Can contributions to $B_s \to l^+l^-\gamma$ from the magnetic-penguin operator with real photons in the standard model be neglected? *

WANG Wen-Yu(王雯宇)[1)] XIONG Zhao-Hua(熊兆华)[2)] ZHOU Si-Hong(周四红)[3)]

Institute of Theoretical Physics, Beijing University of Technology, Beijing 100124, China

Abstract: Using the B_s meson wave function extracted from non-leptonic B_s decays, we reevaluate the rare decays $B_s \to l^+l^- \gamma$, (l=e,μ) in the Standard Model, including two kinds of contributions from the magnetic-penguin operator with virtual and real photons. We find that contributions to the exclusive decays from the magnetic-penguin operator $b \to s\gamma$ with real photons, which were regarded as negligible in the previous literature, are large and the branching ratios $B_s \to l^+l^-\gamma$ are enhanced by a factor of almost 2. With the predicted branching ratios of the order of 10^{-8}, it is expected that these radiative dileptonic decays will be detected in LHC-b and B factories in the near future.

Key words: $B_s \to l^+l^-\gamma$, magnetic-penguin operator, real photons

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1 Introduction

The standard model (SM) of electroweak interactions has been remarkably successful in describing physics below the Fermi scale and is in good agreement with most of the experimental data. Thanks to the efforts of the B factories and the LHC, the exploration of quark-flavor mixing is now entering an interesting new era. Measurements of rare B mesons decays, such as $B \to X_s\gamma$, $B \to X_s l^+l^- (l = e,\mu)$ and $B_s \to \mu^+\mu^-$, $B_s \to l^+l^-\gamma$, are likely to provide sensitive tests of the SM. These decays, induced by flavor changing neutral currents that occur in the SM only at loop level, play an important role in testing higher order effects of the SM and in searching for the physics beyond the SM [1, 2]. Nevertheless, these processes are also important in determining the parameters of the SM and some hadronic parameters in QCD; such as, the CKM matrix elements, the meson decay constant f_{B_s}, and providing information on heavy meson wave functions [3].

The rare radiative inclusive B decays $B \to X_s\gamma$ and $B \to X_s l^+l^- (l = e,\mu)$, as well as the exclusive decay $B_s \to \mu^+\mu^-$, have been studied extensively at the leading logarithm order [4] and higher orders in the SM [5] and various new physics models such as Supersymmetric Models [6], Two-Higgs-Doublet Models [7] and Technicolor Models [8]. Indeed, $B \to X_s\gamma$ is very sensitive to the Wilson coefficient of the magnetic-penguin operator O_7, and $B \to X_s l^+l^-$ depends on the Wilson coefficient of the quark-lepton operators O_9 and O_{10}, while measurement of $B_s \to \mu^+\mu^-$ exerts stringent constraints on the Wilson coefficient of O_{10} and scalar operators in some new physics models. The first evidence of the decay $B_s \to \mu^+\mu^-$ was confirmed at the end of 2012 by LHCb and the result is in good agreement with the Standard Model prediction [9]. Considering that there is non-helicity suppression in $B_s \to \mu^+\mu^-\gamma$ and that a branching ratio as large as that of $B_s \to \mu^+\mu^-$ is expected, the measurement for $B_s \to \mu^+\mu^-\gamma$ can, therefore, be expected as one of the next physics goals at LHCb.

In previous work, predictions for the exclusive decays $B_s \to l^+l^-\gamma$ have been carried out using the light cone sum rule [1, 2], the simple constituent quark model [10], and the B meson distribution amplitude extracted from non-leptonic B decays [11]. Long distance QCD effects describing the neutral vector-meson resonances ϕ and the J/Ψ family have received special attention in Refs. [12–14]. At parton level, $B_s \to l^+l^-\gamma$ decays have been thought to be obtained from the decay $b \to s l^+l^-\gamma$, and further, from $b \to s l^+l^-$ directly. To achieve this, it is necessary to attach a real photon to any charged internal and external line in the Feynman diagrams of $b \to s l^+l^-$,

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1) E-mail: wywang@mail.itp.ac.cn

2) E-mail: xionzh@mail.ihep.ac.cn

3) E-mail: sihongzhou@emails.bjut.edu.cn

with two assumptions: i) contributions from the attachment of the photon to any charged internal propagator are regarded as strongly suppressed by a factor $m_{\rm b}^2/m_{\rm W}^2$ and thus can be safely neglected [1, 2, 10, 11]; and, ii) contributions from the attachment of the real photon with a magnetic-penguin vertex to any charged external lines are always neglected [1, 2] or stated to be negligibly small [11].

We would like to point out that the conclusion of the first statement is correct, but the explanation is not as described [15]. It is easily understood that: 1) contributions from such diagrams with a $\rm W^+W^-\gamma\gamma$ vertex then one photon into a lepton pair are not suppressed, and 2) one of two internal quarks in the effective vertex is off-shell when applying an effective vertex of $\rm b\to s\gamma$ to describe $\rm b\to s\gamma\gamma$. Such off-shell effects are also not suppressed. We have proven that two such non-suppressed effects cancel each other exactly [15]. Therefore, we can use the effective operators for on-shell quarks to safely calculate the total short distance contributions of $\rm B_s\to l^+l^-\gamma$ in the SM.

The second statement seems to be questionable, if the pole of the propagator of the charged line to which the photon is attached may enhance the decay rate greatly, this means that some diagrams cannot be neglected in the calculation. Since the weak radiative B-meson decay is well known to be a sensitive probe of new physics, it is essential to calculate the Standard Model value of its branching ratio as precisely as possible. Although the second contribution has been calculated in Ref. [12], it has mainly concentrated on the long distance effects from the meson resonances, whereas the short distance contribution was incompletely analyzed. Note that only the short-distance contribution can be reliably predicted and is more important than the long-distance contribution from the resonances because the long-distance effect is actually excluded by setting cuts in the experiments, as is done by Belle in their measurement of the branching ratio $\rm B\to X_s l^+l^-$.

In this paper, we will concentrate on the short distance contribution to $\rm B_s \to l^+l^-\gamma$ and check whether the contribution from the magnetic-penguin operator with real photon to $\rm B_s\to l^+l^-$ is negligible or not, and give some remarks, including a comparison with other works. This paper is organized as follows. In Section 2, we present a detailed calculation of the exclusive decays $\rm B_s\to l^+l^-\gamma$, including the full contribution from the magnetic-penguin operator with a real photon. Section 3 contains the numerical results and a comparison with previous works. The conclusions are given in Section 4.

2 The calculation

In order to simplify the decay amplitude for $\rm B_s\to l^+l^-\gamma$, we have to utilize the $\rm B_s$ meson wave function, which is not known from first principals. So far, only the model dependent $\rm B_s$ meson wave function is available. Fortunately, $\rm B_s$ decays [16] have placed strict constraints on the wave function. In this work we use the $\rm B_s$ meson wave function extracted from non-leptonic B decays [17, 18]:

$$\varPhi_{\rm B_s}=\frac{(\not{p}_{\rm B_s}+m_{\rm B_s})\gamma_5}{\sqrt{6}}\ \phi_{\rm B_s}(x), \tag{1}$$

where the distribution amplitude $\phi_{\rm B_s}(x)$ can be expressed [19] as:

$$\phi_{\rm B_s}(x)=N_{\rm B_s}x^2(1-x)^2\exp\left(-\frac{m_{\rm B}^2\ x^2}{2\omega_{\rm b_s}^2}\right) \tag{2}$$

with x being the momentum fraction possessed by the s quark in the $\rm B_s$ meson, and $\omega_{\rm b_s}$ being a parameter determined by experimental data on $\rm B_s$ hadronic decays in the pQCD approach. The normalization constant $N_{\rm B_s}$ can be determined by comparing

$$\begin{aligned}\langle 0|\bar{s}\gamma^{\mu}\gamma_5 b|B_{\rm s}\rangle &= {\rm i}N_{\rm c}\int_0^1{\rm d}x\phi_{\rm B_s}(x){\rm Tr}\left[\gamma^{\mu}\gamma_5\frac{(\not{p}_{\rm B_s}+m_{\rm B_s})\gamma_5}{\sqrt{6}}\right]\\ &= -\frac{4N_{\rm c}}{\sqrt{6}}{\rm i}p_{\rm B_s}^{\mu}\int_0^1\phi_{\rm B_s}(x){\rm d}x\end{aligned}$$

with

$$\langle 0|\bar{s}\gamma^{\mu}\gamma_5 b|B_{\rm s}\rangle=-{\rm i}f_{\rm B_s}p_{\rm B_s}^{\mu}, \tag{3}$$

where $N_{\rm c}=3$ is the color number of the quarks and $f_{\rm B_s}$ is the $\rm B_s$ meson decay constant. Thus we have

$$\int_0^1\phi_{\rm B_s}(x){\rm d}x=\frac{f_{\rm B_s}}{2\sqrt{2N_{\rm c}}}. \tag{4}$$

Let us start with the quark level processes $\rm b\to sl^+l^-$, which are subject to the QCD corrected effective weak Hamiltonian, obtained by integrating out heavy particles; that is, top quark, Higgs, and $\rm W^{\pm}$, Z bosons:

$$\begin{aligned}&\mathcal{H}_{\rm eff}(\rm b\to sl^+l^-)\\ &= -\frac{\alpha_{\rm em}G_{\rm F}}{\sqrt{2}\pi}V_{\rm tb}V_{\rm ts}^*\left\{\left[-\frac{2C_7^{\rm eff}m_{\rm b}}{q^2}\bar{s}{\rm i}\sigma^{\mu\nu}q_{\nu}P_{\rm R}b\right.\right.\\ &\left.\left.+C_9^{\rm eff}\bar{s}\gamma^{\mu}P_{\rm L}b\right]\bar{l}\gamma_{\mu}l+C_{10}(\bar{s}\gamma^{\mu}P_{\rm L}b)\ \bar{l}\gamma_{\mu}\gamma_5 l\right\},\end{aligned} \tag{5}$$

where $P_{\rm L,R}=(1\mp\gamma_5)/2$, q^2 is the dilepton invariant mass squared. The QCD corrected Wilson coefficients $C_7^{\rm eff}$, $C_9^{\rm eff}$ and C_{10} at $\mu=m_{\rm b}$ scale can be found in Ref. [20].

If an additional photon line is attached to any of the charged lines in the diagrams contributing to the Hamiltonian above, we will have the radiative leptonic decays $\rm b\to sl^+l^-\gamma$. Therefore, there are two kinds of diagrams: a photon connecting to the internal propagators, and a photon connecting to the external line. As addressed in the introduction, the contribution from the first kind of

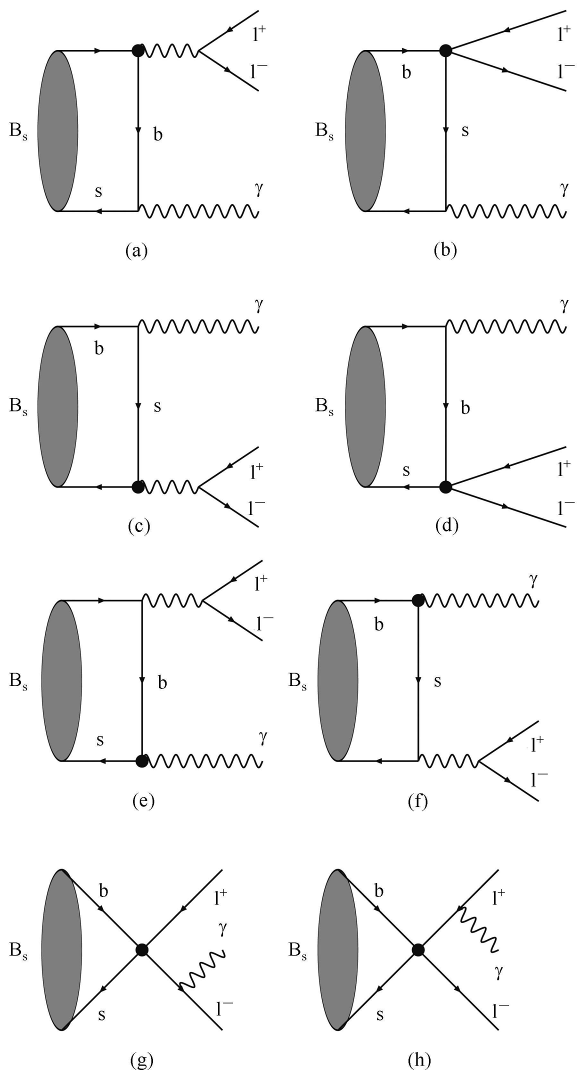

Fig. 1. Feynman diagrams for $B_q\to l^+l^-\gamma$ (q=d,s) in the SM. The black dot in (a), (c) and (e), (f) stands for the magnetic-penguin operator O_7 with virtual and real photons, respectively, and the black dot in (b), (d), (g) and (f) denotes operators O_9 and O_{10}.

diagram can be safely neglected. We will, therefore, only consider the second category of diagrams, which are displayed in Fig. 1.

At first, we recalculate the diagrams (a)–(d) in Fig. 1 with a photon emitted from the external quark lines b or s, using the B_s meson wave function extracted from non-leptonic B_s decays. Note that these diagrams have already been studied in previous literature and are considered as giving the dominant contribution to $B_s\to l^+l^-\gamma$. At the parton level, the amplitudes for the transition $b\to sl^+l^-\gamma$ can be calculated directly from the Hamiltonian of $b\to sl^+l^-$ in (5). For example, the contribution from the magnetic-penguin operator with a virtual photon, as shown in Fig. 1 (a), reads:

$$\mathcal{A}_{\rm a}^{\rm quark} = -{\rm i}ee_{\rm d}\frac{\alpha_{\rm em}G_{\rm F}}{\sqrt{2}\pi}V_{\rm tb}V_{\rm ts}^*\frac{m_{\rm b}}{q^2}C_7^{\rm eff} \times\left(\bar{s}\left[\frac{2p_{\rm s}\cdot\varepsilon^*+\not{\varepsilon}^*\not{k}}{p_{\rm s}\cdot k}\gamma^{\mu}\not{q}P_{\rm R}\right]b\right)\left[\bar{l}\gamma_{\mu}l\right], \quad (6)$$

where $p_{\rm b,s}$, k denotes the momentum of quarks and photon, respectively; ε^* is the vector polarization of photon; and, $e_{\rm d}=-1/3$ is electric charge number of the external quarks. In deriving the above equation, we have used the motion equation for quarks and $q^{\mu}\bar{l}\gamma_{\mu}l=0$.

Using the B_s meson wave function in (1) and (6), we write the amplitude of $B_s\to l^+l^-\gamma$ at the meson level as:

$$A_a = -2{\rm i}G\frac{m_{\rm b}m_{\rm B_s}}{q^2}C_7^{\rm eff}\frac{1}{p_{\rm B_s}\cdot k}\int_0^1\frac{\phi_{\rm B_s}(x)}{x}{\rm d}x \times\left[k_{\mu}q\cdot\varepsilon^*-\varepsilon_{\mu}^*k\cdot q-{\rm i}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}k^{\alpha}q^{\beta}\right]\left[\bar{l}\gamma^{\mu}l\right], \quad (7)$$

where x, $y=1-x$ are the momentum fractions shared by s, b quarks in the B_s. By doing a similar calculation for diagrams (b)–(d), and taking $G=\alpha_{\rm em}^{3/2}G_{\rm F}V_{\rm tb}V_{\rm ts}^*/\sqrt{3}$, the decay amplitude is then obtained as:

$$\begin{aligned}A_{a+b+c+d} = -{\rm i}G\frac{1}{p_{\rm B_s}\cdot k}\Big\{&\left[C_1{\rm i}\epsilon_{\alpha\beta\mu\nu}p_{\rm B_s}^{\alpha}\varepsilon^{*\beta}k^{\nu}\right.\\&\left.+C_2p_{\rm B_s}^{\nu}(\varepsilon_{\mu}^*k_{\nu}-k_{\mu}\varepsilon_{\nu}^*)\right]\bar{l}\gamma^{\mu}l\\&+C_{10}\left[C_+{\rm i}\epsilon_{\alpha\beta\mu\nu}p_{\rm B_s}^{\alpha}\varepsilon^{\beta*}k^{\nu}\right.\\&\left.+C-p_{\rm B_s}^{\nu}(\varepsilon_{\mu}^*k_{\nu}-k_{\mu}\varepsilon_{\nu}^*)\right]\bar{l}\gamma^{\mu}\gamma_5 l\Big\}. \quad (8)\end{aligned}$$

The form factors in Eq. (8) are found to be:

$$\begin{aligned}C_1 &= C_+\left(C_9^{\rm eff}-2\frac{m_{\rm b}m_{\rm B_s}}{q^2}C_7^{\rm eff}\right),\\C_2 &= C_9^{\rm eff}C_--2\frac{m_{\rm b}m_{\rm B_s}}{q^2}C_7^{\rm eff}C_+, \quad (9)\end{aligned}$$

where

$$C_{\pm}=\int_0^1\left(\frac{1}{x}\pm\frac{1}{y}\right)\phi_{\rm B_s}(x){\rm d}x. \quad (10)$$

The expression in (8) is comparable with the amplitude obtained in Ref. [11], except for the coefficient C_2, which is very important for the numerical results. Additionally, Ref. [11] has no factor $1/(p_{\rm B_s}\cdot k)$.

Now we will focus attention on calculating the contributions from diagrams (e) and (f) in Fig. 1, which have been neglected in other works. In these two diagrams, the photon of the magnetic-penguin operator is real, thus its contribution to $B_s\to l^+l^-\gamma$ is different from that of the magnetic-penguin operator with a virtual photon in diagrams (a) and (c). We get the amplitude:

$$A_{e+f} = -\mathrm{i}2GC_7^{\mathrm{eff}}\frac{m_\mathrm{b}m_{\mathrm{B_s}}}{q^2}\frac{1}{p_{\mathrm{B_s}}\cdot q}\overline{C}_+ \times\left[k_\mu q\cdot\varepsilon^* - \epsilon^*_\mu k\cdot q - \mathrm{i}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}k^\alpha q^\beta\right]\left[\bar{l}\gamma^\mu l\right], \quad (11)$$

with coefficients $\overline{C}_+$ obtained by the substitution:

$$\begin{aligned}\overline{C}_+ &= C_+(x\to\bar{x}=x-z-\mathrm{i}\epsilon;\ y\to\bar{y}=y-z-\mathrm{i}\epsilon) \\ &= N_{\mathrm{B_s}}\int_0^1\mathrm{d}x\left(\frac{1}{x-z-\mathrm{i}\epsilon}+\frac{1}{1-x-z-\mathrm{i}\epsilon}\right) \\ &\quad\times x^2(1-x)^2\exp\left[-\frac{m_{\mathrm{B_s}}^2}{2\omega_{\mathrm{B_s}}^2}x^2\right], \end{aligned}\quad (12)$$

where $z=\dfrac{q^2}{2p_{\mathrm{B_s}}\cdot q}$ and the first and second term in (11) denote the contributions from diagrams (e) and (f), respectively. Note that the contribution from diagram (a) is much larger than (c) since $m_{\mathrm{B_s}}\gg\omega_{\mathrm{B_s}}$ (see next section), which is very easily understood in the simple constituent quark model [10]; that is, $\phi_{\mathrm{B_s}}(x)=\delta(x-m_\mathrm{s}/m_{\mathrm{B_s}})$. However, the contributions from diagrams (e) and (f) are comparable. Furthermore, the pole in $\overline{C}_+$ corresponds to the pole of the quark propagator when it is connected by the off-shell photon propagator. Thus, the $\overline{C}_+$ term may enhance the decay rate of $\mathrm{B_s}\to \mathrm{l^+l^-}\gamma$ and its analytic expression reads

$$\begin{aligned}\overline{C}_+ &= 2N_{\mathrm{B_s}}\pi\mathrm{i}z^2(1-z)^2\exp\left[-\frac{m_{\mathrm{B_s}}^2}{2\omega_{\mathrm{B_s}}^2}z^2\right] \\ &\quad+N_{\mathrm{B_s}}\int_0^1\mathrm{d}x\left(\frac{1}{x+z}-\frac{1}{1+x-z}\right) \\ &\quad\times x^2(1+x)^2\exp\left[-\frac{m_{\mathrm{B_s}}^2}{2\omega_{\mathrm{B_s}}^2}x^2\right] \\ &\quad-N_{\mathrm{B_s}}\int_{-1}^1\left(\frac{1}{\frac{1}{x}-z}+\frac{1}{1-\frac{1}{x}-z}\right)\frac{\mathrm{d}x}{x^4} \\ &\quad\times\left(1-\frac{1}{x}\right)^2\exp\left[-\frac{m_{\mathrm{B_s}}^2}{2\omega_{\mathrm{B_s}}^2}\frac{1}{x^2}\right]. \end{aligned}\quad (13)$$

For the contributions from Fig. 1(g) and (h), where the photon is attached to the external lepton lines, consider the fact that: (i) being a pseudoscalar meson, the $\mathrm{B_s}$ meson can only decay through axial currents, so the magnetic penguin operator O_7 's contribution vanishes; (ii) the contribution from operators O_9, O_{10} has the helicity suppression factor $m_\mathrm{l}/m_{\mathrm{B_s}}$. So, for light lepton (electrons and muons), we can safely neglect their contribution.

The total matrix element for the decay $\mathrm{B_s}\to\mathrm{l^+l^-}\gamma$ is obtained as a sum of the amplitudes $A_{a+b+c+d}$ and A_{e+f}. After summing over the spins of leptons and polarizations of the photon, and then performing phase space integration over one of the two Dalitz variables, we get the differential decay width versus the photon energy E_γ,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma} = \frac{\alpha^3G_\mathrm{F}^2}{72\pi^4}|V_{\mathrm{tb}}V_{\mathrm{ts}}^*|^2(m_{\mathrm{B_s}}-2E_\gamma)E_\gamma \times\left[|\overline{C}_1|^2+|\overline{C_2}|^2+C_{10}^2(|C_+|^2+|C_-|^2)\right]. \quad (14)$$

The coefficients $\overline{C}_i$ $(i=1,2)$ can be obtained from the transformation:

$$\overline{C}_i=C_i-\frac{2m_\mathrm{b}m_{\mathrm{B_s}}}{q^2}\frac{p_{\mathrm{B_s}}\cdot k}{p_{\mathrm{B_s}}\cdot q}\overline{C}_+C_7^{\mathrm{eff}}. \quad (15)$$

3 Results and discussions

The decay branching ratios can be easily obtained by integrating over the photon energy. In our numerical calculations, we use the following parameters [21]:

$$\alpha_{\mathrm{em}}=\frac{1}{137},\ G_\mathrm{F}=1.166\times10^{-5}\ \mathrm{GeV}^{-2},\ m_\mathrm{b}=4.2\ \mathrm{GeV},$$
$$|V_{\mathrm{tb}}|=0.999,\ |V_{\mathrm{ts}}|=0.04,\ |V_{\mathrm{td}}|=0.0084$$
$$m_{\mathrm{B_s}}=5.37\ \mathrm{GeV},\ \omega_{\mathrm{B_s}}=0.5,$$
$$f_{\mathrm{B_s}}=0.24\ \mathrm{GeV},\ \tau_{\mathrm{B_s}}=1.47\times10^{-12}\mathrm{s}.$$
$$m_{\mathrm{B_d^0}}=5.28\ \mathrm{GeV},\ \omega_{\mathrm{B_d}}=0.4,$$
$$f_{\mathrm{B_d}}=0.19\ \mathrm{GeV},\ \tau_{\mathrm{B_d}}=1.53\times10^{-12}\mathrm{s}.$$

The ratios of $\mathrm{B_s}\to\mathrm{l^+l^-}\gamma$ with and without the contribution from the magnetic-penguin operator with a real photon are shown in Table 1, together with results of $\mathrm{B_{d,s}}\to\mathrm{l^+l^-}\gamma$ from this work and other models for comparison. The errors shown in Table 1 come from the heavy meson wave function, obtained by varying the parameter $\omega_{\mathrm{B_d}}=0.4\pm0.1$, and $\omega_{\mathrm{B_s}}=0.5\pm0.1$ [11]. Note that, the predicted branching ratios include errors from many parameters, such as the meson decay constant, meson and quark masses etc.

Table 1. Comparison of branching ratios with other model calculations.

branching ratios ($\times10^{-9}$)	our results		quark model		light cone
	excluding Fig. 1(e), (f)*	including Fig. 1(e), (f)	Ref. [11]	Ref. [10]	Ref. [1]
$\mathrm{B_s}\to\mathrm{l^+l^-}\gamma$	$5.16^{+2.42}_{-1.38}$	$10.2^{+4.11}_{-2.51}$	1.90	6.20	2.35
$\mathrm{B_d^0}\to\mathrm{l^+l^-}\gamma$	$0.21^{+0.14}_{-0.06}$	$0.40^{+0.26}_{-0.13}$	0.08	0.82	0.15

*Our expression for the amplitude of diagrams (a) and (b) is different from that in Ref. [11]

Table 2. The form factor values for the matrix elemental $\langle\gamma(k,\varepsilon)|\hat{Q}|B_s(q+k)\rangle = e/m_{B_s}[T_1(q^2)i\epsilon_{\alpha\beta\mu\nu}q^{\alpha}\varepsilon^{*\beta}k^{\nu} + T_2(q^2)q^{\nu}(\varepsilon^*_{\mu}k_{\nu}-k_{\mu}\varepsilon^*_{\nu})]$ at $q^2=0$.

operator $\hat{Q}$	form factor values			
	excluding Fig. 1(e), (f)		including Fig. 1(e), (f)	
$\hat{Q}=\bar{s}\gamma_{\mu}(1-\gamma_5)b$	$T_1(0)=0.077$	$T_2(0)=0.060$	$T_1(0)=0.077$	$T_2(0)=0.060$
$\hat{Q}=\bar{s}\sigma_{\mu\nu}q^{\nu}(1+\gamma_5)b/m_{B_s}$	$T_1(0)=T_2(0)=0.077$		$T_1(0)=T_2(0)=0.173$	

Also note that, in order to compare our result with previous works, we set a cut off for the photon (for instance, $E^{\gamma}_{\min}=50$ MeV in Ref. [12]), which cannot be measured by experiments, and we then redid the calculation. We find that the branching ratio is not sensitive to the cut off.

A couple of remarks follow on the B_s rare exclusive radiative decays:

1) As pointed out in Ref. [3, 11], the branching ratios are proportional to the square of the heavy meson wave function, so the radiative leptonic decays are very sensitive probes for extracting the heavy meson wave functions.

2) The contribution to the exclusive decay from the magnetic-penguin operator with real photons is large, and the branching ratio of $B_s\to l^+l^-\gamma$ is enhanced by a factor of almost 2 compared with when the only contribution is from the magnetic-penguin operator with virtual photons. This increases the expected branching ratio to nearly up to 10^{-8}, implying that the search for $B_s\to l^+l^-\gamma$ can be achieved in near future.

3) Due to the large contributions from the magnetic-penguin operator with real photons, the form factors for the matrix elements $\langle\gamma|\bar{s}\gamma^{\mu}(1\pm\gamma_5)b|B_s\rangle$ and $\langle\gamma|\bar{s}\sigma_{\mu\nu}(1\pm\gamma_5)q^{\nu}b|B_s\rangle$ as a function of dilepton mass squared q^2 are not as simple as $1/(q^2-q_0^2)^2$, where q_0^2 is constant [22]. The $B_s\to\gamma$ transition form factors predicted in this work also have some differences from those in Ref. [12–14]. For instance, Ref. [13] predicted that the form factors $F_{TV}(q^2,0)$, $F_{TA}(q^2,0)$ induced by tensor and pseudotensor currents with emission of the virtual photon, as shown in diagrams (a) and (c) of Fig. 1, are only equal at maximum photon energy, whereas the corresponding formulae in this work have the same expression as $\frac{m_{B_s}}{p_{B_s}\cdot k}\frac{C_+}{\sqrt{6}}\propto 1/(q^2-q_0^2)$ in Eq. (8). Note that with the inclusion of the contribution of more diagrams, the form factor for operator O_7 from $m_{B_s}C_+/\sqrt{6}p_{B_s}\cdot k$ changes to $m_{B_s}\overline{C}_+/\sqrt{6}p_{B_s}\cdot k$, so that the form factors are larger than previous works. A comparison of the form factor values at $q^2=0$ is displayed in Table 2. Further, since the values of $C_9^{\rm eff}$ and $C_7^{\rm eff}$ have opposite signs, including the contributions from new diagrams enhances the branching ratio of $B_s\to l^+l^-\gamma$. More research on $B_s\to l^+l^-\gamma$ may give some hints on these form factors.

At this stage, we think that it is necessary to present a few more comments about the calculation given in Ref. [12]. In order to estimate the contribution from emission of the real photon from the magnetic-penguin operator, the authors of Ref. [12] calculated the form factors $F_{TA,TV}(0,q^2)$ by including the short distance contribution in the limit $q^2\to 0$ and additional long distance contributions from the resonances of vector mesons, such as ρ^0, ω for B_d decay and ϕ for B_s decay. Obviously, this means that the short-distance contributions were not appropriately taken into account. Moreover, if $F_{TA,TV}(0,q^2)=F_{TA,TV}(0,0)$ stands for the short distance contribution, it seems to be double counting since in this case the photons that are emitted from the magnetic-penguin vertex and those that are emitted directly from the quark lines are not distinguishable. We also note that the contributions from weak-annihilation diagrams due to the four-quark operators should be taken into account. The authors of Ref. [12] considered such contributions by taking into account u and c quarks in the loop from the axial anomaly, which is also in the long-distance calculation. They concluded that the anomalous contribution is suppressed because of the power of the heavy quark mass compared to other contributions. We believe that only considering the anomalous contribution from the weak annihilation is not sufficient. A detailed evaluation of the short-distance contributions from the weak-annihilation diagrams will appear in our following work.

4 Conclusion

Using the B_s meson wave function extracted from non-leptonic B_s decays, we evaluated the rare decays $B_s\to\gamma l^+l^-$ in the SM, including two kinds of contributions from the magnetic-penguin operator with virtual and real photons. In contrast to previous works, we found that the contribution to the decays from magnetic-penguin operators with real photons is large, leading to the branching ratios $B_s\to l^+l^-\gamma$ being enhanced by a factor of almost 2. In the current early phase of the LHC era, the exclusive modes with muons in the final states are among the most promising decays for measuring this. Although there are some theoretical challenges in the calculation of the hadronic form factors and non-factorable corrections, with the predicted branching ratios of the order of 10^{-8}, and $B_s\to\mu^+\mu^-$ already measured by LHCb [9], $B_s\to l^+l^-\gamma$ can be expected as the next goal. As one of the main backgrounds in measuring $B_s \to \mu^+\mu^-$, predicting the value of $B_s \to \mu^+\mu^-\gamma$ as precisely as possible is also important. So far, the LHCb collaboration have only adopted the theoretical result in Ref. [12] to estimate the background. Therefore, our predictions for such processes are not only valuable for $B_s \to \mu^+\mu^-$ measurement, but can also be tested in the LHC-b and B factories in the near future.

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