Gauge theory of massless spin-$\frac{3}{2}$ field in de Sitter space-time

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Abstract: On several levels of theoretical physics, especially particle physics and early universe cosmology, de Sitter space-time has become an attractive possibility. The principle of local gauge invariance governs all known fundamental interactions of elementary particles, from electromagnetism and weak interactions to strong interactions and gravity. This paper presents a procedure for defining the gauge-covariant derivative and gauge invariant Lagrangian density in de Sitter ambient space-time formalism. The gauge invariant field equation is then explicitly calculated in detail for a massless spin-$\dfrac{3}{2}$ gauge field.

Keywords: gauge-covariant derivative, de Sitter, ambient space-time, gravitino

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1 Introduction

Recent observational data and gravitational wave detection indicates that the early universe has passed through a de Sitter-like phase [1–3] . The de Sitter phase is the vacuum solution of the Einstein equation using a positive cosmological constant Λ. The de Sitter space-time is a maximally symmetric curved space-time. Other space-times which have this level of symmetry are the anti-de Sitter space-time and the familiar Minkowski space-time; thus, one would expect to extend quantum field theory (QFT) from Minkowski to de Sitter space-time to explain elementary particle physics by the de Sitter group [4–9].

In space-time, particle states are labeled using Poincaré labels, the values of which are closely related to the unitary irreducible representation (UIR) labels of mass and spin. From the group-theoretical perspective, we associate these parameters with the eigenvalues of the Casimir operators. Although the notion of mass is not as clear in de Sitter space-time as in Minkowski space-time in a field-theoretical sense, the Casimir operators of $SO(1,4)$ could resolve the issue. There is a very straightforward link between the Casimir operators and the wave equation, which we shall investigate using ambient space-time formalism. Ambient space-time formalism is a useful tool for making the link between QFT in de Sitter space-time and the group-theoretical approach by, in general, reducing the field equation to a Casimir eigenvalue equation. This connection was established for field theory in space-time in Refs. [10, 11], and led to the canonical form of the covariant particle equations. In this way, the connection between the Wigner UIRs of the Poincaré group and the solutions to the field equation were made explicit.

Rarita and Schwinger derived a wave equation for a massive spin-$\dfrac{3}{2}$ particle (gravitino) in 1941 [12]. The gravitino is the gauge fermion super-symmetric partner of the graviton and can be important in quantum gravity and quantum cosmology. For example, it has been suggested as a candidate for dark matter [13, 14]. Gravitinos can be copiously produced in the ultra-high-temperature region, for example, near the big bang and black hole horizons [15].

The spin-$\dfrac{3}{2}$ field has previously been studied in a de Sitter space-time background [16, 17]. In the present paper, the massless spin-$\dfrac{3}{2}$ field is considered in de Sitter space-time using gauge theory, which is a general class of quantum field theories used to describe elementary particles and their interactions. An interaction is defined using the gauge-covariant derivative, which is defined as a quantity that preserves the gauge invariant transformation of the Lagrangian. The field equation of the massless spin-$\dfrac{3}{2}$ field, or the vector-spinor gauge field, is the gauge invariant in de Sitter space-time and the massless field in Minkowski space-time for Spin $\geqslant 1$. Vector-spinor

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gauge fields are spinor fields and, consequently, their corresponding gauge group must have spinorial generators to justify a set of well-defined gauge-covariant derivatives [9, 18]. A set of anti-commutative generators satisfy a super-algebra; nonetheless, such an algebra would not be closed, since its constituent generators are Grasmanian functions which will have usual functions as their multiplication products, i.e., the anti-commutation of two spinor generators will become a tensor generator. In this case, to obtain a closed super-algebra, the Grasmanian generators must be coupled to the generators of the de Sitter group. Additionally, in the language of gauge theory, one may describe a vector-spinor gauge field as a real force which must be coupled to a spin-2 gauge potential. These two gauge fields can describe a gravitational field. The gauge group in this case is a super-group.

In our previous work [18] we introduced the gauge-covariant derivative, gauge invariant Lagrangian density, and gauge invariant field equation in de Sitter ambient space-time formalism, where the general solution of this field has been studied in Ref. [24]. Because of the complexity of the calculations, the gauge-covariant derivative, gauge invariant Lagrangian density, and gauge invariant field equation are explicitly calculated in the present paper. Section 2 of the current work introduces the notation of de Sitter ambient space-time formalism. Section 3 presents the general framework of gauge theory. Section 4 presents the gauge theory of massless spin-$\frac{3}{2}$. In Section 5, we draw conclusions and the appendices present these conclusions in more detail.

2 Notation

The de Sitter (dS) space-time can be identified by a 4-dimensional hyperboloid embedded in 5-dimensional Minkowskian space-time with the constraint:

$$X_H=\{x\in\mathbb{R}^5|\ xx=\eta_{\alpha\beta}x^\alpha x^\beta=-H^{-2}\},\ \alpha,\beta=0,1,2,3,4, \tag{1}$$

where $\eta_{\alpha\beta}=$diag$(1,-1,-1,-1,-1)$ and H is the Hubble parameter. The de Sitter metric is:

$$\mathrm{d}s^2=\eta_{\alpha\beta}\mathrm{d}x^\alpha\mathrm{d}x^\beta|_{x^2=-H^{-2}}=g^{\mathrm{d}S}_{\mu\nu}\mathrm{d}X^\mu\mathrm{d}X^\nu,\ \mu=0,1,2,3, \tag{2}$$

where the X^μ's are 4 spacetime intrinsic coordinates of the dS hyperboloid. Any geometrical object in this space can be written either in terms of four local coordinates X^μ (intrinsic space notation) or five global coordinates x^α (ambient space notation). The de Sitter group has two Casimir operators:

$$Q^{(1)}=-\frac{1}{2}L_{\alpha\beta}L^{\alpha\beta},\ \alpha,\beta=0,1,2,3,4, \tag{3}$$

$$Q^{(2)}=-W_\alpha W^\alpha\ ,\ W_\alpha=\frac{1}{8}\epsilon_{\alpha\beta\gamma\delta\eta}L^{\beta\gamma}L^{\delta\eta}, \tag{4}$$

where the symbol $\epsilon_{\alpha\beta\gamma\delta\eta}$ stands for the usual antisymmetric tensor and $L_{\alpha\beta}=M_{\alpha\beta}+S_{\alpha\beta}$ are ten infinitismal generators of the de Sitter group. The orbital part $M_{\alpha\beta}$is:

$$M_{\alpha\beta}=-i(x_\alpha\partial_\beta-x_\beta\partial_\alpha)=-i(x_\alpha\partial^\top_\beta-x_\beta\partial^\top_\alpha), \tag{5}$$

where $\partial^\top_\beta=\theta_\beta^{\ \alpha}\partial_\alpha$ is the transverse derivative $(x\cdot\partial^\top=0)$ and $\theta_{\alpha\beta}=\eta_{\alpha\beta}+H^2x_\alpha x_\beta$ is considered as the projection tensor in ambient space-time notation. The spinorial part $S_{\alpha\beta}$ with half-integer spin $s=l+\frac{1}{2}$ is defined by:

$$S^{(s)}_{\alpha\beta}=S^{(l)}_{\alpha\beta}+S^{(\frac{1}{2})}_{\alpha\beta}, \tag{6}$$

in which the first term acts on a tensor field as follows:

$$S^{(l)}_{\alpha\beta}\Psi_{\gamma_1\cdots\gamma_l}=-i\sum_{i=1}^{l}\left(\eta_{\alpha\gamma_i}\Psi_{\gamma_1\cdots(\gamma_i\to\beta)\cdots\gamma_l}-\eta_{\beta\gamma_i}\Psi_{\gamma_1\cdots(\gamma_i\to\alpha)\cdots\gamma_l}\right), \tag{7}$$

and the second term is:

$$S^{(\frac{1}{2})}_{\alpha\beta}=-\frac{i}{4}[\gamma_\alpha,\gamma_\beta]. \tag{8}$$

The five γ-matrices are the generators of the Clifford algebra based on the metric $\eta^{\alpha\beta}$:

$$\{\gamma^\alpha,\gamma^\beta\}=2\eta^{\alpha\beta}\mathbb{I}_{4\times4}, \tag{9}$$

and their four-dimensional matrix representations are [19, 20]:

$$\gamma^1=\begin{pmatrix}0 & i\sigma^1\\ i\sigma^1 & 0\end{pmatrix},\gamma^2=\begin{pmatrix}0 & -i\sigma^2\\ -i\sigma^2 & 0\end{pmatrix},$$

$$\gamma^3=\begin{pmatrix}0 & i\sigma^3\\ i\sigma^3 & 0\end{pmatrix},$$

$$\gamma^0=\begin{pmatrix}\mathbb{I}_{2\times2} & 0\\ 0 & -\mathbb{I}_{2\times2}\end{pmatrix},\gamma^4=\begin{pmatrix}0 & \mathbb{I}_{2\times2}\\ -\mathbb{I}_{2\times2} & 0\end{pmatrix},$$

$$\gamma^{\alpha\dagger}=\gamma^0\gamma^\alpha\gamma^0\qquad(\gamma^4)^2=-\mathbb{I}\qquad(\gamma^0)^2=\mathbb{I} \tag{10}$$

where $\mathbb{I}$ and σ^i's are the unit matrix and the Pauli matrices, respectively. The action of the Casimir operator $Q^{(1)}_j$ on a vector-spinor field $\Psi_\alpha(x)$, is [16]:

$$Q^{(1)}_j\Psi(x)=\left(-\frac{1}{2}M_{\alpha\beta}M^{\alpha\beta}+\frac{i}{2}\gamma_\alpha\gamma_\beta M^{\alpha\beta}-\frac{11}{2}\right)\Psi_\alpha(x)$$
$$-2\partial_\alpha x\cdot\Psi(x)+2x_\alpha\partial\cdot\Psi(x)+\gamma_\alpha(\gamma\cdot\Psi(x)). \tag{11}$$

The "scalar" Casimir operator is defined as $Q^{(1)}_0=-\frac{1}{2}M^{\alpha\beta}M_{\alpha\beta}=-\partial^\top_\alpha\partial^{\alpha\top}$.

3 General formulation of gauge theory

This section briefly discusses a general formulation of gauge theory of the type needed for super-gravity. An infinitesimal symmetry transformation is determined by a set of parameters which we denote in general as ϵ^A, $A=1,\cdots,m$ where m is the number of independent transformations, and operation $\delta(\epsilon)$ depends linearly on the parameter and acts on the fields of the dynamical system under study. For global symmetries, the parameters do not depend on the space-time point at which the symmetry operation is applied. Because the symmetry operation is linear in ϵ, we can write it in general as:

$$\delta(\epsilon)=\epsilon^A T_A, \tag{12}$$

in which T_A is an operator on the space-time of fields. It describes the symmetry transformation with the parameter stripped. T_A satisfies the following commutation relation:

$$[T_A,T_B]=f_{AB}^C T_C, \tag{13}$$

wheref_{AB}^C 's are the structure constants of the algebra. The notation and the formalism presented above apply to all types of symmetry; internal symmetry, space-time symmetry, and super-symmetry can be viewed as special cases. We now want to consider field theories in which the Lagrangian contains both gauge fields B_μ^A with $\mu=0,1,2,3$ and other fields ϕ^i, where i labels the fields, whose transformation rules are:

$$\delta(\epsilon)\phi^i(x)=\epsilon^A(T_A\phi^i)(x). \tag{14}$$

One very important covariant quantity is the covariant derivative of a field ϕ^i, for which the gauge transformation rule has the form (14):

$$D_\mu\phi^i\equiv(\partial_\mu-\delta(B_\mu))\phi^i=(\partial_\mu-B_\mu^A T_A)\phi^i, \tag{15}$$

and the notation $\delta(B_\mu)$ means that the covariant derivative is constructed using the specific prescription to subtract the gauge transform of the field from the gauge field itself as the symmetry parameter. For each symmetry, there is a field space-time generator T_A, but the parameters $\epsilon^A(x)$ are arbitrary functions in space-time. To realize local symmetry in Lagrangian field theory, one needs a gauge field, which we will generically denote as $B_\mu^A(x)$, which transforms as:

$$\delta(\epsilon)B_\mu^A\equiv\partial_\mu\epsilon^A+\epsilon^C B_\mu^B f_{BC}^A. \tag{16}$$

The proof for this equation is provided in Appendix A. We can use the covariant derivative to define the next important set of quantities in any gauge theory of algebra. For each generator of the algebra, curvature $R_{\mu\nu}^A$ is a second rank anti-symmetric tensor. The curvature reads:

$$[D_\mu,D_\nu]=-R_{\mu\nu}^A T_A,$$

$$R_{\mu\nu}^A=\partial_\mu B_\nu^A-\partial_\nu B_\mu^A+B_\mu^A B_\nu^B f_{AB}^C. \tag{17}$$

The proof for this is provided in Appendix A. Next, we try to obtain the action using this formalism.

4 Gauge theory of massless spin-$\frac{3}{2}$

We now consider the vector-spinor gauge field $\Psi_\alpha(x)$. The principle of gauge invariance asserts that the interactions of different fields with a specific gauge field should be defined using the definition of gauge-covariant derivatives. The gauge potential in the present case is a spinor field which satisfies the Grassmann algebra. Correspondingly, the symmetry group involved includes spinorial generators (generators with anti-commutation relations). Assuming that there are N vector-spinor gauge fields (Ψ_α^A, with $A=1,\cdots,N$), the gauge-covariant derivative can be defined as:

$$D_\beta^\Psi=\nabla_\beta^\top+i\left({\Psi_\beta}^A\right)^\dagger\gamma^0\mathcal{Q}_A, \tag{18}$$

where $\nabla_\beta^\top$ is the transverse-covariant derivative. The transverse-covariant derivative makes a tensor-spinor field of rank $l+1$ from a tensor-spinor field of rank l on the de Sitter ambient space-time formalism [9]:

$$\nabla_\beta^\top\Psi_{\alpha_1\cdots\alpha_l}\equiv\left(\partial_\beta^\top+\gamma_\beta^\top\not{x}\right)\Psi_{\alpha_1\cdots\alpha_l} -H^2\sum_{n=1}^{l}x_{\alpha_n}\Psi_{\alpha_1\cdots\alpha_{n-1}\beta\alpha_{n+1}\cdots\alpha_l}, \tag{19}$$

$$\tilde{\nabla}_\beta^\top\tilde{\Psi}_{\alpha_1\cdots\alpha_l}\equiv\partial_\beta^\top\tilde{\Psi}_{\alpha_1\cdots\alpha_l}-H^2\sum_{n=1}^{l}x_{\alpha_n}\tilde{\Psi}_{\alpha_1\cdots\alpha_{n-1}\beta\alpha_{n+1}\cdots\alpha_l}, \tag{20}$$

where the conjugate spinor is $\tilde{\Psi}_\alpha\equiv\Psi_\alpha^\dagger\gamma^0$; in addition, $\not{x}=\gamma_\alpha x^\alpha$ and $\gamma_\alpha^\top=\theta_\alpha^\beta\gamma_\beta$. Generators $\mathcal{Q}_A$ are spinor-like, satisfying some anti-commutation relations. It is evident that the super-algebra in de Sitter ambient space-time formalism will naturally appear. A brief discussion of the simple case of $N=1$ is instructive. The gauge-covariant derivative:

$$D_\beta^\Psi=\nabla_\beta^\top+i\left(\Psi_\beta\right)^\dagger\gamma^0\mathcal{Q}=\nabla_\beta^\top+i\left(-\bar{\Psi}_\beta\gamma^4\right)^i\mathcal{Q}_i,$$

where $i=1,\cdots,4$ is the spinorial index and $\bar{\Psi}_\beta=\Psi_\beta^\dagger\gamma^0\gamma^4$. In order to acquire a rank-1 tensor field for the covariant derivative, a spinor generator $\mathcal{Q}$ must be added. In this case, the super-algebra between the Grassmanian generators is not closed, because the product of two Grassmanian numbers becomes a normal number. To obtain a closed super-algebra, these generators must be coupled to the de Sitter group generators $L_{\alpha\beta}$. In other words the vector-spinor gauge field Ψ_β must be coupled to the tensor gauge field $\mathcal{K}_\beta^{\gamma\delta}$ [9]. $\mathcal{K}_\beta^{\gamma\delta}$ is a massless spin-2 rank-3 mixed-symmetric tensor field. The $N=1$ super-algebra in de Sitter ambient space-time formalism has been calculated [21] as:

$$\{\mathcal{Q}_i,\mathcal{Q}_j\}=\left(S^{(\frac{1}{2})}_{\alpha\beta}\gamma^4\gamma^2\right)_{ij}L^{\alpha\beta}, \tag{21}$$

$$[\mathcal{Q}_i,L_{\alpha\beta}]=\left(S^{(\frac{1}{2})}_{\alpha\beta}\mathcal{Q}\right)_i,\ [\tilde{\mathcal{Q}}_i,L_{\alpha\beta}]=-\left(\tilde{\mathcal{Q}}S^{(\frac{1}{2})}_{\alpha\beta}\right)_i, \tag{22}$$

$$[L_{\alpha\beta},L_{\gamma\delta}]=-i(\eta_{\alpha\gamma}L_{\beta\delta}+\eta_{\beta\delta}L_{\alpha\gamma}-\eta_{\alpha\delta}L_{\beta\gamma}-\eta_{\beta\gamma}L_{\alpha\delta}), \tag{23}$$

where $\tilde{\mathcal{Q}}_i=(\mathcal{Q}^t\gamma^4,C)_i$, and Q^t is the transpose of $\mathcal{Q}$. Charge conjugation C is defined in Ref. [22]. It can be shown that $\tilde{\mathcal{Q}}\gamma^4\mathcal{Q}$ is a scalar field under the de Sitter transformation [22]. To define the gauge-covariant derivative, one must use the presented $N=1$ super-algebra; hence, the gauge fields are $\mathcal{H}_\alpha^{\ A}\equiv(\mathcal{K}_\beta^{\ \gamma\delta},\Psi_\beta^{\ i})$, and the generators are $Z_A\equiv(L_{\alpha\beta},\mathcal{Q}_i)$. The gauge-covariant derivative can be defined as:

$$D^{\mathcal{H}}_\beta=\nabla^\top_\beta+i\mathcal{H}_\beta^{\ A}Z_A. \tag{24}$$

One can rewrite the $N=1$ super-algebra as:

$$[Z_A,Z_B\}=\mathcal{C}_{BA}^{\ \ C}Z_C,$$

where $[Z_A,Z_B\}$ is a commutation or an anti-commutation relation and $\mathcal{C}_{BA}^{\ \ C}$ is the structure constant of the algebra. Under a local infinitesimal gauge transformation generated by $\epsilon^A(x)Z_A$ (Appendix A):

$$\delta_\epsilon\mathcal{H}_\beta^{\ A}=D^{\mathcal{H}}_\beta\epsilon^A=\nabla^\top_\beta\epsilon^A+\mathcal{C}_{BC}^{\ \ A}\mathcal{H}_\beta^{\ C}\epsilon^B. \tag{25}$$

The covariant derivative can be used to define the next important set of quantities in any gauge theory of an algebra. According to the general framework, one can obtain:

$$-[D^{\mathcal{H}}_\alpha,D^{\mathcal{H}}_\beta\}=R_{\alpha\beta}^{\ \ A}Z_A,$$

where the curvature R is (Appendix A):

$$\begin{aligned}R_{\alpha\beta}^{\ \ A}=&\nabla^\top_\alpha\mathcal{H}_\beta^{\ A}-\nabla^\top_\beta\mathcal{H}_\alpha^{\ A}\\&+\mathcal{H}_\beta^{\ B}\mathcal{H}_\alpha^{\ C}\mathcal{C}_{BC}^{\ \ A},\ \ x^\alpha R_{\alpha\beta}^{\ \ A}=0=x^\beta R_{\alpha\beta}^{\ \ A}.\end{aligned} \tag{26}$$

For the spinorial part, the curvature is:

$$R_{\alpha\beta}^{\ \ i}=\nabla^\top_\alpha\Psi_\beta^{\ i}-\nabla^\top_\beta\Psi_\alpha^{\ i}+\mathcal{H}_\beta^{\ B}\mathcal{H}_\alpha^{\ C}\mathcal{C}_{BC}^{\ \ i},$$

where the transverse-covariant derivative becomes:

$$\nabla^\top_\alpha\Psi_\beta=\partial^\top_\alpha\Psi_\beta+\gamma^\top_\alpha\ \not{x}\Psi_\beta-x_\beta\Psi_\alpha.$$

Next, we obtain the Lagrangian in ambient space formalism by using the gauge principle and defining the gauge covariant derivative. The interactions between the elementary systems in the universe are governed by the gauge principle and formulated through the gauge-covariant derivative, which is defined as a quantity that preserves the gauge invariant transformation of the Lagrangian. The super-gauge invariant action, or the super-gravity Lagrangian in de Sitter ambient space-time formalism, is [9, 23]:

$$S_g=\int \mathrm{d}\mu(x)R_{\alpha\beta}^{\ \ A}g_{AB}R^{\alpha\beta B},$$

where g_{AB} is the numerical constant matrix and $\mathrm{d}\mu(x)$ is the de Sitter invariant volume element [9]. For the vector-spinor field part, the action is given by:

$$S_g[\Psi,\mathcal{K}]=\int \mathrm{d}\mu(x)\left(\tilde{R}^i\right)_{\alpha\beta}\left(R^i\right)^{\alpha\beta},$$

where

$$\tilde{R}^i_{\alpha\beta}=\tilde{\nabla}^\top_\alpha\tilde{\Psi}_\beta^{\ i}-\tilde{\nabla}^\top_\beta\tilde{\Psi}_\alpha^{\ i}+\mathcal{H}_\beta^{\ \mathcal{B}}\mathcal{H}_\alpha^{\ \mathcal{C}}\mathcal{C}_{\mathcal{BC}}^{\ \ \ i}.$$

The transverse covariant-derivative acts on the conjugate spinor in the following form [9, 18]:

$$\tilde{\nabla}^\top_\beta\tilde{\Psi}_\alpha\equiv\partial^\top_\beta\tilde{\Psi}_\alpha-x_\alpha\tilde{\Psi}_\beta. \tag{27}$$

In the approximation of the linear field equation, we consider action:

$$\begin{aligned}&S[\Psi,\tilde{\Psi}]\\&\simeq\int \mathrm{d}\mu(x)\left[\left(\tilde{\nabla}^\top_\alpha\tilde{\Psi}_\beta-\tilde{\nabla}^\top_\beta\tilde{\Psi}_\alpha\right)\left(\nabla^{\top\alpha}\Psi^\beta-\nabla^{\top\beta}\Psi^\alpha\right)\right].\end{aligned} \tag{28}$$

The field equation of the vector-spinor field in the linear approximation can be obtained by using the Euler-Lagrange equation as [Appendix B]:

$$(x_\alpha-\partial^\top_\alpha)\left(\nabla^{\top\alpha}\Psi^\beta-\nabla^{\top\beta}\Psi^\alpha\right)=0. \tag{29}$$

This equation of motion in terms of the Casimir operator can be rewritten as [Appendix C]:

$$\left(Q^{(1)}_{\frac{3}{2}}+\frac{5}{2}\right)\Psi_\alpha+\nabla^\top_\alpha\partial^\top\cdot\Psi=0. \tag{30}$$

It has been shown that Eq. (30) is completely consistent with the vector-spinor field that is calculated on the basis of the group theory approach [9, 16, 17]. It has also been shown that gauge transformations leave the action and field equation invariant. One can then prove the following identities:

$$Q^{(1)}_{\frac{3}{2}}\nabla^\top_\alpha=\nabla^\top_\alpha Q^{(1)}_{\frac{1}{2}}\ ,\ \partial^\top\cdot\nabla^\top\psi=-\left(Q^{(1)}_{\frac{1}{2}}+\frac{5}{2}\right)\psi. \tag{31}$$

ψ is an arbitrary spinor field and:

$$Q^{(1)}_{\frac{1}{2}}\psi=\left(Q_0+\not{x}\ \not{\partial}^\top-\frac{5}{2}\right)\psi.$$

One can use these identities to show that the field equation (30) is invariant for the following gauge transformation:

$$\Psi_\alpha\longrightarrow\Psi^g_\alpha=\Psi_\alpha+\nabla^\top_\alpha\psi.$$

The vector-spinor Lagrangian density is then invariant under the following gauge transformations [Appendix D]:

$$\Psi_\alpha \longrightarrow \Psi^g_\alpha = \Psi_\alpha + \nabla^\top_\alpha \psi,$$
$$\tilde{\Psi}_\alpha \longrightarrow \tilde{\Psi}^g_\alpha = \tilde{\Psi}_\alpha + \partial^\top_\alpha \tilde{\psi}, \quad \tilde{\nabla}^\top_\alpha \tilde{\psi} = \partial^\top_\alpha \tilde{\psi}.$$

Because the action is invariant under a gauge transformation, we can choose to fix any gauge we want. The gauge fixing terms are added to the Lagrangian and, in the linear approximation, the gauge fixing field equation becomes:

$$\left(Q^{(1)}_{\frac{3}{2}} + \frac{5}{2}\right)\Psi_\alpha + c\nabla^\top_\alpha \partial^\top \cdot \Psi = 0. \tag{32}$$

The selection of the gauge fixing parameter c determines the space of the gauge solutions. The general solution has been calculated in our previous work [18, 24].

5 Conclusions

The theory of gauge fields is universally recognized to constitute a supporting pillar of fundamental physics. In the present paper, the gauge invariant field equation for a massless spin-$\frac{3}{2}$ gauge field in de Sitter space-time is reformulated using the definition of the gauge-covariant derivative and gauge invariant Lagrangian density in de Sitter ambient space-time formalism. In the framework of gauge theory, the vector-spinor gauge field Ψ_α can be considered as a potential of a possible new force in nature, but this gauge field must be coupled to the gauge potential $\mathcal{K}_\beta^{\gamma\delta}$. As a result, Ψ_α can be considered to be a new sector of the gravitational field. This means that the gravitational field can be decomposed into three parts: the background $\theta_{\alpha\beta}$, the gravitational waves $\mathcal{K}_\beta^{\gamma\delta}$, and Ψ_α. This result can be used to construct a unitary form of super-gravity by coupling the vector-spinor gauge field with the massless spin-2 gauge field in de Sitter ambient space-time formalism. This will be considered in a forthcoming paper.

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Appendix A

General formulation

Here, we prove equation (16). Since gauge transformations commute with covariant differentiation on fields ϕ for which the algebra is closed, so we can write

$$\delta(\epsilon) D_\mu \phi = \epsilon^A D_\mu (T_A \phi). \tag{A1}$$

We expand the two sides of the above equation. First the left side:

$$I = \delta(\epsilon)(\partial_\mu - \delta(B_\mu))\phi = \underbrace{\delta(\epsilon)\partial_\mu \phi}_{II} - \underbrace{\delta(\epsilon)\delta(B_\mu)\phi}_{III}, \tag{A2}$$

$$\begin{aligned} II &= \delta(\epsilon)\partial_\mu \phi = \partial_\mu(\delta(\epsilon)\phi) \\ &= \partial_\mu(\epsilon^A T_A \phi) = (\partial_\mu \epsilon^A)(T_A\phi) + \epsilon^A \partial_\mu (T_A \phi), \end{aligned} \tag{A3}$$

$$\begin{aligned} III &= \delta(\epsilon)\delta(B_\mu)\phi = \delta(\epsilon)(B^A_\mu T_A \phi) \\ &= \delta(\epsilon) B^A_\mu (T_A\phi) + B^A_\mu \delta(\epsilon)(T_A\phi), \end{aligned} \tag{A4}$$

$$\begin{aligned} \Longrightarrow I = {} & (\partial_\mu \epsilon^A)(T_A\phi) + \epsilon^A \partial_\mu (T_A\phi) - \delta(\epsilon) B^A_\mu (T_A \phi) \\ & - B^A_\mu \delta(\epsilon)(T_A\phi). \end{aligned} \tag{A5}$$

The right side of equation (A1) is:

$$\epsilon^A D_\mu (T_A\phi) = \epsilon^A [\partial_\mu (T_A\phi) - \delta(B_\mu)(T_A\phi)], \tag{A6}$$

by the following auxiliary relation:

$$\epsilon^A \delta(B_\mu)(T_A\phi) - B^A_\mu \delta(\epsilon)(T_A\phi) = \epsilon^B B^A_\mu f^C_{AB}(T_C\phi). \tag{A7}$$

Equation (A1) becomes:

$$\Longrightarrow (\partial_\mu \epsilon^A)(T_A\phi) + \epsilon^B B^A_\mu f^C_{AB}(T_C\phi) = \delta(\epsilon) B^A_\mu (T_A\phi). \tag{A8}$$

Therefore, we have:

$$\Longrightarrow \delta(\epsilon) B^A_\mu = \partial_\mu (\epsilon^A) + \epsilon^B B^A_\mu f^C_{AB}, \tag{A9}$$

which is equation (16). Now, we prove equation (17).

$$[D_\mu, D_\nu] = -R^A_{\mu\nu} T_A, \tag{A10}$$

$$\begin{aligned} (D_\mu D_\nu - D_\nu D_\mu)\phi = D_\mu D_\nu \phi - D_\nu D_\mu \phi = (\partial_\mu - \delta(B_\mu)) D_\nu \phi \\ -(\partial_\nu - \delta(B_\nu)) D_\mu \phi, \end{aligned} \tag{A11}$$

$$\Longrightarrow = \partial_\mu (D_\nu \phi) - \delta(B_\mu) D_\nu \phi - \partial_\nu (D_\mu \phi) + \delta(B_\nu) D_\mu \phi, \tag{A12}$$

$$\begin{aligned} \Longrightarrow = {} & \partial_\mu (\partial_\nu - \delta(B_\nu))\phi - \delta(B_\mu)(\partial_\nu - \delta(B_\nu))\phi \\ & - \partial_\nu (\partial_\mu - \delta(B_\mu))\phi + \delta(B_\nu)(\partial_\mu - \delta(B_\mu))\phi, \end{aligned} \tag{A13}$$

$$\begin{aligned} \Longrightarrow = {} & \partial_\mu \partial_\nu \phi - \partial_\mu (\delta(B_\nu)\phi) - \delta(B_\mu)\partial_\nu \phi + \delta(B_\mu)\delta(B_\nu) \\ & - \partial_\nu \partial_\mu \phi + \partial_\nu (\delta(B_\mu)\phi) \\ & + \delta(B_\nu)\partial_\mu \phi - \delta(B_\nu)\delta(B_\mu). \end{aligned} \tag{A14}$$

By the following auxiliary relations:

$$\begin{aligned} [\delta(\epsilon_1), \delta(\epsilon_2)]\phi &= \left[\epsilon^A_1 T_A, \epsilon^B_2 T_B\right]\phi = \epsilon^A_2 \epsilon^B_1 [T_A, T_B]\phi \\ &= \epsilon^A_2 \epsilon^B_1 f^C_{AB} T_C, \end{aligned} \tag{A15}$$

$$[\delta(B_\mu),\delta(B_\nu)]\,\phi=B_\nu^A B_\mu^B f_{AB}^C T_C, \tag{A16}$$

we have:

$$\Longrightarrow=-\partial_\mu(\delta(B_\nu)\phi)-\delta(B_\mu)\partial_\nu\phi+\partial_\nu(\delta(B_\mu)\phi)$$
$$+\delta(B_\nu)\partial_\mu\phi+B_\nu^A B_\mu^B f_{AB}^C T_C, \tag{A17}$$

$$\Longrightarrow=-\partial_\mu(B_\nu^A T_A\phi)-\delta(B_\mu)\partial_\nu\phi+\partial_\nu(B_\mu^A T_A\phi)$$
$$+\delta(B_\nu)\partial_\mu\phi+B_\nu^A B_\mu^B f_{AB}^C T_C, \tag{A18}$$

$$\Longrightarrow=-(\partial_\mu B_\nu^A)(T_A\phi)-B_\nu^A(\partial_\mu T_A\phi)-B_\mu^A T_A\partial_\nu\phi$$
$$+(\partial_\nu B_\mu^A)(T_A\phi)+B_\mu^A(\partial_\nu T_A\phi)$$
$$+B_\nu^A T_A\partial_\mu\phi+B_\nu^A B_\mu^B f_{AB}^C T_C, \tag{A19}$$

$$\Longrightarrow=-(\partial_\mu B_\nu^A)(T_A\phi)+(\partial_\nu B_\mu^A)(T_A\phi)+B_\nu^A B_\mu^B f_{AB}^C T_C, \tag{A20}$$

hence

$$[D_\mu,D_\nu]=-(\partial_\mu B_\nu^A)(T_A\phi)+(\partial_\nu B_\mu^A)(T_A\phi)$$
$$+B_\nu^A B_\mu^B f_{AB}^C T_C, \tag{A21}$$

finally

$$R_{\mu\nu}^A=\partial_\mu B_\nu^A-\partial_\nu B_\mu^A+B_\mu^A B_\nu^B f_{AB}^C, \tag{A22}$$

which is equation (17). Now, if T_A replaced by iZ_A, B_μ^A replaced by $\mathcal{H}_\nu^A$ and ∂_μ replaced by $\nabla_\alpha^\top$, we obtain equations (25) and (26).

Appendix B

The Euler-Lagrange equation

From the action, the Lagrangian density is

$$\mathcal{L}=\left(\tilde{\nabla}_\alpha^\top\tilde{\Psi}_\beta-\tilde{\nabla}_\beta^\top\tilde{\Psi}_\alpha\right)\left(\nabla^{\top\alpha}\Psi^\beta-\nabla^{\top\beta}\Psi^\alpha\right), \tag{B1}$$

where

$$\left(\tilde{\nabla}_\alpha^\top\tilde{\Psi}_\beta-\tilde{\nabla}_\beta^\top\tilde{\Psi}_\alpha\right)=\left(\partial_\alpha^\top\tilde{\Psi}_\beta-x_\beta\tilde{\Psi}_\alpha-\partial_\beta^\top\tilde{\Psi}_\alpha+x_\alpha\tilde{\Psi}_\beta\right). \tag{B2}$$

Using the Euler-Lagrange equation

$$\frac{\delta\mathcal{L}}{\delta\tilde{\Psi}_m}-\partial_l^\top\frac{\delta\mathcal{L}}{\delta(\partial_l^\top\tilde{\Psi}_m)}=0, \tag{B3}$$

we obtain

$$\frac{\delta\mathcal{L}}{\delta\tilde{\Psi}_m}=(x_\alpha\delta_\beta^m-x_\beta\delta_\alpha^m)\left(\nabla^{\top\alpha}\Psi^\beta-\nabla^{\top\beta}\Psi^\alpha\right), \tag{B4}$$

and

$$\frac{\delta\mathcal{L}}{\delta(\partial_l^\top\tilde{\Psi}_m)}=(\delta_\alpha^l\delta_\beta^m-\delta_\beta^l\delta_\alpha^m)\left(\nabla^{\top\alpha}\Psi^\beta-\nabla^{\top\beta}\Psi^\alpha\right). \tag{B5}$$

If $\beta=m$, we have

$$\frac{\delta\mathcal{L}}{\delta\tilde{\Psi}_m}=x_\alpha\left(\nabla^{\top\alpha}\Psi^m-\nabla^{\top m}\Psi^\alpha\right), \tag{B6}$$

$$\frac{\delta\mathcal{L}}{\delta(\partial_l^\top\tilde{\Psi}_m)}=\delta_\alpha^l\left(\nabla^{\top\alpha}\Psi^m-\nabla^{\top m}\Psi^\alpha\right)=\left(\nabla^{\top l}\Psi^m-\nabla^{\top m}\Psi^l\right). \tag{B7}$$

Then the Euler-Lagrange equation leads immediately to the following field equation

$$(x_\alpha-\partial_\alpha^\top)\left(\nabla^{\top\alpha}\Psi^\beta-\nabla^{\top\beta}\Psi^\alpha\right)=0, \tag{B8}$$

which is equation (29).

Appendix C

The equation of motion expressed in terms of the Casimir operator

We write the equation of motion (29) in terms of the Casimir operator:

$$(x_\alpha-\partial_\alpha^\top)\left(\nabla^{\top\alpha}\Psi^\beta-\nabla^{\top\beta}\Psi^\alpha\right)=0, \tag{C1}$$

where

$$\left(\nabla^{\top\alpha}\Psi^\beta-\nabla^{\top\beta}\Psi^\alpha\right)$$
$$=\left(\partial^{\top\alpha}\Psi^\beta+\gamma^\alpha\not{x}\Psi^\beta-\partial^{\top\beta}\Psi^\alpha-\gamma^\beta\not{x}\Psi^\alpha\right).$$

Equation (C1) is divided into two parts as follows:

$$x_\alpha\left(\partial^{\top\alpha}\Psi^\beta+\gamma^\alpha\not{x}\Psi^\beta-\partial^{\top\beta}\Psi^\alpha-\gamma^\beta\not{x}\Psi^\alpha\right)=0, \tag{C2}$$

$$I=\partial_\alpha^\top\left(\partial^{\top\alpha}\Psi^\beta+\gamma^\alpha\not{x}\Psi^\beta-\partial^{\top\beta}\Psi^\alpha-\gamma^\beta\not{x}\Psi^\alpha\right). \tag{C3}$$

So we can write:

$$\Longrightarrow I=\partial_\alpha^\top\partial^{\top\alpha}\Psi^\beta+\partial_\alpha^\top(\gamma^\alpha\not{x}\Psi^\beta)-\partial_\alpha^\top\partial^{\top\beta}\Psi^\alpha-\partial_\alpha^\top(\gamma^\beta\not{x}\Psi^\alpha), \tag{C4}$$

$$\Longrightarrow I=-Q_0\Psi^\beta+\gamma^\alpha\gamma^\rho\partial_\alpha^\top(x_\rho\Psi^\beta)-\partial_\alpha^\top\partial^{\top\beta}\Psi^\alpha-\gamma^\beta\gamma^\rho\partial_\alpha^\top(x_\rho\Psi^\alpha). \tag{C5}$$

According to our definitions and the auxiliary relations:

$$x\cdot\Psi=0,\quad x\cdot\partial^\top=0,\quad Q_0=-\partial_\alpha^\top\partial^{\top\alpha},$$
$$Q_{\frac{3}{2}}^{(1)}\Psi_\alpha=(Q_0+\not{x}\,\not{\partial}^\top-3)\Psi_\alpha+2x_\alpha\partial^\top\cdot\Psi+\gamma^\alpha\,\not{\Psi},$$
$$\partial_\alpha^\top x_\rho=\eta_{\alpha\rho}+x_\alpha x_\rho,\quad \partial_\alpha^\top x^\rho=\delta_\alpha^\rho+x_\alpha x^\rho,\ \gamma\cdot\gamma=5\ ,\ \not{x}\cdot\not{x}=-1,$$
$$[\partial_\alpha^\top,\partial_\beta^\top]=x_\beta\partial_\alpha^\top-x_\alpha\partial_\beta^\top,$$

after doing some calculation we have:

$$\Longrightarrow I=-Q_0\Psi^\beta+\gamma^\alpha\gamma_\alpha\Psi^\beta+\not{x}\not{x}\Psi^\beta+\gamma^\alpha\not{x}\partial_\alpha^\top\Psi^\beta-\partial_\alpha^\top\partial^{\top\beta}\Psi^\alpha$$
$$-\gamma^\beta\gamma_\alpha\Psi^\alpha-\gamma^\beta x_\alpha\not{x}\Psi^\alpha$$
$$-\gamma^\beta\not{x}\partial^\top\cdot\Psi, \tag{C6}$$

$$\Longrightarrow I=-(Q_0\Psi_\alpha+\not{x}\,\not{\partial}^\top\Psi_\alpha-3\Psi_\alpha-\frac{5}{2}\Psi_\alpha+2x_\alpha\partial^\top\cdot\Psi$$
$$+\gamma^\alpha\,\not{\Psi})-(\partial_\alpha^\top+\gamma_\alpha\not{x}-x_\alpha)\partial^\top\cdot\Psi-\frac{5}{2}\Psi_\alpha, \tag{C7}$$

and finally, the above equation can be written as

$$\left(Q_{\frac{3}{2}}^{(1)}+\frac{5}{2}\right)\Psi_\alpha+\nabla_\alpha^\top\partial^\top\cdot\Psi=0, \tag{C8}$$

which is equation (30).

Appendix D

Gauge invariance

The Lagrangian density is:

$$\mathcal{L}=\left(\tilde{\nabla}_\alpha^\top\tilde{\Psi}_\beta-\tilde{\nabla}_\beta^\top\tilde{\Psi}_\alpha\right)\left(\nabla^{\top\alpha}\Psi^\beta-\nabla^{\top\beta}\Psi^\alpha\right).$$

The above Lagrangian density is invariant under the following gauge transformations:

$$\Psi_\alpha\longrightarrow\Psi_\alpha^g=\Psi_\alpha+\nabla_\alpha^\top\psi, \tag{D1}$$

$$\tilde{\Psi}_\alpha\longrightarrow\tilde{\Psi}_\alpha^g=\tilde{\Psi}_\alpha+\partial_\alpha^\top\tilde{\psi}. \tag{D2}$$

The Lagrangian density is divided into two parts, as follow:

$$\begin{aligned}A&=\left(\nabla^{\top\alpha}\Psi^\beta-\nabla^{\top\beta}\Psi^\alpha\right)\\&=\left(\partial^{\top\alpha}\Psi^\beta+\gamma^\alpha\not{x}\Psi^\beta-\partial^{\top\beta}\Psi^\alpha-\gamma^\beta\not{x}\Psi^\alpha\right),\end{aligned} \tag{D3}$$

$$\begin{aligned}B&=\left(\tilde{\nabla}_\alpha^\top\tilde{\Psi}_\beta-\tilde{\nabla}_\beta^\top\tilde{\Psi}_\alpha\right)\\&=\left(\partial_\alpha^\top\tilde{\Psi}_\beta-x_\beta\tilde{\Psi}_\alpha-\partial_\beta^\top\tilde{\Psi}_\alpha+x_\alpha\tilde{\Psi}_\beta\right),\end{aligned} \tag{D4}$$

where all parts have their own gauge transformation. Under the gauge transformation (D1), the first part becomes:

$$\begin{aligned}A^g=&\,\partial^{\top\alpha}(\Psi^\beta+\nabla^{\top\beta}\psi)+\gamma^\alpha\not{x}(\Psi^\beta+\nabla^{\top\beta}\psi)\\&-\partial^{\top\beta}(\Psi^\alpha+\nabla^{\top\alpha}\psi)-\gamma^\beta\not{x}(\Psi^\alpha+\nabla^{\top\alpha}\psi)\\=&\,\partial^{\top\alpha}\Psi^\beta+\gamma^\alpha\not{x}\Psi^\beta-\partial^{\top\beta}\Psi^\alpha-\gamma^\beta\not{x}\Psi^\alpha+\partial^{\top\alpha}\nabla^{\top\beta}\psi\\&+\gamma^\alpha\not{x}\nabla^{\top\beta}\psi-\partial^{\top\beta}\nabla^{\top\alpha}\psi-\gamma^\beta\not{x}\nabla^{\top\alpha}\psi.\end{aligned}$$

By using the following relations

$$\begin{aligned}\partial^{\top\alpha}\nabla^{\top\beta}\psi=&\,\partial^{\top\alpha}\partial^{\top\beta}\psi+\gamma^\beta\gamma^\alpha\psi+\gamma^\beta x^\alpha\not{x}\psi+\gamma^\beta\not{x}\partial^{\top\alpha}\psi\\&-\eta^{\alpha\beta}\psi-x^\alpha x^\beta\psi-x^\beta\partial^{\top\alpha}\psi,\end{aligned}$$

$$\begin{aligned}\partial^{\top\beta}\nabla^{\top\alpha}\psi=&\,\partial^{\top\beta}\partial^{\top\alpha}\psi+\gamma^\alpha\gamma^\beta\psi+\gamma^\alpha x^\beta\not{x}\psi+\gamma^\alpha\not{x}\partial^{\top\beta}\psi\\&-\eta^{\beta\alpha}\psi-x^\beta x^\alpha\psi-x^\alpha\partial^{\top\beta}\psi,\end{aligned}$$

$$\gamma^\alpha\not{x}\nabla^{\top\beta}\psi=\gamma^\alpha\not{x}\partial^{\top\beta}\psi+\gamma^\alpha\not{x}x^\beta\psi+\gamma^\alpha\gamma^\beta\psi,$$

$$\gamma^\beta\not{x}\nabla^{\top\alpha}\psi=\gamma^\beta\not{x}\partial^{\top\alpha}\psi+\gamma^\beta\not{x}x^\alpha\psi+\gamma^\beta\gamma^\alpha\psi,$$

we have:

$$\partial^{\top\alpha}\nabla^{\top\beta}\psi+\gamma^\alpha\not{x}\nabla^{\top\beta}\psi-\partial^{\top\beta}\nabla^{\top\alpha}\psi-\gamma^\beta\not{x}\nabla^{\top\alpha}\psi=0. \tag{D5}$$

Therefore, by using (D5), we can see that(D3) is invariant under (D1). Similarly for (D4), under the transformation (D2), we have:

$$\begin{aligned}B^g=&\,\partial_\alpha^\top(\tilde{\Psi}_\beta+\partial_\beta^\top\tilde{\psi})-x_\beta(\tilde{\Psi}_\alpha+\partial_\alpha^\top\tilde{\psi})\\&-\partial_\beta^\top(\tilde{\Psi}_\alpha+\partial_\alpha^\top\tilde{\psi})+x_\alpha(\tilde{\Psi}_\beta+\partial_\beta^\top\tilde{\psi})\\=&\,\partial_\alpha^\top\tilde{\Psi}_\beta-x_\beta\tilde{\Psi}_\alpha-\partial_\beta^\top\tilde{\Psi}_\alpha+x_\alpha\tilde{\Psi}_\beta+\partial_\alpha^\top\partial_\beta^\top\tilde{\psi}\\&-x_\beta\partial_\alpha^\top\tilde{\psi}-\partial_\beta^\top\partial_\alpha^\top\tilde{\psi}+x_\alpha\partial_\beta^\top\tilde{\psi}.\end{aligned}$$

Using the identity

$$[\partial_\alpha^\top,\partial_\beta^\top]=x_\beta\partial_\alpha^\top-x_\alpha\partial_\beta^\top,$$

or equivalently

$$\partial_\alpha^\top\partial_\beta^\top\tilde{\psi}-x_\beta\partial_\alpha^\top\tilde{\psi}-\partial_\beta^\top\partial_\alpha^\top\tilde{\psi}+x_\alpha\partial_\beta^\top\tilde{\psi}=0,$$

one can see that (D4) is also invariant under the gauge transformation (D2).

References

1 P. A. R. Ade, R. W. Aikin, D. Barkats et al, Phys. Rev. Lett., **112**: 241101 (2014)
2 S. Perlmutter, G. Aldering, G. Goldhaber et al, Astrophys. J, **517**: 565 (1999)
3 A. G. Riess, Alexei V. Filippenko, Peter Challis et al, Astron. J, **116**: 1009 (1998)
4 T. S. Bunch, P. C. W. Davies, Proc. R. Soc. Lond. A, **360**: 117 (1978)
5 B. Allen, Phys. Rev. D, **32**: 3136 (1985)
6 J. Bros, U. Moschella, Rev. Math. Phys., **8**: 327 (1996)
7 N. A. Chernikov, E. A. Tagirov, Ann. Inst. Henri Poincaré IX, 109 (1968)
8 M.V. Takook, Annals of Physics, **367** (2016)
9 M.V. Takook [arXiv:1403.1204v2]
10 L. L. Foldy, Physical Review, **102**(2): 568–581 (1956)
11 A. Chakrabarti, Journal of Mathematical Physics,**4**(10): 1215–1222 (1963)
12 W. Rarita, J. Schwinger, Phys. Rev., **60**: 61 (1941)
13 H. Pagels, J.R. Primack, Phys. Rev. Lett., **48**: 223 (1982)
14 T. Moroi, H. Murayama, and M. Yamaguchi, Phys. Lett. B, **303**: 289 (1993)
15 K. Kohri, T. Moroi, and A. Yotsuyanagi, Phys. Rev. D, **73**: 123511 (2006)
16 M. V. Takook, A. Azizi, and E. Babaian, Eur. Phys. J. C, **72**: 2026 (2012)
17 N. Fatahi, M. V. Takook, and M.R. Tanhayi, Eur. Phys. J. C, **74**: 3111 (2014)
18 S. Parsamehr, M. Enayati, and M. V. Takook, Eur. Phys. J. C, **76**: 260 (2016)
19 M. V. Takook, *Théorie quantique des champs pour des systèmes élémentaires "massifs" et de "masse nulle" sur l'espace- temps de de Sitter*, Ph.D Thesis (Paris: l'université Paris VI,1997) (in French)
20 P. Bartesaghi, J. P. Gazeau, U. Moschella, M.V. Takook, Class. Quant. Grav., **18**: 4373 (2001)
21 A. Pahlavan, S. Rouhani, and M. V. Takook, Phys. Lett. B, **627**: 217 (2005)
22 S. Moradi, S. Rouhani, and M. V. Takook , Phys. Lett. B, **613**: 74 (2005)
23 S. W. MacDowell, F. Mansouri, Phys. Rev. Lett., **38**: 739 (1977)
24 S. Parsamehr, Int. J. Theor Phys., **55**: 1043 (2016)