Constraining the UT angle γ by CP violation parameters in $B^0 \rightarrow \pi^+\pi^-^*$

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Abstract: We calculate the tree and penguin amplitudes in the $B^0 \to \pi^+\pi^-$ decay channel employing the perturbative QCD factorization approach. Using the amplitudes as input with the theoretical uncertainties sufficiently considered, we constrain the UT angle γ to $53^{\circ} \leqslant \gamma \leqslant 70^{\circ}$, from the measurements of the CP violation parameters $C_{\pi^+\pi^-}$ and $S_{\pi^+\pi^-}$ in $B^0 \to \pi^+\pi^-$. The U-spin breaking effect between $B^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ is estimated to be around 30%.

Keywords: CP violation, γ extraction, B meson decay

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1 Introduction

In the Standard Model, quark mixing is described by the Cabbibo-Kobayashi-Maskawa (CKM) matrix [1], in which the nonzero phase angle induces charge conjugation and parity (CP) violation in weak interactions. For recent developments on the CKM matrix, one can refer to the review in Ref. [2]. It is important to examine the unitarity of the CKM matrix, since any deviation would indicate new physics beyond the Standard Model. The three angles of the well-known unitarity triangle (UT), which are defined by $\alpha \equiv \arg[-(V_{\rm td}V_{\rm tb}^*)/(V_{\rm ud}V_{\rm ub}^*)], \ \beta \equiv \arg[-(V_{\rm td}V_{\rm tb}^*)/(V_{\rm td}V_{\rm tb}^*)]$ and $\gamma \equiv \arg[-(V_{\rm ud}V_{\rm ub}^*)/(V_{\rm cd}V_{\rm cb}^*)]$, have been measured by experiments and the present averages are [3]

$$\alpha = (85.4^{+3.9}_{-3.8})^{\circ}, \quad \sin 2\beta = 0.682 \pm 0.019, \quad \gamma = (68.0^{+8.0}_{-8.5})^{\circ}. \tag{1}$$

The angle γ is the least known of the three angles. Methods have been proposed to extract γ from the tree-dominated modes B \rightarrow DK, including the GLW method [4], the ADS method [5], and the Dalitz-plot method [6], with different final states of D decays. Combining the B \rightarrow DK measurements performed by Belle, BaBar, CDF and LHCb [7], the CKMfitter group [8] obtained the above average for γ . Recently, the LHCb collaboration made two new measurements [9]. Alternatively, γ can also be determined by the U-spin analysis on the two-body charmless B decays, $B^0 \rightarrow \pi^+\pi^-$ and $B_s^0 \rightarrow K^+K^-$

[10]. A combination with the channels $B^0 \to \pi^0 \pi^0$ and $B^+ \to \pi^+ \pi^0$ makes the analysis more sophisticated [11]. Recently, following the method proposed in Ref. [11], the LHCb collaboration performed the U-spin and isospin analysis and obtained [12]

$$\gamma = (63.5^{+7.2}_{-6.7})^{\circ},\tag{2}$$

which has a smaller central value than the world average in Eq. (1).

In this letter, we constrain the UT angle γ from $B^0 \to \pi^+\pi^-$, with the help of a factorization approach to calculate the tree and penguin amplitudes. Similar ideas have been used to constrain α from $B^0 \to \pi^+\pi^-$ [13], and to constrain γ from $B_s^0 \to D_s^{\pm}K^{\mp}$ [14]. However, neither of them has given a strong constraint on the corresponding UT angle, for lack of precisely measured experimental results at that time. Recently, the CP violation parameters in $B^0 \to \pi^+\pi^-$ have been precisely measured [15], and the weighted averages of the results are given by [12],

$$C_{\pi^{+}\pi^{-}} = -0.30 \pm 0.05, \quad S_{\pi^{+}\pi^{-}} = -0.66 \pm 0.06, \quad (3)$$

with the statistical correlation $\rho(C_{\pi^+\pi^-}, S_{\pi^+\pi^-}) = -0.007$. The high precision of the parameters indicates the possibility that our constraint on γ is comparable to the world average in Eq. (1) and the results given in Ref. [12]. The method can also be applied to $B^0_s \to K^+K^-$.

The rest of the paper is organized as follows. In Sec. 2, the relevant formulas for the CP violation parameters

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in the channels $B^0 \to \pi^+\pi^-$ and $B^0_s \to K^+K^-$ are listed. In Sec. 3, we introduce our strategy for the numerical analysis and obtain the constraints on γ from the two channels, between which the U-spin breaking effect is also estimated. In Sec. 4, we conclude.

2 Theoretical formalism

For $B^0 \to \pi^+\pi^-$, the relevant effective Hamiltonian is given by

$$\mathcal{H}_{\text{eff}} = V_{\text{ub}}^* V_{\text{ud}} [C_1 O_1 + C_2 O_2] - V_{\text{tb}}^* V_{\text{td}} \sum_{n=3}^{10} C_n O_n + \text{h.c.},$$
(4)

where $O_{1,2(3-10)}$ are the tree (penguin) 4-quark operators, and C_{1-10} are the corresponding Wilson coefficients. After we apply some factorization approach to calculate the hadronic matrix elements $\langle \pi^+\pi^-|O_i|{\bf B}^0\rangle$, the amplitude of ${\bf B}^0\to\pi^+\pi^-$ can be expressed as

$$\mathcal{A}(\mathbf{B}^{0} \to \pi^{+} \pi^{-}) = V_{\rm ub}^{*} V_{\rm ud} \mathcal{T} - V_{\rm tb}^{*} V_{\rm td} \mathcal{P}$$

$$= V_{\rm ub}^{*} V_{\rm ud} (\mathcal{T} + \mathcal{P}) \left(1 + \frac{V_{\rm cb}^{*} V_{\rm cd}}{V_{\rm ub}^{*} V_{\rm ud}} \frac{\mathcal{P}}{\mathcal{T} + \mathcal{P}} \right),$$
(5)

where \mathcal{T} and \mathcal{P} are the tree and penguin amplitudes, respectively. Defining

$$de^{i\theta} \equiv \frac{|V_{cb}^* V_{cd}|}{|V_{cb}^* V_{cd}|} \frac{\mathcal{P}}{\mathcal{T} + \mathcal{P}},\tag{6}$$

with d and θ real-valued, we obtain the expression for the CP violation parameters

$$\begin{split} C_{\pi^+\pi^-} &= -\frac{2d\sin\theta\sin\gamma}{1+d^2-2d\cos\theta\cos\gamma},\\ S_{\pi^+\pi^-} &= -\frac{\sin(2\beta+2\gamma)-2d\cos\theta\sin(2\beta+\gamma)+d^2\sin(2\beta)}{1+d^2-2d\cos\theta\cos\gamma}. \end{split}$$
 (7)

For $C_{\pi^+\pi^-}$ and $S_{\pi^+\pi^-}$, we have accepted the convention in [12],

$$C_{\pi^{+}\pi^{-}} \equiv \frac{1 - |\lambda_{\pi^{+}\pi^{-}}|^{2}}{1 + |\lambda_{\pi^{+}\pi^{-}}|^{2}}, \quad S_{\pi^{+}\pi^{-}} \equiv \frac{2 \text{Im} \lambda_{\pi^{+}\pi^{-}}}{1 + |\lambda_{\pi^{+}\pi^{-}}|^{2}},$$

$$\lambda_{\pi^{+}\pi^{-}} \equiv \frac{q}{p} \frac{\mathcal{A}(\bar{B}^{0} \to \pi^{+}\pi^{-})}{\mathcal{A}(\bar{B}^{0} \to \pi^{+}\pi^{-})},$$
(8)

where q and p are the coefficients in the mass eigenstates $p|\mathbf{B}^0\rangle \pm q|\bar{\mathbf{B}}^0\rangle$.

Similarly for $B_s^0 \to K^+K^-$, one has

$$C_{\mathrm{K^{+}K^{-}}} \approx \frac{2\widetilde{d'}\sin\theta'\sin\gamma}{1+\widetilde{d'}^{2}+2\widetilde{d'}\cos\theta'\cos\gamma},$$

$$S_{\mathrm{K^+K^-}} \approx$$

$$-\frac{\sin(-2\beta_{s}+2\gamma)+2\widetilde{d}'\cos\theta'\sin(-2\beta_{s}+\gamma)+\widetilde{d}'^{2}\sin(-2\beta_{s})}{1+\widetilde{d}'^{2}+2\widetilde{d}'\cos\theta'\cos\gamma},$$
(9)

where the real-valued parameters are defined by

$$\widetilde{d}' \equiv \frac{|V_{cs}||V_{ud}|}{|V_{cd}||V_{us}|} d', \quad d'e^{i\theta'} \equiv \frac{|V_{cb}^*V_{cd}|}{|V_{ub}^*V_{ud}|} \frac{\mathcal{P}'}{\mathcal{T}' + \mathcal{P}'}, \quad (10)$$

with \mathcal{T}' (\mathcal{P}') representing the tree (penguin) amplitude in $B_s^0 \to K^+K^-$. $\beta_s \equiv \arg[-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)]$ gives the mixing phase in the B_s^0 - \bar{B}_s^0 mixing system.

3 Numerical analysis

The present average of the UT angle β is given in Eq. (1), which has a two-fold ambiguity $2\beta \to \pi - 2\beta$. A series of measurements [16] prefer that $\cos 2\beta$ is positive, so we accept

$$\beta = (21.50^{+0.75}_{-0.74})^{\circ}. \tag{11}$$

Choosing the sample values for d and θ , $de^{i\theta} = 0.23e^{i2.4}$, we can then obtain the γ dependence of $C_{\pi^+\pi^-}$ and $S_{\pi^+\pi^-}$, as shown in Fig. 1. The experimental 1 σ allowed regions are also displayed. Fig. 1(b) shows that $S_{\pi^+\pi^-}$ is very sensitive to the change of the angle γ , and on the other hand, precise measurements for $S_{\pi^+\pi^-}$ have been performed. This indicates that γ is potentially strongly constrained in our method, though there are considerable theoretical uncertainties in any factorization approach.

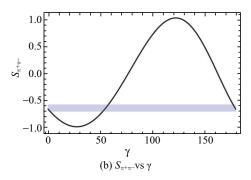


Fig. 1. (color online) The solid curves correspond to the sample choice d=0.23 and $\theta=2.4$. The light blue bands show the experimentally 1 σ allowed regions $-0.35 \leqslant C_{\pi^+\pi^-} \leqslant -0.25$ and $-0.72 \leqslant S_{\pi^+\pi^-} \leqslant -0.60$, respectively.

In the perturbative QCD (PQCD) approach based on the transverse momentum factorization [17], hadronic matrix elements are factorized into convolutions of the calculable hard kernels and the non-perturbative meson wave functions which are, however, universal. For recent theoretical developments, see Refs. [18–21]. The PQCD approach has been applied in analysis on hadronic B meson decays, successfully making predictions for both branching ratios and CP violation [22, 23]. Especially for $B^0 \to \pi^+\pi^-$, the PQCD prediction of the branching ratio is $(5.8^{+3.0+0.5+0.4}_{-2.1-0.4-0.3}) \times 10^{-6}$ [23], which is consistent with the experimental result $(5.12 \pm 0.19) \times 10^{-6}$ [3]. Therefore, we employ the PQCD approach to calculate the tree and penguin amplitudes here. The formulas for calculating the leading-order decay amplitudes are given by Eqs. (50–61) in Ref. [23]. We also include the next-toleading-order corrections to the $B \to \pi$ transition form factors, of which the twist-2 and -3 contributions have been studied in Refs. [19] and [20], respectively.

To perform a reliable analysis, we need to sufficiently take into account the uncertainties introduced by the calculation of the hadronic matrix elements. In the calculation, we adopt the updated non-asymptotic distribution amplitudes [24],

$$\begin{split} \phi_{\pi}^{A}(x) &= \frac{f_{\pi}}{2\sqrt{6}} 6x(1-x)[1 + a_{2}^{\pi}C_{2}^{3/2}(2x-1) + a_{4}^{\pi}C_{4}^{3/2}(2x-1)], \\ \phi_{\pi}^{P}(x) &= \frac{f_{\pi}}{2\sqrt{6}} [1 + 30\eta_{3}^{\pi}C_{2}^{1/2}(2x-1) - 3\eta_{3}^{\pi}\omega_{3}^{\pi}C_{4}^{1/2}(2x-1)], \\ \phi_{\pi}^{T}(x) &= \frac{f_{\pi}}{2\sqrt{2N_{c}}} (1 - 2x)\{1 + \frac{1}{2}\eta_{3}^{\pi}(10 - \omega_{3}^{\pi})C_{2}^{3/2}(2x-1) \\ &- 15\eta_{3}^{\pi}(10 - \omega_{3}^{\pi})x(1-x)\}, \end{split}$$

where $C_n^{\alpha}(2x-1)$ are the well known Gegenbauer polynomials with x the longitudinal momentum fraction of the quark in pion. The values of the Gegenbauer moments, a_2^{π} and a_4^{π} , have been determined in the global fit to the data of the pion electromagnetic form factor [25], which yields

$$a_2^{\pi} = 0.17 \pm 0.08, \quad a_4^{\pi} = 0.06 \pm 0.10.$$
 (13)

To keep it safe, we double the error bars in the numerical analysis. In Ref. [21] where the joint resummation was performed for the pion transition form factor in the transverse-momentum factorization formalism, the authors found that their prediction for the form factor with $a_2^{\pi} = 0.05$ agrees well with the experimental data. Our choice for the range of a_2^{π} covers this value. As for the other non-perturbative parameters η_3^{π} and ω_3^{π} , we accept the values taken in Ref. [24], also with doubled error bars. The shape parameter in the distribution amplitude of the B⁰ meson [26] is taken value in the range

$$\omega_b \in [0.36, 0.44].$$
 (14)

We also consider the uncertainties caused by the unknown next-to-leading-order corrections characterized by the choice that $\Lambda_{\rm QCD} \in [0.20, 0.30]$ and a 20% variation of the factorization scale. Taking values for the theoretical parameters randomly in the ranges covering their uncertainties, we perform the PQCD calculation and obtain 99 points of (d, θ) , which are shown in Fig. 2. At each point of (d, θ) , we perform the global fit of β and γ to the experimental results of the CP violation parameters in Eq. (3) and that of β in Eq. (11). Then, we combine the 1 σ allowed regions of all fits at the 99 points, and regard it as our constraint on γ and β . As shown in Fig. 3, the constraint on γ is

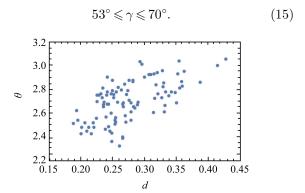


Fig. 2. (color online) Plots for (d, θ) calculated with the random theoretical parameters ranging in the allowed regions.

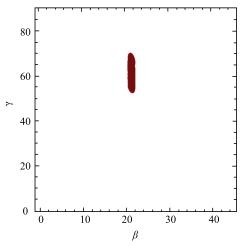


Fig. 3. (color online) The 68% C.L. region for β - γ with the theoretical uncertainties considered.

Although the constraint on γ obtained in this work is comparable to the world average in PDG2014 [3], it covers a broad range. It suffers from both experimental and theoretical uncertainties. On the theoretical side, the uncertainties mainly originate from two sources, the scale dependence of the decay amplitudes and the wave functions. The scale dependence can be reduced significantly at the complete NLO correction of α_s , while the

wave functions are expected to be precisely determined in a global fit of the PQCD results to all of the available data. After the two theoretical developments in the PQCD framework are accomplished, the precision of the angle γ extracted with this method will be remarkably improved.

We can also perform a similar analysis to $B_s^0 \to K^+K^-$ using Eqs. (9) and (10), though the measurements of its CP violation parameters are much less precise. The averaged experimental results are given by [12]

$$C_{K^+K^-} = 0.14 \pm 0.11, \quad S_{K^+K^-} = 0.30 \pm 0.13, \quad (16)$$

with the statistical correlation $\rho(C_{K^+K^-}, S_{K^+K^-}) = 0.02$. To improve the precision of the γ determination, β_s is expressed in terms of β and γ . However, the $B_s^0 \to K^+K^-$ constraint $20^\circ \leqslant \gamma \leqslant 150^\circ$ is still too loose.

As a byproduct, we also estimate the U-spin breaking effect in the two channels $B^0\to\pi^+\pi^-$ and $B^0_s\to K^+K^-,$ which is parameterized by

$$d'e^{i\theta'} = de^{i\theta}(1 + re^{i\theta_r}). \tag{17}$$

The PQCD result is

$$r = 0.3 \pm 0.1, \quad \theta_r = -1.2 \pm 0.2.$$
 (18)

In Ref. [12], the U-spin breaking effect is parameterized by two relative magnitudes r_D and r_G with the corresponding phase shifts θ_{r_D} and θ_{r_G} ,

$$d'e^{i\theta'} = de^{i\theta} \frac{1 + r_G e^{i\theta_{r_G}}}{1 + r_D e^{i\theta_{r_D}}}.$$
 (19)

By comparing Eqs. (17) and (19), one finds the relation between the two parametrizations, $re^{i\theta_r} = (1 + r_G e^{i\theta_{r_G}})/(1+r_D e^{i\theta_{r_D}})-1$. Assuming the parameters range within the region

$$r_D, r_G \in [0, 0.5], \quad \theta_{r_D}, \theta_{r_G} \in [-\pi, \pi],$$
 (20)

the authors obtained $\gamma = (63.5^{+7.2}_{-6.7})^{\circ}$. This region can fully cover the PQCD result (including the uncertainties), so we conclude that the assumption about the U-spin breaking in Ref. [12] is reasonable.

4 Conclusion

We extract the UT angle γ from the precise experimental results of $C_{\pi^+\pi^-}$ and $S_{\pi^+\pi^-}$ given in [12], with the tree and penguin amplitudes in $B^0 \to \pi^+\pi^-$ calculated in the PQCD approach. Including the theoretical uncertainties, we constrain $53^\circ \leqslant \gamma \leqslant 70^\circ$ at 68% probability. Through a similar method, the angle γ is also constrained in the range $20^\circ - 150^\circ$ by the measurements of $C_{K^+K^-}$ and $S_{K^+K^-}$. The U-spin breaking effect between the two channels is found to be smaller than 50%, which indicates that the results in Ref. [12] are reliable.

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