(\boldsymbol{k})=\sqrt{2E^i_2(\boldsymbol{k})/(W-E^i_1(\boldsymbol{k})+E^i_2(\boldsymbol{k}))}$. The normalized wave function can be related to the vertex as $|\phi^i_{\lambda_1\lambda'_2}\rangle=N^i|\psi^i_{\lambda_1\lambda_2}\rangle=N^i(F^i)^{-1}G^i_0\ |\Gamma^i_{\lambda_1\lambda_2}\rangle$ with the normalization factor $N^i(\boldsymbol{k})=\sqrt{2E^i_1(\boldsymbol{k})E^i_2(\boldsymbol{k})/(2\pi)^5W}$.

A partial wave expansion can reduce the 3-dimensional integral equation to a one-dimensional equation,

$$\begin{aligned}&(W-E^k_1(|\boldsymbol{k}|)-E^k_2(|\boldsymbol{k}|))\phi_k(|\boldsymbol{k}|)\\&=\sum_l\int\frac{|\boldsymbol{k}'|^2d|\boldsymbol{k}'|}{(2\pi)^3}V_{kl}(|\boldsymbol{k}'|,|\boldsymbol{k}'|)\phi_l(|\boldsymbol{k}'|),\end{aligned} \quad (11)$$

where k/l is the number of wave functions with a certain spin-parity.

3 Lagrangians and potential

For a loosely bound system, long-range interaction through the π exchange should be more important than short-range interaction through heavier mesons. Moreover, in the isovector sector the isospin factors are $-1/2$ and $1/2$ for ρ and ω mesons, respectively [28]. The cancelation between the contributions from these two mesons introduces further suppression of the short-range interaction. Hence, the heavier mesons, ρ and ω, are not considered in this paper. The σ exchange which mediates the medium range interaction is included as in Ref. [17]. We will find that the σ exchange is negligible compared with π exchange.

The effective Lagrangians describing the interaction between the light pseudoscalar meson $\mathbb{P}$ and heavy flavor mesons are constructed with the help of the chiral symmetry and heavy quark symmetry [32, 33],

$$\begin{aligned}\mathcal{L}_{D^*D^*\mathbb{P}}=&\mathrm{i}\frac{g}{f_\pi}\big[-\mathrm{i}\epsilon_{\alpha\mu\nu\lambda}D^{*\mu}_b\overleftrightarrow{\partial}^{\alpha}D^{*\lambda\dagger}_a\partial^\nu\mathbb{P}_{ba}\\&+\mathrm{i}\epsilon_{\alpha\mu\nu\lambda}\tilde{D}^{*\mu\dagger}_a\overleftrightarrow{\partial}^{\alpha}\tilde{D}^{*\lambda}_b\partial^\nu\mathbb{P}_{ab}\big],\end{aligned} \quad (12)$$

$$\begin{aligned}\mathcal{L}_{D_1D_1\mathbb{P}}=&\mathrm{i}\frac{5k}{6f_\pi}\big[\mathrm{i}\epsilon_{\alpha\mu\nu\lambda}D^{\mu}_{1b}\overleftrightarrow{\partial}^{\alpha}D^{\lambda\dagger}_{1a}\partial^\nu\mathbb{P}_{ba}\\&-\mathrm{i}\epsilon_{\alpha\mu\nu\lambda}\tilde{D}^{\mu\dagger}_{1a}\overleftrightarrow{\partial}^{\alpha}\tilde{D}^{\lambda}_{1b}\partial^\nu\mathbb{P}_{ab}\big],\end{aligned} \quad (13)$$

$$\begin{aligned}\mathcal{L}_{D^{(\prime)*}D\mathbb{P}}=&\frac{2g^{(\prime)}\sqrt{m_Dm_{D^{(\prime)*}}}}{f_\pi}\\&\cdot\big[-(D_bD^{(\prime)*\dagger}_{a\lambda}+D^{(\prime)*}_{b\lambda}D^\dagger_a)\partial^\lambda\mathbb{P}_{ba}\\&+(\tilde{D}^{(\prime)*\dagger}_{a\lambda}\tilde{D}_b+\tilde{D}^\dagger_a\tilde{D}^{(\prime)*}_{b\lambda})\partial^\lambda\mathbb{P}_{ab}\big],\end{aligned} \quad (14)$$

$$\begin{aligned}\mathcal{L}_{D_1D^{(\prime)*}\mathbb{P}}=&\mathrm{i}\sqrt{\frac{2}{3}}\frac{h^{(\prime)}_1+h^{(\prime)}_2}{\Lambda_\chi f_\pi}\sqrt{m_{D_1}m_{D^{(\prime)*}}}\\&\cdot\Bigg\{\Bigg[-\frac{1}{4m_{D_1}m_{D^{(\prime)*}}}D^{\alpha}_{1b}\overleftrightarrow{\partial}^{\rho}\overleftrightarrow{\partial}^{\lambda}D^{(\prime)*\dagger}_{\alpha a}\\&\cdot\partial_\rho\partial_\lambda\mathbb{P}_{ba}-D^{\alpha}_{1b}D^{(\prime)*\dagger}_{\alpha a}\partial_\rho\partial_\rho\mathbb{P}_{ba}\\&+3D^{\alpha}_{1b}D^{(\prime)*\dagger\beta}_{a}\partial_\alpha\partial_\beta\mathbb{P}_{ba}\Bigg]\\&-\Bigg[-\frac{1}{4m_{D_1}m_{D^{(\prime)*}}}D^{(\prime)*\dagger}_{\alpha a}\overleftrightarrow{\partial}^{\rho}\overleftrightarrow{\partial}^{\lambda}D^{\alpha}_{1b}\\&\cdot\partial_\rho\partial_\lambda\mathbb{P}_{ab}-D^{(\prime)*\dagger}_{\alpha a}D^{\alpha}_{1b}\partial_\rho\partial_\rho\mathbb{P}_{ab}\\&+3D^{(\prime)*\dagger\beta}_{a}D^{\alpha}_{1b}\partial_\alpha\partial_\beta\mathbb{P}_{ab}\Bigg]\Bigg\},\end{aligned} \quad (15)$$

which corresponds to $D=(D^0,D^+,D^+_s)$ and $\tilde{D}=(\bar{D}^0,D^-,D^-_s)$. The coupling constant g can be extracted from the experimental D^* width with a value $g=0.59$ [32]. Falk and Luke obtained an approximate relation $k=g$ in the quark model [34]. With the available experimental information, Casalbuoni and coworkers extracted $h'=(h_1+h_2)/\Lambda_\chi=0.55\ \mathrm{GeV}^{-1}$ [33]. The coupling constant for D'^* decaying into $D\pi$ and $D_1\pi$ can be extracted from the decay widths obtained in the quark model as $\Gamma_{D'^*\to D\pi}=10.84$ MeV and $\Gamma_{D'^*\to D_1\pi}=0.28$ MeV [35]. The values are $g'=0.086$

and $h''=(h_1'+h_2')/\Lambda_\chi=0.42$ GeV^{-1}. The relative phases between the Lagrangians are not fixed, which will be discussed later.

The σ exchange which mediates the medium range interaction is included as in Ref. [17]. The Lagrangians for the scalar σ meson read,

$$\mathcal{L}_{P^*P^*\sigma}=2g_\sigma[D^*D^*\sigma+\tilde{D}^*\tilde{D}^*\sigma], \quad (16)$$
$$\mathcal{L}_{PP\sigma}=2g_\sigma[-D^*D^*\sigma-\tilde{D}^*\tilde{D}^*\sigma], \quad (17)$$
$$\mathcal{L}_{P_1P_1\sigma}=2g'_\sigma[-D_1D_1\sigma-\tilde{D}_1\tilde{D}_1\sigma]. \quad (18)$$

The coupling constant $g_\sigma=g'_\sigma=-\dfrac{1}{2\sqrt{6}}g_\pi$ with $g_\pi=3.73$ [36].

With the above Lagrangians, we can obtain the potential for direct and cross diagrams,

$$\mathcal{V}^{ij}_{\lambda_1\lambda_2,\lambda_1'\lambda_2'}(p_1,p_2;p_1',p_2')=I^{ij}_d\mathcal{V}^{d\ ij}_{\lambda_1\lambda_2,\lambda_1'\lambda_2'}(p_1,p_2;p_1',p_2') + I^{ij}_c\mathcal{V}^{c\ ij}_{\lambda_1,\lambda_2,\lambda_2'\lambda_1'}(p_1,p_2;p_2',p_1'), \quad (19)$$

where $p^{(\prime)}_{1,2}$ is the initial (final) momentum for constituent 1 or 2. The flavor factor $I^{ij}_{c,d}$ is listed in Table 1.

Table 1. The flavor factors I^{ij}_d and I^{ij}_c for direct and cross diagrams and different exchange mesons.

isospin	1		0		1	0
exchange	π	σ	π	σ	π	π
$D^*D_1\to D^*D_1$	$-\frac{1}{2}$	1	$\frac{3}{2}$	1	$-\frac{1}{2}c$	$\frac{3}{2}c$
$DD'^*\to DD'^*$	0	1	0	1	$-\frac{1}{2}c$	$\frac{3}{2}c$
$D^*D_1\to DD'^*$	$-\frac{1}{2}$	0	$\frac{3}{2}$	0	0	0
$DD'^*\to D^*D_1$	$-\frac{1}{2}$	0	$\frac{3}{2}$	0	0	0

The form factor is introduced to compensate the off-shell effect of heavy mesons, and is also required by the convergence of the equation. It is also convenient to interpret the form factors as self-energies, which is important in this work due to the large decay width of the heavier constituent, D_1/D'^* [30]. In this work, we adopt

$$f(q^2)=\left[\frac{n\Lambda^4}{n\Lambda^4+(m^2-q^2)^2}\right]^n. \quad (20)$$

Here $n>2$ is adopted to make the equation convergent. We will present the results with $n\to\infty$, that is, an exponential type of form factor $f(q^2)\to \mathrm{e}^{-(m^2-q^2)^2/\Lambda^4}$, also to show the sensitivity of results to n. In the propagator of the exchange meson we make a replacement $q^2\to-|q^2|$ to remove the singularities as in Ref. [31]. The form factor for the light meson is chosen as a monopole type $f(q^2)=(\Lambda^2-m^2)/(\Lambda^2+|q^2|)$. The cut-off can be related to the radius of the hadron $r^2=\dfrac{6}{f(0)}\dfrac{df(q^2)}{dq^2}\Big|_{q^2=0}$, which is about $0.5-1$ fm for a meson. The cut-off is about $1.4-2.7$ GeV for exponential type or $0.5-1$ GeV for monopole type. Such an estimation is very rough, so in this work we choose the cut-off as a free parameter from 0.8−2 GeV.

4 Numerical results

To search for the bound state from the $D^*\bar{D}_1$-$D\bar{D}'^*$ interactions, the integral equation will be solved following the procedure in Ref. [27]. After discretizing $|\boldsymbol{k}|$ and $|\boldsymbol{k}'|$ by Gaussian quadrature, the recursion method in Refs. [37] is adopted to solve the nonlinear spectral problem. The numerical results are presented in Fig. 1. To

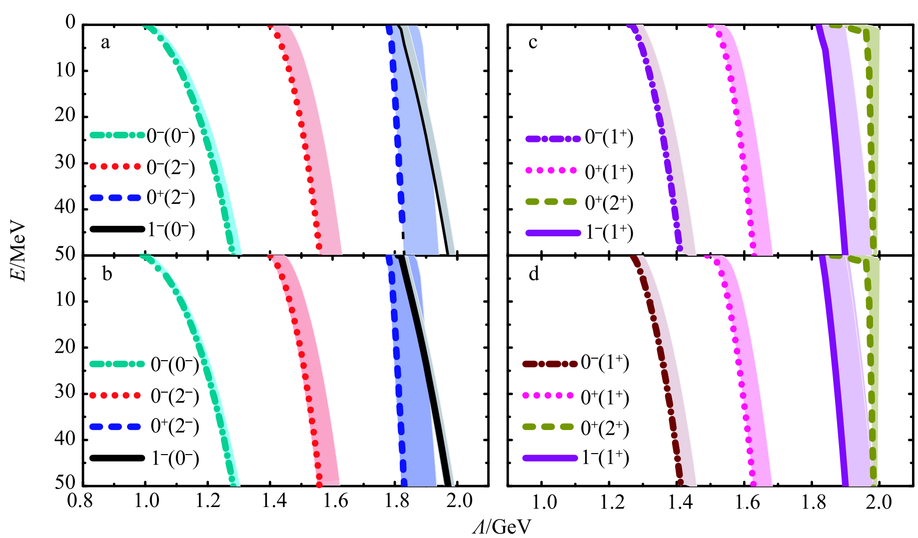

Fig. 1. (color online) The binding energies E for the $D^*\bar{D}_1$ system (patterns (a) and (c)) and $D^*\bar{D}_1-D\bar{D}'^*$ system (patterns (b) and (d)) with the variation of cut-off Λ. The lines are for the results with $n=2$ in form factor in Eq. (20) and the bands for results with $n=2\to\infty$.

show the sensitivity of the results to parameter n in the form factor in Eq. (20), the results with $n=2\to\infty$ are also presented as solid bands. The results suggest the binding energies are not sensitive to n. In this work, all quantum number $J\leqslant 2$ will be considered in the range of cut-offs $0.8<\Lambda<2$ GeV.

In Fig. 1(b) and (d), the coupled-channel results with both configurations, $D^*\bar{D}_1$-$D\bar{D}'^*$, are presented, and are almost the same as these with the configuration $D^*\bar{D}_1$ only (Fig. 1(a) and Fig. 1(c)), which suggests that the $D\bar{D}'^*$ interaction is much weaker than the $D^*\bar{D}_1$ interaction and transitions between $D^*\bar{D}_1$ and $D\bar{D}'^*$ are negligible. The S wave $D\bar{D}'^*$ system carries spin-parity 1^+, which is consistent with the new experimental results, and the $D\bar{D}'^*$ threshold is very close to the $Z(4430)$ mass measured in the new LHCb experiment [3]. However, in our calculation, no bound state solution is found from the $D\bar{D}'^*$ interaction with a coupling constant $h''=0.42$ GeV^{-1}. In this work, the coupling constant h'' is determined from the decay width predicted in the quark model [35]. So, we increase the value of h''^2 to check if the results are sensitive to h''^2 , and find that even with $10h''^2$ there is no bound state produced from the $D\bar{D}'^*$ interaction.

Different from Ref. [17], the π exchange is dominant in the $D^*\bar{D}_1$ interaction in our model, and the effect of σ exchange is negligible. In the π exchange, the contributions from $D^*\bar{D}_1\to D_1\bar{D}^*$ diagram, *i. e.*, the cross diagram, is much more important than the contribution from the direct diagram $D^*\bar{D}_1\to D^*\bar{D}_1$. Hence, the contribution from the cross diagram $D^*\bar{D}_1\to D_1\bar{D}^*$ of the π exchange is dominant in the coupled $D^*\bar{D}_1-D\bar{D}'^*$ interaction. Since diagram $D^*\bar{D}_1\to D_1\bar{D}^*$ is composed of two $D^*D_1\pi$ vertices, the phase of the Lagrangian will be canceled. Hence, its dominance guarantees that the results are not sensitive to the relative phases of the Lagrangians, which are not well fixed.

There exists a bound solution with quantum number $J^P=0^-$ with cut-off about 1.8 GeV (see Fig. 1(a) and Fig. 1(b)). Such an S wave $D^*\bar{D}_1$ molecular state has been related to the $Z(4430)$ with the assumption that it carries spin parity $J^P=0^-$. However, the new experimental results favor quantum number 1^+, which corresponds to a P wave $D^*\bar{D}_1$ bound state. In the isovector sector, only two bound states are produced from the $D^*\bar{D}_1$ interaction. One of them has quantum number $I^G(J^P)=1^-(1^+)$ which is consistent with the experimental observed quantum number of the $Z(4430)$, $J^P=1^+$.

For the coupled $D^*\bar{D}_1$-$D\bar{D}'^*$ system, the cross diagram contribution from the π exchange for channel $D^*\bar{D}_1\to D_1\bar{D}^*$ is dominant. So the results are only sensitive to the square of the coupling constant, h'^2, for $D_1\to D^*\pi$. The value $h'=0.55$ GeV^{-1} in Ref. [33] is extracted from the old experimental data, which corresponds to decay width $\Gamma_{tot}(D_1(2420))\approx 6$ MeV [33]. Compared with the new suggested value in the PDG, 25 ± 6 MeV [21], the largest possible value of h' is about 1 GeV^{-1}. It is of interest to check the variation of results, especially for bound states with the $Z(4430)$ quantum numbers, with the variation of coupling constant h'. The results are presented in Fig. 2.

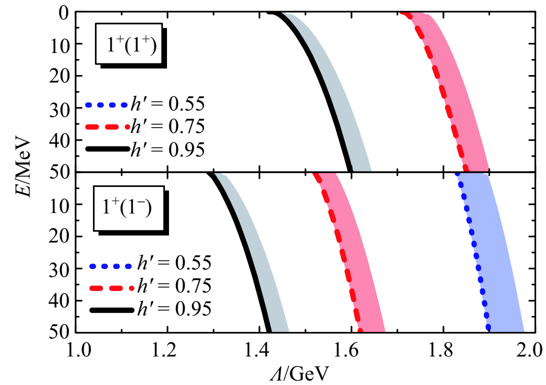

Fig. 2. (color online) The binding energies E for coupled $D^*\bar{D}_1$-$D\bar{D}'^*$ system with the variation of cut-off Λ. The lines are for the results with $n=2$ in form factor in Eq. (20) and the bands for results with $n=2\to\infty$.

With larger h', bound states are generated from the $D^*\bar{D}_1$-$D\bar{D}'^*$ interactions with smaller cut-offs. For example, with a coupling constant $h'=0.95$, the isovector bound states with $J^{PC}=1^{++}$ and $J^{PC}=1^{+-}$ are generated with cut-offs about 1.3 GeV and 1.5 GeV, respectively.

5 Summary

The new experimental results released by the LHCb Collaboration exclude the S wave $D^*\bar{D}_1$ molecular state interpretation with quantum number $J^P=0^-$ for the $Z(4430)$. In this paper we discuss the possibility to interpret the $Z_c(4430)$ as $D^*\bar{D}_1$ or $D\bar{D}'^*$ molecular state with quantum number $J^P=1^+$.

Isovector bound state solutions with spin-parity $J^P=1^+$ are found from the $D^*\bar{D}_1(2420)$ interaction, which may be related to the charged charmonium-like state $Z(4430)$. Different from the Belle experiment [1], the new observed mass of $Z(4430)$ is above the $D^*\bar{D}_1(2420)$ threshold. However, considering the current large uncertainties and broad width, further more precise measurements are expected. The $Z(4430)$ is still a candidate for the $D^*\bar{D}_1(2420)$ molecular state. On the theoretical side, it is interesting to consider the possibility of interpreting the $Z(4430)$ as a resonance from the $D^*\bar{D}_1(2420)$ interaction [28], which can provide a mass above the threshold and is still consistent with the conclusion in this work.

There is no bound state solution found from the

$D\bar{D}'^*(2600)$ interaction. A calculation with the coupled $D^*\bar{D}_1$-$D\bar{D}'^*$ interaction is also performed, and it is found that the results are almost the same as those obtained from the $D^*\bar{D}_1$ configuration only.

The current work is performed with the assumption that only channels with thresholds close to the mass of the $Z(4430)$ are important. A more comprehensive study with more coupled channels and more sophisticated treatment of the non-zero width of the excited D meson will be helpful to further understand the internal structure of the $Z(4430)$ and the molecular states with two excited D mesons.

In this work many molecular states are found from the $D^*\bar{D}_1(2420)$ interaction, but only one of them can be related to the observed $Z(4430)$. This is not surprising because those states are not obtained with the same cut-off, which should be the same for a given interaction. Hence, the states obtained in this work do not exist simultaneously. Besides, the effect of some molecular state predicted states may be too small to be observed in current experiments. Further more precise experiments are expected to check their existence.

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