Quadrupole-octopole alignment of CMB related to the primordial power spectrum with dipolar modulation in anisotropic spacetime*

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Abstract: The WMAP and Planck observations show that the quadrupole and octopole orientations of the CMB might align with each other. We reveal that the quadrupole-octopole alignment is a natural implication of the primordial power spectrum in an anisotropic spacetime. The primordial power spectrum is presented with a dipolar modulation. We obtain the privileged plane by employing the "power tensor" technique. At this plane, there is maximum correlation between quadrupole and octopole. The probability for the alignment is much larger than that in the isotropic universe. We find that this model would lead to deviations from the statistical isotropy only for low-ℓ multipoles.

Key words: CMB mode alignment, anisotropic inflation, Finsler geometry

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1 Introduction

The universe is assumed to be statistically isotropic at large scales in the standard model of cosmology. Recently, however, evidence has appeared for possible deviations from the statistical isotropy of the CMB temperature fluctuations [1, 2]. One refers to the alignment of the quadrupole and octopole orientations. Based on the one-year WMAP data [3], Tegmark et al. [4] and Copi et al. [5–7] originally proposed that there is a quadrupole-octopole alignment of CMB anisotropy. The recent nine-year WMAP results [1] also show that the quadrupole-octopole alignment is about 3°. The Planck 2013 results [2] confirm the evidence for this alignment while there is a misalignment between the quadrupole and octopole orientations by an amount between 9° and 13°. However, the quadrupole and octopole orientations are distributed uniformly in the isotropic universe. The possibility for the quadrupole-octopole alignment is expected to be much smaller.

The significance of the quadrupole-octopole alignment based on the Planck dataset becomes smaller than that based on the WMAP dataset. Nevertheless, we cannot simply drop the proposition that the universe might be statistically anisotropic at large scales. The reason is that there are several other "anomalies" of the CMB anisotropy at low-ℓ multipoles. For instance, the hemispherical asymmetry [8–10], the parity asymmetry [11–22], the dipolar power modulation [23–26], and so on. These anomalies might be related to the deviation from isotropy if they occupy certain cosmological origins [2]. There is a useful approach called the "power tensor" technique [27–29] to study the deviations from the isotropy in the CMB. The power tensor is a generalization of the power spectrum. In the limit of isotropy, its ensemble expectation would reduce to the power spectrum. However, it could not be diagonal in the anisotropic universe. The correlation between two power tensors directly describes the level of alignment between two multipoles. In this paper, we employ this method to study the quadrupole-octopole alignment in an anisotropic spacetime.

The deviations from isotropy may be related with a privileged direction in the universe. We have proposed an anisotropic inflation model in the Randers spacetime to study the dipole modulation of the CMB temperature fluctuations [30]. In the Randers spacetime [31], the line element includes an extra one-form in addition to the Riemannian line element. The one-form would reduce the number of Killing vectors. Locally, this one-form has been extensively studied in very special relativity (VSR) [32]. The local Randers metric was proved to possess

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symmetries of the group $TE(2)$ [33, 34]. Thus, the Randers spacetime has fewer symmetries than the Riemannian Friedmann-Robertson-Walker (FRW) spacetime does. There is a privileged direction in the Randers spacetime. It would induce the primordial power spectrum with a dipolar modulation if the inflationary universe is of Randers type [30]. This initial spectrum with anisotropy is compatible with the Planck 2013 results.

The Randers spacetime belongs to Finsler geometry [35, 36]. Finsler geometry gets rid of the quadratic restriction on the line element. It is a natural generalization of Riemann geometry and includes Riemann geometry as a special case. It is intrinsically anisotropic. The isometric transformation shows that there are less symmetries in Finsler geometry than in Riemann geometry [33, 37, 38]. Thus, Finsler geometry could be a reasonable candidate to account for the deviations from isotropy at large scales in the universe. In this paper, we employ the Randers-Finsler spacetime to study the quadrupole-octopole alignment. The anisotropic inflation model and the primordial power spectrum with dipolar modulation are introduced briefly. By the "power tensor" technique, we study the correlations between two multipoles. We obtain eigenvectors with the maximum eigenvalue of the correlation matrix of two power tensors. Several quantities, which describe the correlations between two multipoles, are evaluated.

The rest of the paper is arranged as follows. In Section 2, we introduce the "power tensor" technique to study the statistical anisotropy. The correlation matrix of two power tensors is defined to study the statistical correlation between two multipoles. In Section 3, we briefly review the inflation model with anisotropy in the Randers-Finsler spacetime. The primordial power spectrum is presented with a dipolar modulation. In Section 4, we study the implications of the primordial power spectrum with anisotropy on the mode alignments of CMB anisotropy, especially the quadrupole-octopole alignment. Conclusions and remarks are listed in Section 5. In addition, there are two appendixes for calculating physical quantities explicitly.

2 The "power tensor" technique

Generically, the CMB temperature fluctuation $\frac{\delta T}{T}(\hat{\boldsymbol{p}},\boldsymbol{x},\eta)$ could be decomposed in terms of the spherical harmonics $Y_{\ell m}(\hat{\boldsymbol{p}})$, namely,

$$\frac{\delta T}{T}(\hat{\boldsymbol{p}},\boldsymbol{x},\eta)=\sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{\ell}a_{\ell m}Y_{\ell m}(\hat{\boldsymbol{p}}), \tag{1}$$

where $\hat{\boldsymbol{p}}$ denotes the observed direction by an observer at the location $\boldsymbol{x}$, and η denotes the conformal time. We assume that the CMB anisotropy satisfies the Gaussian distribution, even though the statistical anisotropy might be related to non-Gaussianity [39–47]. The variance of harmonic coefficients $a_{\ell m}$ gives the two-point correlator. It is given by

$$\langle a_{\ell m}a^*_{\ell' m'}\rangle=C_{\ell\ell' mm'}, \tag{2}$$

where the angle brackets stand for the ensemble average. The correlators $C_{\ell\ell' mm'}$ might be not diagonal in the anisotropic case. In the case of statistical isotropy, it reduces to the power spectrum $C_{\ell\ell' mm'}=C_\ell\delta_{\ell\ell'}\delta_{mm'}$.

At the largest scales, the CMB temperature fluctuations are linear. They could be given by

$$\frac{\delta T}{T}(\hat{\boldsymbol{p}},\boldsymbol{x},\eta)=\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}}\mathcal{R}(\boldsymbol{k},\eta_i)\Delta(k,\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}},\eta)\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}, \tag{3}$$

where $k=|\boldsymbol{k}|$, and $\mathcal{R}(\boldsymbol{k},\eta_i)$ is the initial curvature perturbation at the initial time η_i. Here $\Delta(k,\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}},\eta)$ denotes the transfer function, which characterizes the change of amplitude of the CMB temperature fluctuation from the conformal time η_i to today η. It could be expanded in terms of Legendre polynomials,

$$\Delta(k,\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}},\eta)=\sum_\ell(-\mathrm{i})^\ell\left(\frac{2\ell+1}{4\pi}\right)P_\ell(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}})\Delta_\ell(k,\eta). \tag{4}$$

According to Eq. (1)–(4), the correlators $C_{\ell\ell' mm'}$ could be calculated as[1)]

$$C_{\ell\ell' mm'}=(-\mathrm{i})^{\ell-\ell'}\int\mathrm{d}^3\boldsymbol{k}\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(\boldsymbol{k})\Delta_\ell(k,\eta)\Delta^*_{\ell'}(k,\eta)\times Y^*_{\ell m}(\hat{\boldsymbol{k}})Y_{\ell' m'}(\hat{\boldsymbol{k}}). \tag{5}$$

Here, the primordial power spectrum $\mathcal{P}_{\mathcal{R}}(\boldsymbol{k})$ is given by

$$\langle\mathcal{R}(\boldsymbol{k})\mathcal{R}^*(\boldsymbol{k}')\rangle=\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(\boldsymbol{k}), \tag{6}$$

which encodes all the properties of statistical anisotropy.

In this paper, we employ the "power tensor" technique [27–29] to study the possible statistical anisotropy of the CMB temperature fluctuations in an anisotropic spacetime. The power tensor is defined by a second-rank matrix as

$$A_{ij}(\ell)=N(\ell)\sum_{m,m',m''}\langle\ell m|J_i|\ell m'\rangle\langle\ell m''|J_j|\ell m\rangle a_{\ell m'}a^*_{\ell m''}, \tag{7}$$

where the coefficient $N(\ell)$ refers[2)]

$$N(\ell)=\frac{1}{\ell(\ell+1)(2\ell+1)}\ . \tag{8}$$

1) See Appendix A for an explicit calculation.

2) This is slightly different from the original definition in Refs. [28, 29]. One should note that the power tensor requires the condition $\langle A_{ij}(\ell)\rangle=\frac{1}{3}C_\ell\delta_{ij}$ in the limit of statistical isotropy. The definition here satisfies this condition.

Here, J_i denotes the i-th component of angular momentum operator, and the state $|\ell m\rangle$ stands for the spin-ℓ representation. The power tensor is real and symmetric. The privileged direction of the ℓ-multipole is given by the eigenvector of $\langle A_{ij}(\ell)\rangle$ with the maximum eigenvalue. However, it is possible that $\langle A_{ij}(\ell)\rangle$ has no preferred directions when the statistical anisotropy has certain particular behaviors. For instance, the initial power spectrum $\mathcal{P}_{\mathcal{R}}(\boldsymbol{k})$ may acquire a dipolar modulation [29, 30]. The two-point correlators could reduce to the isotropic form, since a dipole is point-parity asymmetric, see Appendix A. In this case, the multipoles would acquire preferred directions occasionally. One should note that the privileged direction of a multipole is not statistically significant. Only the alignment of two multipoles reveals the possible statistical anisotropy.

There is a useful way to describe the alignment of two multipoles. It refers to the correlator of two power tensors with different multipoles [29]. Thus, the correlation of two power tensors is defined as[1)]

$$S(\ell,\ell')=\frac{3}{C_\ell C_{\ell'}}\mathrm{Tr}(\langle A^+(\ell)A(\ell')\rangle_{ij})-1 \ . \tag{9}$$

This quantity describes the level of alignment between two multipoles. It is directly a statistically significant quantity. In the isotropic case, $S(\ell,\ell')$ would vanish. Even though $S(\ell,\ell')$ could give the magnitude of correlations, it is direction-blind since it cannot determine the direction (or the plane) of the maximum correlation. Statistically, however, the alignment of two multipoles would lie on the direction(s) of the maximum correlation. Fortunately, it is easy to overcome the above problem. One can define a second-rank real symmetric matrix as

$$S_{ij}(\ell,\ell')=\frac{3}{C_\ell C_{\ell'}}\langle A^+(\ell)A(\ell')\rangle_{ij}-\frac{1}{3}\delta_{ij} \ . \tag{10}$$

In the following, we call this matrix the "correlation matrix" of power tensors, or "correlation matrix" for short. We can obtain the eigenvalues and the related eigenvectors of $S_{ij}(\ell,\ell')$. The eigenvectors with the maximum eigenvalue would give the privileged directions, which refer to the maximum correlation of the two multipoles. In Section 4, we will follow this approach to study the quadrupole–octopole alignment of the CMB anisotropy in an anisotropic spacetime.

3 Primordial power spectra with anisotropy

Recently, an anisotropic inflation model has been proposed explicitly in Ref. [30]. This model suggested that the spacetime of the very early universe may be of Randers-Finsler type. The Friedmann equation as well as its inflationary solution was presented at first order. The primordial power spectrum was obtained with direction dependence. For example, it might acquire a dipolar modulation at the first-order approximation. In this section, we briefly review the main results of the anisotropic inflation model.

Finsler geometry [35, 36] is defined by the Finsler structure $F(x,y)$, where x denotes a position and $y=\mathrm{d}x/\mathrm{d}\tau$ a fibre coordinate. The Finsler structure is a smooth positive function on the tangent bundle. It satisfies the property of positive homogeneity of degree one,

$$F(x,\lambda y)=\lambda F(x,y), \ \text{ for } \lambda>0 \ . \tag{11}$$

The Finsler metric is defined by

$$g_{\mu\nu}=\frac{\partial}{\partial y^\mu}\frac{\partial}{\partial y^\nu}\left(\frac{1}{2}F^2\right) \ . \tag{12}$$

The Randers spacetime [31] is a class of Finsler spacetime. The Randers(-Finsler) structure is given as

$$F(x,y)=\alpha(x,y)+\beta(x,y) \ , \tag{13}$$

where

$$\alpha(x,y) \ = \ \sqrt{\tilde{a}_{\mu\nu}(x)y^\mu y^\nu} \ , \tag{14}$$

$$\beta(x,y) \ = \ \tilde{b}_\mu(x)y^\mu \ . \tag{15}$$

Here α denotes a Riemann structure and $\tilde{a}_{\mu\nu}$ the Riemann metric. β denotes a 1-form, which may be related to a vector field. In the following discussions, we choose $\tilde{a}_{\mu\nu}$ the spatially flat FRW metric, i.e., $\tilde{a}_{\mu\nu}=\mathrm{diag}(1,-a^2(t),-a^2(t),-a^2(t))$. The 1-form β has just the temporal component

$$\tilde{b}_\mu=B(x^3)\delta^0_\mu \ , \tag{16}$$

where $B(x^3)$ is a function of only the third spatial coordinate x^3.

In the osculating Riemannian approach [48–50], the Finsler structure could be related to a certain osculating Riemannian metric, namely, $g_{\mu\nu}(x)=g_{\mu\nu}(x,y(x))$. The fibre coordinate y could be viewed as a function of the spacetime position x. We are only interested in the evolution of the very early universe. After our calculation of the connection and the curvature tensor, Einstein's gravitational equation could be obtained as [30, 50]

$$\mathrm{Ric}_{\mu\nu}-\frac{1}{2}g_{\mu\nu}S=8\pi G T_{\mu\nu} \ , \tag{17}$$

where $\mathrm{Ric}_{\mu\nu}$ denotes the Ricci tensor, S denotes the scalar curvature, and

$$T_{\mu\nu}(\phi)=\partial_\mu\phi\partial_\nu\phi-g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi-V(\phi)\right) \tag{18}$$

1) This form of the correlation has a slight difference from the original one in Ref. [29]. Here, we have only extracted the anisotropic effects. In the original definition, however, $S(\ell,\ell')$ is given by the correlator of two power tensors. Thus, the isotropic effects are still contained in the original definition. One should note that these two definitions would point to the same physical results.

is the energy-momentum tensor of the inflaton field. Here $V(\phi)$ is a potential of the inflaton.

At a very early time, the universe might have undergone an era of exponential expansion, which is called inflation [51–55]. The scale factor expands as $a(t)\sim \mathrm{e}^{Ht}$ where H denotes the Hubble constant. The inflation could be decomposed as $\phi=\phi^{(0)}(t)+\delta\phi(t,\boldsymbol{x})$. Here $\phi^{(0)}(t)$ denotes the zero-order part which drives the inflation. The 00 component of Einstein's gravitational equation gives the Friedmann equation [30]

$$3\left(\frac{\dot{a}}{a}\right)^2+\left(\frac{1}{a}\right)^2\frac{3B'^2-4B''(1+B)}{4(1+B)}=8\pi G T_{00}(\phi^{(0)})\ , \quad (19)$$

where the dot and the prime denote derivatives with respect to t and x^3, respectively. At first order, the above Friedmann equation could have an inflationary solution [30]

$$a(t)\ \propto\ \mathrm{e}^{Ht}\ , \quad (20)$$

$$B(x^3)\ =\ r_{\rm c}^{-1}x^3+\mathcal{O}(x^3)\ , \quad (21)$$

where $r_{\rm c}$ stands for a spatial scale for cutoff. This cutoff scale would be comparable to the observable scale of the universe. One should note that $B(x^3)$ depends only on x^3 such that the inflationary universe does not deserve the spherical symmetry. Thus, the universe would have undergone a phase of anisotropic inflation.

The anisotropic inflation should leave certain anisotropic imprints on the observable universe [56]. The primordial power spectrum has been calculated in Ref. [30]. It could acquire a dipolar modulation besides the isotropic power spectrum. To obtain the primordial power spectrum, one should study the equation of motion for the first-order perturbation $\delta\phi(t,\boldsymbol{x})$ of the inflaton [57–63], see also Refs. [64, 65]. The equation of motion for $\delta\phi(t,\boldsymbol{x})$ could be given as [30]

$$\delta\ddot{\phi}+3H\delta\dot{\phi}-\frac{1+B}{a^2}\nabla^2\delta\phi-\frac{3}{2}\frac{B'}{a^2}\delta\phi'=0. \quad (22)$$

In momentum space, this equation becomes

$$\delta\ddot{\phi}_{\boldsymbol{k}}+3H\delta\dot{\phi}_{\boldsymbol{k}}+\frac{k_{\rm eff}^2}{a^2}\delta\phi_{\boldsymbol{k}}=0, \quad (23)$$

where the effective wavenumber $k_{\rm eff}$ is given by

$$k_{\rm eff}^2=k^2\left(1+B-\mathrm{i}\frac{3B'}{2k}(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{x}}^3)\right). \quad (24)$$

At the super-horizon scales, therefore, we obtain the primordial power spectrum of the form [30]

$$\mathcal{P}_{\delta\phi_{\boldsymbol{k}}}=\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{|k_{\rm eff}|}\right)^3. \quad (25)$$

As a first-order approximation, thus, the initial power spectrum (6) of comoving curvature perturbation could be obtained as

$$\mathcal{P}_{\mathcal{R}}\simeq\left(\frac{H}{\dot{\phi}}\right)^2\mathcal{P}_{\delta\phi_{\boldsymbol{k}}}\simeq\mathcal{P}_{\mathcal{R}}^{\rm iso}(k)(1-3B), \quad (26)$$

where $\mathcal{P}_{\mathcal{R}}^{\rm iso}(k)\simeq\left(\frac{H}{2\pi}\right)^2$. We have dropped the terms of higher orders in B. By using Eq. (21), the initial power spectrum (26) could be rewritten as

$$\mathcal{P}_{\mathcal{R}}(\boldsymbol{k})=\mathcal{P}_{\mathcal{R}}^{\rm iso}(k)\left(1+\frac{k_{\rm c}}{k}(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}})\right). \quad (27)$$

We have used the relation $x\sim k^{-1}$ in the above equation, and extra constants have been absorbed into $k_{\rm c}$. Here $\hat{\boldsymbol{n}}\equiv\hat{\boldsymbol{x}}^3$ denotes the privileged direction in general. The parameter $k_{\rm c}$ denotes a critical wavenumber, which refers to the range of the statistical anisotropy. By comparing (27) with (A11), we obtain a relation $g(k)=k_{\rm c}k^{-1}$. This prediction implies that the anisotropic effects would be significant at large scales only. It is compatible with the observable constraints at present [30, 40].

4 Implications on the quadrupole-octopole alignment

In the above section, we have briefly reviewed the inflation model and the anisotropic power spectrum in Randers spacetime. It was revealed that the power spectrum acquires a dipolar modulation. In this section, we apply this dipole-modulated power spectrum to study the quadrupole-octopole alignment of the CMB anisotropy.

We are only interested in the case $\ell'=\ell+1$. The initial power spectrum has been presented in Eq. (27). Thus, the correlators $C_{\ell\ell' mm'}$'s in Eq. (5) could be computed as

$$C_{\ell\ell' mm'}\ =\ C^{\rm iso}_{\ell\ell' mm'}+C^{\rm aniso}_{\ell\ell' mm'}, \quad (28)$$

$$C^{\rm iso}_{\ell\ell' mm'}\ =\ \delta_{\ell\ell'}\delta_{mm'}\int\frac{\mathrm{d}k}{k}\mathcal{P}_{\mathcal{R}}^{\rm iso}(k)\Delta_\ell(k,\eta)\Delta^*_\ell(k,\eta), \quad (29)$$

$$C^{\rm aniso}_{\ell\ell' mm'}\ =\ (-\mathrm{i})^{\ell-\ell'+1}\zeta_{\ell m;\ell' m'}\int\frac{\mathrm{d}k}{k}\mathcal{P}_{\mathcal{R}}^{\rm iso}(k)\left(\frac{k_{\rm c}}{k}\right)\times\Delta_\ell(k,\eta)\Delta^*_{\ell'}(k,\eta). \quad (30)$$

Eq. (29) corresponds to the isotropic case, while Eq. (30) describes violations of the statistical isotropy. Here, the coefficient $\zeta_{\ell m;\ell' m'}$ is given by

$$\zeta_{\ell m;\ell' m'}\ =\ \sqrt{\frac{4\pi}{3}}\int\mathrm{d}\Omega_{\hat{\boldsymbol{k}}}Y_{10}(\hat{\boldsymbol{k}})Y^*_{\ell m}(\hat{\boldsymbol{k}})Y_{\ell' m'}(\hat{\boldsymbol{k}}) \quad (31)$$

$$=\ \delta_{m',m}\left(\delta_{\ell',\ell+1}\sqrt{\frac{(\ell-m+1)(\ell+m+1)}{(2\ell+1)(2\ell+3)}}+\delta_{\ell',\ell-1}\sqrt{\frac{(\ell-m)(\ell+m)}{(2\ell+1)(2\ell-1)}}\right). \quad (32)$$

There is a useful relation for the CMB anisotropy, namely, $a_{\ell m}=(-1)^m a^*_{\ell -m}$. Therefore, we could obtain $\bar{C}_{\ell\ell' mm'}\equiv\langle a_{\ell m}a_{\ell' m'}\rangle$,

$$\bar{C}_{\ell\ell' mm'} = \bar{C}^{\rm iso}_{\ell\ell' mm'}+\bar{C}^{\rm aniso}_{\ell\ell' mm'}, \tag{33}$$

$$\bar{C}^{\rm iso}_{\ell\ell' mm'} = (-1)^m\delta_{\ell\ell'}\delta_{mm'}\int\frac{{\rm d}k}{k}\mathcal{P}^{\rm iso}_{\mathcal{R}}(k) \times\Delta_\ell(k,\eta)\Delta^*_\ell(k,\eta)\ , \tag{34}$$

$$\bar{C}^{\rm aniso}_{\ell\ell' mm'} = (-1)^m(-{\rm i})^{\ell-\ell'+1}\bar{\zeta}_{\ell m;\ell' m'} \times\int\frac{{\rm d}k}{k}\mathcal{P}^{\rm iso}_{\mathcal{R}}(k)\left(\frac{k_{\rm c}}{k}\right)\Delta_\ell(k,\eta)\Delta^*_{\ell'}(k,\eta). \tag{35}$$

Here $\bar{\zeta}_{\ell m;\ell' m'}$ could be obtained by replacing $\delta_{m',m}$ in Eq. (32) with $\delta_{m',-m}$.

By a not too tedious calculation, we get the correlation matrix of power tensors in Eq. (10)

$$S_{ij}(\ell,\ell') = \frac{3N(\ell)N(\ell')}{C_\ell C_{\ell'}}\sum_{m_n=-\ell}^{\ell}\sum_{m'_n=-\ell'}^{\ell'}J_{ij}J \times(\langle a^*_{\ell m_2}a_{\ell' m'_2}\rangle\langle a_{\ell m_3}a^*_{\ell' m'_3}\rangle +\langle a^*_{\ell m_2}a^*_{\ell' m'_3}\rangle\langle a_{\ell m_3}a_{\ell' m'_2}\rangle), \tag{36}$$

where

$$J_{ij} \equiv \langle\ell m_3|J_i|\ell m_1\rangle^*\langle\ell' m'_3|J_j|\ell' m'_1\rangle, \tag{37}$$

$$J \equiv \sum_k\langle\ell m_1|J_k|\ell m_2\rangle^*\langle\ell' m'_1|J_k|\ell' m'_2\rangle. \tag{38}$$

The explicit expressions of J and J_{ij} can be found in Appendix B. Here, we have used the expression of four-point correlators of $a_{\ell m}$'s as

$$\langle a^*_{\ell m_2}a_{\ell m_3}a_{\ell' m'_2}a^*_{\ell' m'_3}\rangle = \langle a^*_{\ell m_2}a_{\ell m_3}\rangle\langle a_{\ell' m'_2}a^*_{\ell' m'_3}\rangle +\langle a^*_{\ell m_2}a_{\ell' m'_2}\rangle\langle a_{\ell m_3}a^*_{\ell' m'_3}\rangle +\langle a^*_{\ell m_2}a^*_{\ell' m'_3}\rangle\langle a_{\ell m_3}a_{\ell' m'_2}\rangle. \tag{39}$$

Note that the term $\langle a^*_{\ell m_2}a_{\ell m_3}\rangle\langle a_{\ell' m'_2}a^*_{\ell' m'_3}\rangle$ would not induce the statistical anisotropy in the case of the dipole-modulated initial power spectrum, see Appendix A. It is easy to check that its effect cancels the isotropic term $\frac{1}{3}\delta_{ij}$ at the right hand side of Eq. (10). Thus, one need calculate only the other two terms at the right hand side of Eq. (39). Finally, the correlation matrix (10) (or (36)) can be given as

$$S_{ij}(\ell,\ell') = \frac{3N(\ell)N(\ell')}{C_\ell C_{\ell'}}\Big(\sum_{mm'}J_{ij}J \times(\zeta_{\ell m_2;\ell' m'_2}\zeta^*_{\ell m_3;\ell' m'_3}+\bar{\zeta}_{\ell m_2;\ell' m'_3}\bar{\zeta}^*_{\ell m_3;\ell' m'_2})\Big) \times\left(\int\frac{{\rm d}k}{k}\mathcal{P}^{\rm iso}_{\mathcal{R}}(k)\left(\frac{k_{\rm c}}{k}\right)\Delta_\ell(k,\eta)\Delta^*_{\ell'}(k,\eta)\right)^2. \tag{40}$$

At large scales, the transfer function is approximately given by $\Delta_\ell(k)\propto J_\ell(k(\eta-\eta_{ls}))\approx J_\ell(k\eta)$. Here, η and η_{ls} denote the conformal time today and at the last scattering, respectively. The last approximation arises from $\eta_{ls}\ll\eta$. We could compute all the components of $S_{ij}(\ell,\ell')$ for a given ℓ. The eigenvalues and eigenvectors could also be calculated directly as was mentioned in Section 2. Thus, the privileged direction is singled out for the maximum correlation of the two multipoles.

We are interested in the quadrupole-octopole alignment of CMB anisotropy. After a lengthy computation, it is shown that the correlation matrix $S_{ij}(2,3)$ is diagonal in the coordinates chosen in this paper. Analytically, Eq. (40) for $\ell=2$ is calculated as

$$S_{ij}(2,3)\simeq(k_{\rm c}\eta)^2\begin{pmatrix}0.0652 & 0 & 0\\ 0 & 0.0652 & 0\\ 0 & 0 & 0.0235\end{pmatrix}. \tag{41}$$

As was mentioned above, $k_{\rm c}$ denotes the critical wavenumber of the statistical anisotropy, and η is the conformal time of the universe today. Only when the critical scale $k_{\rm c}^{-1}\gg\eta$, the statistical isotropy recovers in the observable universe. However, the observations of CMB anisotropy from WMAP and Planck showed certain possible evidence for the statistical anisotropy at large scales. This reveals that the critical scale should be comparable with the present scale of the observable universe, namely, $k_{\rm c}\eta\sim1$.

There is the other significant issue which refers to the privileged direction of the quadrupole-octopole alignment. From Eq. (41), the correlation matrix acquires the maximum eigenvalue along both x^1 and x^2 directions. This result reveals that the quadrupole and the octopole have the maximum correlation in the plane perpendicular to the privileged direction $\hat{\boldsymbol{n}}$. Thus, the quadrupole and octopole orientations should lie in the plane with the uniform distribution. We cannot determine which direction is preferred in the plane. Actually, the nine-year WMAP data show that there is a 3° misalignment between the quadrupole and the octopole [1]. The Planck 2013 results show a larger misalignment of the amount between 9° and 13° [2]. Statistically, all the directions at this plane are equally likely in this model. In the isotropic universe, however, the quadrupole and octopole orientations are distributed uniformly on the celestial sphere. Thus, there is a much larger possibility for the quadrupole-octopole alignment in this anisotropic model than that in the isotropic model.

Another significant issue refers to the decreasing rate of the anisotropic effect. To describe this issue, we could compute the trace of the correlation matrix $S_{ij}(\ell,\ell+1)$. The correlation $S(\ell,\ell+1)$ could be written in the form

$$S(\ell,\ell+1)=s(\ell)(k_{\rm c}\eta)^2, \tag{42}$$

where $s(\ell)$ describes the magnitude of the correlation between the two multipoles directly. The numerical com-

putation shows that $s(\ell)$ is a decreasing function of ℓ, see Fig. 1.

It reveals that the anisotropic effects dominate at large angular scales (small ℓ) but decreases at small angular scales (large ℓ). On the other hand, the above result could also be revealed via the primordial power spectrum with anisotropy. The anisotropic contribution in Eq. (27) is inversely proportional to the wavenumbers k's. Thus, it affects dominantly the modes at large scales (small k). For small scales (large k), it decreases. These predictions imply that the statistical anisotropy discussed above affects only the low-ℓ multipoles of the CMB anisotropy.

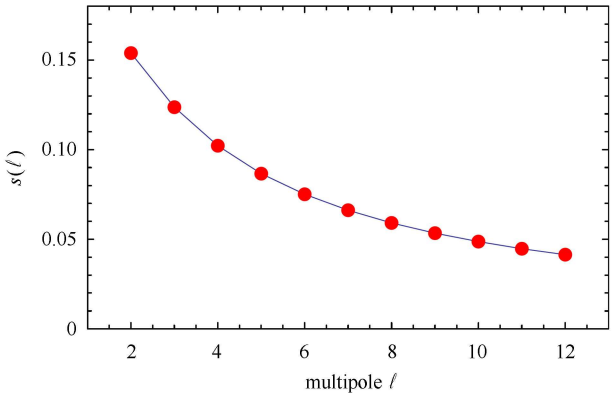

Fig. 1. (color online) $s(\ell)$ is a decreasing function of multipoles ℓ's.

5 Conclusions and remarks

In this paper, we resolved the quadrupole-octopole alignment of CMB anisotropy via the primordial power spectrum with anisotropy. First, the inflationary universe was assumed to be of the Randers-Finsler type. In the Randers spacetime, there is a privileged direction which characterizes the anisotropy of the spacetime. We obtained the initial power spectrum with a dipolar modulation in this spacetime. The dipolar modulation contributes only to the large-scale modes at the universe. The "power tensor" technique was used to analyze the induced anisotropic effects. In this model, however, the anisotropic contribution vanishes for the power tensor of a ℓ-multipole. Thus, the correlation matrix of power tensors was defined in our analysis. This is a statistically significant quantity directly. It describes the direct correlation between any two multipoles. Thus, its eigenvectors with the maximum eigenvalue would lead to the privileged directions of alignments of the two multipoles.

After a lengthy calculation, we obtained the correlation matrix $S_{ij}(2,3)$ for $\ell=2$ and $\ell'=3$. It was found to be diagonal in the chosen coordinates. It acquires the maximum eigenvalue at the plane perpendicular to the privileged direction in the Randers spacetime. This result reveals that the quadrupole and octopole orientations would lie in this plane with the uniform distribution. However, we cannot determine which direction is preferred for the alignment. In the isotropic universe, by contrast, the quadrupole and octopole orientations are distributed uniformly at the celestial sphere. Thus, the possibility for the quadrupole-octopole alignment is much larger in the anisotropic model than that in the isotropic model. In addition, we noted that the anisotropic effect contributes only to the low-ℓ multipoles. Fig. 1 shows that the correlation between two neighboring multipoles decreases rapidly for large ℓ's. This is consistent with the present observations.

Originally, the initial power spectrum with the dipolar modulation was used to resolve the dipole modulation of CMB temperature fluctuations at large-angular scales. Interestingly, the direction for the CMB dipole modulation was found to be nearly perpendicular to the quadrupole-octopole alignment [2]. There might be certain relevance between these two kinds of deviations from isotropy in the CMB. Nevertheless, we need more precise observations to test this proposition in future, since the existing datasets have large uncertainties. In addition, one should note that Finsler geometry is intrinsically anisotropic. Any cosmological process would acquire similar anisotropic properties in Finsler spacetime. Here we studied the inflationary phase of the universe in Randers spacetime. We found that there could be certain imprints of such an anisotropic inflation on the CMB temperature fluctuations. It would be interesting to test these imprints in future CMB observations.

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Appendix A

Calculation of the correlator $C_{\ell\ell' mm'}$

In this appendix, we would like to give an explicit calculation of the two-point correlator $C_{\ell\ell' mm'}$. One could also find a similar derivation in Ref. [66]. First, we could obtain the expression for $a_{\ell m}$ by inverting Eq. (1)

$$a_{\ell m}=\int \mathrm{d}\Omega_{\hat{\boldsymbol{p}}}\frac{\delta T}{T}(\hat{\boldsymbol{p}},\boldsymbol{x},\eta)Y^*_{\ell m}(\hat{\boldsymbol{p}}). \tag{A1}$$

By substituting Eq. (A1) into Eq. (2), one obtains

$$\begin{aligned}C_{\ell\ell' mm'} &= \int \mathrm{d}\Omega_{\hat{\boldsymbol{p}}}\mathrm{d}\Omega_{\hat{\boldsymbol{p}}'}\left\langle \frac{\delta T}{T}(\hat{\boldsymbol{p}},\boldsymbol{x},\eta)\frac{\delta T^*}{T}(\hat{\boldsymbol{p}}',\boldsymbol{x},\eta)\right\rangle \\ &\quad\times Y^*_{\ell m}(\hat{\boldsymbol{p}})Y_{\ell' m'}(\hat{\boldsymbol{p}}').\end{aligned} \tag{A2}$$

The CMB temperature fluctuations have evolved from the initial curvature perturbations according to Eq. (3). Using Eq. (3) and Eq. (6), we could get

$$\begin{aligned}C_{\ell\ell' mm'} &= \int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(\boldsymbol{k})\int \mathrm{d}\Omega_{\hat{\boldsymbol{p}}}\mathrm{d}\Omega_{\hat{\boldsymbol{p}}'}\Delta(k,\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}},\eta) \\ &\quad\times \Delta^*(k,\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}}',\eta)Y^*_{\ell m}(\hat{\boldsymbol{p}})Y_{\ell' m'}(\hat{\boldsymbol{p}}').\end{aligned} \tag{A3}$$

The transfer function $\Delta(k,\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}},\eta)$ has been decomposed in terms of Legendre polynomials in Eq. (4). By substituting Eq. (4) into Eq. (A3), one obtains

$$\begin{aligned}C_{\ell\ell' mm'} &= \int \mathrm{d}^3\boldsymbol{k}\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(\boldsymbol{k})\sum_{\ell''\ell'''}(-\mathrm{i})^{\ell''-\ell'''}\Delta_{\ell''}(k,\eta) \\ &\quad\times\Delta^*_{\ell'''}(k,\eta)\int \mathrm{d}\Omega_{\hat{\boldsymbol{p}}}\mathrm{d}\Omega_{\hat{\boldsymbol{p}}'}\left(\frac{2\ell''+1}{4\pi}\right)P_{\ell''}(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}}) \\ &\quad\times\left(\frac{2\ell'''+1}{4\pi}\right)P^*_{\ell'''}(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}}')Y^*_{\ell m}(\hat{\boldsymbol{p}})Y_{\ell' m'}(\hat{\boldsymbol{p}}'),\end{aligned} \tag{A4}$$

$$\begin{aligned}&= \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(\boldsymbol{k})\sum_{\ell'' m''}\sum_{\ell''' m'''}(-\mathrm{i})^{\ell''-\ell'''} \\ &\quad\times\Delta_{\ell''}(k,\eta)\Delta^*_{\ell'''}(k,\eta)Y^*_{\ell'' m''}(\hat{\boldsymbol{k}})Y_{\ell''' m'''}(\hat{\boldsymbol{k}}) \\ &\quad\times\int \mathrm{d}\Omega_{\hat{\boldsymbol{p}}}Y^*_{\ell m}(\hat{\boldsymbol{p}})Y_{\ell'' m''}(\hat{\boldsymbol{p}}) \\ &\quad\times\int \mathrm{d}\Omega_{\hat{\boldsymbol{p}}'}Y^*_{\ell''' m'''}(\hat{\boldsymbol{p}}')Y_{\ell' m'}(\hat{\boldsymbol{p}}'),\end{aligned} \tag{A5}$$

$$\begin{aligned}&= \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(\boldsymbol{k})((-\mathrm{i})^{\ell-\ell'}\Delta_{\ell}(k,\eta) \\ &\quad\times\Delta^*_{\ell'}(k,\eta)Y^*_{\ell m}(\hat{\boldsymbol{k}})Y_{\ell' m'}(\hat{\boldsymbol{k}})).\end{aligned} \tag{A6}$$

Here Eq. (A6) is just Eq. (5) in the main text. From (A4) to (A5), we have used the fact that Lengendre polynomial could be expanded in terms of the spherical harmonics, i.e.,

$$P_\ell(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}})=\frac{4\pi}{2\ell+1}\sum_{m=-\ell}^{\ell}Y^*_{\ell m}(\hat{\boldsymbol{k}})Y_{\ell m}(\hat{\boldsymbol{p}})\ . \tag{A7}$$

From (A5) to (A6), we used the orthogonal relation of spherical harmonics, namely,

$$\int \mathrm{d}\Omega_{\hat{\boldsymbol{p}}}Y^*_{\ell m}(\hat{\boldsymbol{p}})Y_{\ell' m'}(\hat{\boldsymbol{p}})=\delta_{\ell\ell'}\delta_{mm'}. \tag{A8}$$

In the limit of statistical isotropy, Eq. (A6) will reduce to the diagonal form,

$$C^{iso}_{\ell\ell' mm'}=C_\ell\delta_{\ell\ell'}\delta_{mm'}, \tag{A9}$$

where the variance C_ℓ is given by

$$C_\ell=\int\frac{\mathrm{d}k}{k}\mathcal{P}_{\mathcal{R}}(k)|\Delta_\ell(k,\eta)|^2. \tag{A10}$$

This is Eq. (29) in the main text.

In the following, we discuss a special case where $\langle A_{ij}(\ell)\rangle$ has no preferred directions while the initial perturbations are anisotropic. In this case, one should analyze the correlation matrix of power tensors to determine the possible alignment of two multipoles as was discussed in Eq. (9) and Eq. (10). As an example, we discuss the simplest case that the initial power spectrum has a dipolar modulation, namely, [29, 30]

$$\mathcal{P}_{\mathcal{R}}(\boldsymbol{k})=\mathcal{P}^{\mathrm{iso}}_{\mathcal{R}}(k)\Big(1+g(k)(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}})\Big), \tag{A11}$$

where $\mathcal{P}^{\mathrm{iso}}_{\mathcal{R}}(k)$ denotes the isotropic part and $g(k)$ is a function of k. Here $\hat{\boldsymbol{n}}$ denotes the privileged direction derived from certain anisotropic spacetime. Let the third spatial direction x^3 align with $\hat{\boldsymbol{n}}$. We could use the polar coordinate system in the momentum space. The spherical harmonics are given as

$$Y_{\ell m}(\theta,\phi)=\mathrm{e}^{\mathrm{i}m\phi}\sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}}P^m_\ell(\cos\theta), \tag{A12}$$

where θ is the angle between $\hat{\boldsymbol{k}}$ and z-direction. The expression of $\langle A_{ij}(\ell)\rangle$ can be written as

$$\langle A_{ij}(\ell)\rangle=N(\ell)\sum_{m,m',m''}\langle\ell m|J_i|\ell m'\rangle\langle\ell m''|J_j|\ell m\rangle C_{\ell\ell m'm''}. \tag{A13}$$

To obtain $C_{\ell\ell m'm''}$, one need only calculate the anisotropic effect as

$$\begin{aligned}C^{\mathrm{aniso}}_{\ell\ell m'm''} &\equiv C_{\ell\ell m'm''}-C^{\mathrm{iso}}_{\ell\ell m'm''} \\ &= \int\frac{\mathrm{d}k}{k}\mathcal{P}^{\mathrm{iso}}_{\mathcal{R}}(k)g(k)|\Delta_\ell(k,\eta)|^2I,\end{aligned} \tag{A14}$$

where I denotes the integral as

$$I=\int \mathrm{d}\Omega_{\hat{\boldsymbol{k}}}(\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{k}})Y^*_{\ell m'}(\hat{\boldsymbol{k}})Y_{\ell m''}(\hat{\boldsymbol{k}}). \tag{A15}$$

We can show that I always vanishes. In the polar coordinates, I could be rewritten as

$$I=\int_0^{2\pi}\mathrm{d}\phi\int_0^{\pi}\sin\theta\mathrm{d}\theta\cos\theta Y^*_{\ell m'}(\theta,\phi)Y_{\ell m''}(\theta,\phi). \tag{A16}$$

By substituting (A12) into the above equation, we obtain

$$I\propto\delta_{m'm''}\int_1^{-1}\mathrm{d}\zeta\zeta|P_\ell^{m'}(\zeta)|^2, \tag{A17}$$

where $\zeta=\cos\theta$. It is obvious that the above integral always vanishes. In this way, $C_{\ell\ell m'm''}$ reduces to the isotropic case $C_\ell\delta_{m'm''}$. Thus, the dipolar modulation of the initial perturbations does not affect the eigenvalues of $\langle A_{ij}(\ell)\rangle$. Nevertheless, it could influence the correlation matrix of power tensors. This issue has been discussed in Section 2 and Section 4.

Appendix B

Expressions of J and J_{ij}

In this Appendix, we give the explicit expressions of J and J_{ij}. The representation of so(3) algebra is assumed in the following computation. Especially, we use

$$J_\pm = J_1\pm\mathrm{i}J_2, \tag{B1}$$

$$J_\pm|\ell m\rangle = d_\pm(\ell,m)|\ell m\rangle, \tag{B2}$$

$$J_3|\ell m\rangle = m|\ell m\rangle, \tag{B3}$$

where

$$d_\pm(\ell,m)=\sqrt{(\ell\mp m)(\ell\pm m+1)}. \tag{B4}$$

According to the definition (38), J can be obtained as [29]

$$\begin{aligned}J =& \frac{1}{2}[d_-(\ell,m_1)d_+(\ell',m_2')\delta_{m_2,m_1-1}\delta_{m_1',m_2'+1}\\&+d_+(\ell,m_1)d_-(\ell',m_2')\delta_{m_2,m_1+1}\delta_{m_1',m_2'-1}\\&+2m_1m_1'\delta_{m_1,m_2}\delta_{m_1',m_2'}].\end{aligned} \tag{B5}$$

Similarly, the components of J_{ij} defined in (37) are listed as follows:

$$\begin{aligned}J_{11} =& \frac{1}{4}[d_-(\ell,m_3)d_+(\ell',m_1')\delta_{m_1,m_3-1}\delta_{m_3',m_1'+1}\\&+d_+(\ell,m_3)d_-(\ell',m_1')\delta_{m_1,m_3+1}\delta_{m_3',m_1'-1}\\&+d_-(\ell,m_3)d_-(\ell',m_1')\delta_{m_1,m_3-1}\delta_{m_3',m_1'-1}\\&+d_+(\ell,m_3)d_+(\ell',m_1')\delta_{m_1,m_3+1}\delta_{m_3',m_1'+1}],\end{aligned} \tag{B6}$$

$$\begin{aligned}J_{22} =& \frac{1}{4}[d_-(\ell,m_3)d_+(\ell',m_1')\delta_{m_1,m_3-1}\delta_{m_3',m_1'+1}\\&+d_+(\ell,m_3)d_-(\ell',m_1')\delta_{m_1,m_3+1}\delta_{m_3',m_1'-1}\\&-d_-(\ell,m_3)d_-(\ell',m_1')\delta_{m_1,m_3-1}\delta_{m_3',m_1'-1}\\&-d_+(\ell,m_3)d_+(\ell',m_1')\delta_{m_1,m_3+1}\delta_{m_3',m_1'+1}],\end{aligned} \tag{B7}$$

$$J_{33} = m_3m_1'\delta_{m_1,m_3}\delta_{m_1',m_3'},$$

$$\begin{aligned}J_{12} =& \frac{1}{4\mathrm{i}}[d_-(\ell,m_3)d_+(\ell',m_1')\delta_{m_1,m_3-1}\delta_{m_3',m_1'+1}\\&-d_+(\ell,m_3)d_-(\ell',m_1')\delta_{m_1,m_3+1}\delta_{m_3',m_1'-1}\\&-d_-(\ell,m_3)d_-(\ell',m_1')\delta_{m_1,m_3-1}\delta_{m_3',m_1'-1}\\&+d_+(\ell,m_3)d_+(\ell',m_1')\delta_{m_1,m_3+1}\delta_{m_3',m_1'+1}],\end{aligned} \tag{B8}$$

$$\begin{aligned}J_{21} =& \frac{1}{4\mathrm{i}}[d_-(\ell,m_3)d_+(\ell',m_1')\delta_{m_1,m_3-1}\delta_{m_3',m_1'+1}\\&-d_+(\ell,m_3)d_-(\ell',m_1')\delta_{m_1,m_3+1}\delta_{m_3',m_1'-1}\\&+d_-(\ell,m_3)d_-(\ell',m_1')\delta_{m_1,m_3-1}\delta_{m_3',m_1'-1}\\&-d_+(\ell,m_3)d_+(\ell',m_1')\delta_{m_1,m_3+1}\delta_{m_3',m_1'+1}],\end{aligned} \tag{B9}$$

$$\begin{aligned}J_{13} =& \frac{1}{2}m_1'\delta_{m_1',m_3'}[d_-(\ell,m_3)\delta_{m_1,m_3-1}\\&+d_+(\ell,m_3)\delta_{m_1,m_3+1}],\end{aligned} \tag{B10}$$

$$\begin{aligned}J_{31} =& \frac{1}{2}m_3\delta_{m_1,m_3}[d_-(\ell',m_1')\delta_{m_3',m_1'-1}\\&+d_+(\ell',m_1')\delta_{m_3',m_1'+1}],\end{aligned} \tag{B11}$$

$$\begin{aligned}J_{23} =& \frac{1}{2\mathrm{i}}m_1'\delta_{m_1',m_3'}[d_+(\ell,m_3)\delta_{m_1,m_3+1}\\&-d_-(\ell,m_3)\delta_{m_1,m_3-1}],\end{aligned} \tag{B12}$$

$$\begin{aligned}J_{32} =& \frac{1}{2\mathrm{i}}m_3\delta_{m_1,m_3}[d_+(\ell',m_1')\delta_{m_3',m_1'+1}\\&-d_-(\ell',m_1')\delta_{m_3',m_1'-1}].\end{aligned} \tag{B13}$$

By substituting the above expressions of J and J_{ij} into Eq. (40), we can obtain the correlation matrix between any two neighboring multipoles in the Randers spacetime.

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