Bubble contributions to scalar correlators with mixed actions *

FU Zi-Wen(傅子文)[1)]

Key Laboratory of Radiation Physics and Technology (Sichuan University), Ministry of Education;
Institute of Nuclear Science and Technology, Sichuan University, Chengdu 610064, China

Abstract: Within mixed-action chiral perturbation theory (MAχPT), Sasa's derivation of the bubble contribution to scalar a_0 meson is extended to those of scalar κ and σ mesons. We revealed that the κ bubble has two double poles and the σ bubble contains a quadratic-in-t^2 growth factor stemming from the multiplication of two double poles for a general mass tuning of valence quarks and sea quarks. The corresponding preliminary analytical expressions in MAχPT with 2+1 chiral valence quarks and 2+1 staggered sea quarks will be helpful for lattice studies of scalar mesons.

Key words: bubble contribution, scalar meson, MAχPT, SχPT

PACS: 11.15.Ha, 12.38.Gc **DOI:** 10.1088/1674-1137/38/6/063102

1 Introduction

Although the nature of the lowest scalar meson has been unveiled [1], there are long-lasting debates on the traits of the lowest scalar meson [2–7]. The tetraquark interpretation has easily realized the experimental mass ordering $m_{a_0(980)} > m_\kappa$ by lattice QCD, while $\bar{q}q$ states hardly explain it [7–11]. However, the tetraquark interpretation has been sharply criticized for overlooking some unresolved physical issues, such as chiral symmetry breaking and the non-trivial vacuum state [12], etc. This question can be partially reconciled if the masses of $\bar{q}q$ states with $I=1/2$,1 and 0 are robustly calculated on lattice. We call these states κ, a_0 and σ mesons, respectively.

The a_0 meson was studied in staggered fermion [13, 14]. The propagators are discovered to hold states with masses below the likely combinations of two physical mesons [13, 14], which can be nicely interpreted by the bubble contribution to a_0 correlator (for simplicity, we call it "a_0 bubble", likewise for "κ bubble" and "σ bubble") derived by Sasa in mixed-action chiral perturbation theory (MAχPT) [15].

With 2+1 Asqtad-improved staggered sea quarks [16], we studied the κ meson [6], and rectified the taste-symmetry breaking by extending the analyses of σ and a_0 mesons [15, 17–20] to κ meson in staggered chiral perturbation theory (SχPT) [21, 22]. We realized that the κ bubble should be involved in a fit of κ correlator for the MILC medium coarse ($a\approx 0.15$ fm) and coarse ($a\approx 0.12$ fm) lattice ensembles. Moreover, these bubble contributions in SχPT offer a test of lattice artifacts due to the fourth-root trick [15, 17]. Additionally, SχPT predicts that these lattice artifacts vanish in the continuum limit, merely remaining as physical thresholds [15, 17–22]. To check this, we specially studied scalar mesons at a MILC fine ($a\approx 0.09$ fm) lattice ensemble [19, 22].

Lattice studies with staggered fermions are cheaper than those of other discretizations, which allow lattice studies with the smaller dynamical quark masses or finer lattice spacings. However, this benefit is accompanied by an extra theoretical complication. Each staggered quark exists in four tastes [23], and the staggered meson comes in sixteen tastes, and the taste symmetry breaking at $a\neq 0$ gives rise to discretization errors of $\mathcal{O}(a^2)$ [24]. Since these errors are usually not negligible, these unphysical effects predicted by SχPT should be neatly removed from lattice data to extract the desired physical quantities.

Theoretically, a lattice study with domain-wall (DW) quarks is simpler than that with staggered quarks since they do not come in multiple species [25, 26]. This makes MAχPT expressions for bubble contributions to scalar mesons simple and continuum-like [15, 27]. Moreover, they keep proper chiral symmetry up to exponentially small corrections at $a\neq 0$ [28]. We have noticed a pioneering study on a_0 meson with DW fermions [29].

MAχPT for Ginsparg-Wilson type quarks on a stag-

Received 22 July 2013

∗ Supported by Fundamental Research Funds for the Central Universities (2010SCU23002)

1) E-mail: fuziwen@scu.edu.cn

gered sea was derived [27]. Other quantities, such as the a_0 bubble, have since been added in MAχPT [15, 30–33]. It is worth mentioning that only a few extra parameters have entered the relevant chiral formulas. One of these parameters was determined by the MILC Collaboration [14]. Aubin et al. estimated another parameter which is inherent in a mixed-action case [29]. These parameters can be used to study other quantities: for example, bubble contributions to scalar mesons.

In this work we extend Sasa's derivation of a_0 bubble [15] to those of κ and σ correlators in MAχPT with 2+1 chiral valence quarks and 2+1 staggered sea quarks, and use the two above-mentioned published parameters [14, 29] to elucidate our analytical expressions. We have found that κ and σ bubbles have particular features.

2 MAχPT

At leading-order quark mass expansion, the mixed action chiral Lagrangian is described by $N_{\rm V}$ Ginsparg-Wilson valence quarks and $N_{\rm S}$ staggered sea quarks [27]. Each staggered sea quark comes in four tastes, and each Ginsparg-Wilson valence quark owns a bosonic ghost partner, and these bosons are expressed by the field

$$\Sigma=\exp(2{\rm i}\Phi/f),$$

which is an element of $U(4N_{\rm S}{+}N_{\rm V}|N_{\rm V})$, and Φ is a matrix gathering pseudoscalar fields. For instance, in the case $N_{\rm V}=3$, $N_{\rm S}=3$, Φ is arranged by [27, 30]

$$\begin{pmatrix}
U & \pi^+ & K^+ & Q_{\rm ux} & Q_{\rm uy} & Q_{\rm uz} & \cdots & \cdots & \cdots \\
\pi^- & D & K^0 & Q_{\rm dx} & Q_{\rm dy} & Q_{\rm dz} & \cdots & \cdots & \cdots \\
K^- & \bar{K}^0 & S & Q_{\rm sx} & Q_{\rm sy} & Q_{\rm sz} & \cdots & \cdots & \cdots \\
Q^\dagger_{\rm ux} & Q^\dagger_{\rm dx} & Q^\dagger_{\rm sx} & X & P^+ & T^+ & R^\dagger_{\rm \tilde{x}x} & R^\dagger_{\rm \tilde{y}x} & R^\dagger_{\rm \tilde{z}x} \\
Q^\dagger_{\rm uy} & Q^\dagger_{\rm dy} & Q^\dagger_{\rm sy} & P^- & Y & T^0 & R^\dagger_{\rm \tilde{x}y} & R^\dagger_{\rm \tilde{y}y} & R^\dagger_{\rm \tilde{z}y} \\
Q^\dagger_{uz} & Q^\dagger_{\rm dz} & Q^\dagger_{\rm sz} & T^- & \bar{T}^0 & Z & R^\dagger_{\rm \tilde{x}z} & R^\dagger_{\rm \tilde{y}z} & R^\dagger_{\rm \tilde{z}z} \\
\cdots & \cdots & \cdots & R_{\rm \tilde{x}x} & R_{\rm \tilde{x}y} & R_{\rm \tilde{x}z} & \tilde{X} & \tilde{P}^+ & \tilde{T}^+ \\
\cdots & \cdots & \cdots & R_{\rm \tilde{y}x} & R_{\rm \tilde{y}y} & R_{\rm \tilde{y}z} & \tilde{P}^- & \tilde{Y} & \tilde{T}^0 \\
\cdots & \cdots & \cdots & R_{\rm \tilde{z}x} & R_{\rm \tilde{z}y} & R_{\rm \tilde{z}z} & \tilde{T}^- & \tilde{\bar{T}}^0 & \tilde{Z}
\end{pmatrix},$$

where we label the sea quarks by u, d, and s and the corresponding valence quarks and valence ghosts by x, y, z and x̃, ỹ, z̃, respectively. The sea-quark bound state fields are U, π^+, K^+, etc., P^+, T^+, T^0, X, Y and Z are the x$\bar{\rm y}$, x$\bar{\rm z}$, y$\bar{\rm z}$, x$\bar{\rm x}$, y$\bar{\rm y}$ and z$\bar{\rm z}$ valence bound states, respectively; and, $\tilde{P}^+$, $\tilde{T}^+$, $\tilde{T}^0$, $\tilde{X}$, $\tilde{Y}$ and $\tilde{Z}$ are the relevant combinations of valence ghost quarks. Those labeled by R's are the (fermionic) bound state composed of one valence and one ghost quark. Similarly, $Q_{\rm Fv}$ stands for the bosonic mixed bound state F$\bar{\rm v}$, where F $\in\{$u,d,s$\}$, and v$\in\{$x,y,z$\}$. The mixed ghost-sea pseudo-Goldstone bosons indicated by ellipses are not used in this work.

The valence-valence mesons obey mass relations [27]:

$$M^2_{\rm vv'}=\mu(m_{\rm v}+m_{\rm v'}), \tag{1}$$

where three valence quarks have $m_{\rm x}$, $m_{\rm y}$ and $m_{\rm z}$ (which is the same for the corresponding valence ghost flavors), respectively. We are only interested in the degenerate up and down valence quarks masses; that is, $m_{\rm x}=m_{\rm y}\neq m_{\rm z}$ (2+1) case.

For a meson of taste b made up of sea quarks F and F′ (F≠F′), the tree-level results give [24]

$$M^2_{\rm FF',b}=\mu(m_{\rm F}+m_{\rm F'})+a^2\Delta(\xi_{\rm b}), \tag{2}$$

where the staggered sea quarks u, d, s own masses $m_{\rm u}$, $m_{\rm d}$ and $m_{\rm s}$, respectively, and $\Delta(\xi_b)$ is different for each of $SO(4)$-taste irreps: P, V, A, T, I [24]. A new operator relating the valence and sea sectors yields a taste breaking parameter $a^2\Delta_{\rm Mix}$ of the valence-sea pion mass, for a F$\bar{\rm v}$ meson with field $Q_{\rm Fv}$, whose mass is given by [27]

$$M^2_{\rm Fv}=\mu(m_{\rm F}+m_{\rm v})+a^2\Delta_{\rm Mix}, \tag{3}$$

where parameter $\Delta_{\rm Mix}$ can be measured via lattice QCD.

The connected propagators for valence-valence mesons with v, v′=x, y, z, x̃, ỹ, z̃ are given [27]

$$\langle\Phi_{\rm vv'}|\Phi_{\rm v'v}\rangle=\frac{\epsilon_{\rm v}}{k^2+M^2_{\rm v,v'}},\quad \epsilon_{\rm x,y,z}=1,\epsilon_{\rm \tilde{x},\tilde{y},\tilde{z}}=-1. \tag{4}$$

The flavor-neutral propagators appearing in the expression for the bubble contributions to scalar mesons are only those with two valence quarks [27],

$$\langle\Phi_{\rm vv}|\Phi_{\rm v'v'}\rangle_{\rm disc} = -\frac{1}{3}\frac{(k^2+M^2_{\rm U_I})(k^2+M^2_{\rm S_I})}{(k^2+M^2_{\rm v,v})(k^2+M^2_{\rm v',v'})(k^2+M^2_{\eta_{\rm I}})}, \tag{5}$$

where it is convenient to use $m_0^2\to\infty$ to decouple the $\eta'_{\rm I}$, and we are only interested in 2+1 case [14],

$$m^2_{\pi^0_{\rm I}}=m^2_{\rm U_I}=m^2_{\rm D_I},\quad m^2_{\eta_{\rm I}}=\frac{1}{3}m^2_{\rm U_I}+\frac{2}{3}m^2_{\rm S_I},$$

here $M^2_{\rm U_I}=M^2_{\rm U_5}+a^2\Delta_{\rm I}$, $M^2_{\rm S_I}=M^2_{\rm S_5}+a^2\Delta_{\rm I}$. It is interesting to note that the sea-sea pseudo-Goldstone bosons in the above expressions are taste singlets.

The propagators for valence-sea mesons with F=u, d, s and v=x, y, z are given by

$$\langle\Phi_{\rm vF}|\Phi_{\rm Fv}\rangle=\frac{1}{k^2+M^2_{\rm v,F}}. \tag{6}$$

It is important to note that propagators (4), (5) and (6) rest only on taste breaking parameters $a^2\Delta_{\rm I}$ and $a^2\Delta_{\rm Mix}$.

3 Scalar bubble term in MAχPT

The simulations with chiral valence quarks on top of MILC staggered sea quarks is feasible and charming. The relevant effective theory has been developed [27]. Following the original derivations and notations [15, 27, 29, 30], we here deduce κ bubble and σ bubble in MχPT with 2+1 chiral valence quarks and 2+1 MILC staggered sea quarks ($m_u=m_d\neq m_s$). Since the a_0 bubble is derived in Ref. [15], and its time Fourier transform is provided in Eq. (11) of Ref. [29], we will directly quote these results.

3.1 κ bubble

The bubble contribution to κ correlator is denoted in Ref. [21]. Applying the Wick contractions, we have

$$\begin{aligned}B^{\mathrm{M}\chi\mathrm{PT}}_{2+1,\kappa} = \mu^2\Big[&2\langle\Phi_{\mathrm{xx}}|\Phi_{\mathrm{zz}}\rangle\langle\Phi_{\mathrm{xz}}|\Phi_{\mathrm{zx}}\rangle\\ &+\sum_{\mathrm{v=x,y,z,\tilde{x},\tilde{y},\tilde{z}}}\langle\Phi_{\mathrm{xv}}|\Phi_{\mathrm{vx}}\rangle\langle\Phi_{\mathrm{vz}}|\Phi_{\mathrm{zv}}\rangle\\ &+\sum_{\mathrm{F=u,d,s}}\langle\Phi_{\mathrm{xF}}|\Phi_{\mathrm{Fx}}\rangle\langle\Phi_{\mathrm{Fz}}|\Phi_{\mathrm{zF}}\rangle\Big],\end{aligned}\tag{7}$$

where the third term is already considered to reduce four tastes per sea quark to one [15][1)]. The bubble contribution is secured by inserting relevant propagators into (7)

$$\begin{aligned}B^{\mathrm{M}\chi\mathrm{PT}}_{2+1,\kappa}(p) = \mu^2\sum_k\Big\{&-\frac{1}{(k+p)^2+M^2_{\mathrm{x,z}}}\\ &\times\Big[\frac{2}{3}\frac{1}{(k^2+M^2_{\mathrm{x,x}})(k^2+M^2_{\mathrm{z,z}})}\\ &\times\frac{(k^2+M^2_{\mathrm{U_I}})(k^2+M^2_{\mathrm{S_I}})}{k^2+M^2_{\eta_\mathrm{I}}}\\ &+\frac{1}{3}\frac{1}{(k^2+M^2_{\mathrm{x,x}})^2}\frac{(k^2+M^2_{\mathrm{U_I}})(k^2+M^2_{\mathrm{S_I}})}{k^2+M^2_{\eta_\mathrm{I}}}\\ &+\frac{1}{3}\frac{1}{(k^2+M^2_{\mathrm{z,z}})^2}\frac{(k^2+M^2_{\mathrm{U_I}})(k^2+M^2_{\mathrm{S_I}})}{k^2+M^2_{\eta_\mathrm{I}}}\Big]\\ &+2\frac{1}{(k+p)^2+M^2_{\mathrm{x,u}}}\frac{1}{k^2+M^2_{\mathrm{z,u}}}\\ &+\frac{1}{(k+p)^2+M^2_{\mathrm{x,s}}}\frac{1}{k^2+M^2_{\mathrm{z,s}}}\Big\}.\end{aligned}\tag{8}$$

It is helpful to perform a partial fraction decomposition, Eq. (8) can then be simplified to a form

$$\begin{aligned}B^{\mathrm{M}\chi\mathrm{PT}}_{2+1,\kappa}(p) = \mu^2\sum_k\Big\{&-\frac{1}{(k+p)^2+M^2_{\mathrm{x,z}}}\times\Big[\frac{g_1}{k^2+M^2_{\eta_\mathrm{I}}}\\ &+\frac{g_2}{k^2+M^2_{\mathrm{x,x}}}+\frac{g_3}{k^2+M^2_{\mathrm{z,z}}}+\frac{g_4}{(k^2+M^2_{\mathrm{x,x}})^2}\\ &+\frac{g_5}{(k^2+M^2_{\mathrm{z,z}})^2}\Big]+\frac{2}{(k+p)^2+M^2_{\mathrm{x,u}}}\frac{1}{k^2+M^2_{\mathrm{z,u}}}\\ &+\frac{1}{(k+p)^2+M^2_{\mathrm{x,s}}}\frac{1}{k^2+M^2_{\mathrm{z,s}}}\Big\},\end{aligned}\tag{9}$$

where

$$\begin{aligned}g_1 &= \frac{1}{3}\times\frac{(M^2_{\mathrm{U_I}}-M^2_{\eta_\mathrm{I}})(M^2_{\mathrm{S_I}}-M^2_{\eta_\mathrm{I}})}{(M^2_{\mathrm{x,x}}-M^2_{\eta_\mathrm{I}})(M^2_{\mathrm{z,z}}-M^2_{\eta_\mathrm{I}})}\\ &\quad\times\Big[2+\frac{M^2_{\mathrm{z,z}}-M^2_{\eta_\mathrm{I}}}{M^2_{\mathrm{x,x}}-M^2_{\eta_\mathrm{I}}}+\frac{M^2_{\mathrm{x,x}}-M^2_{\eta_\mathrm{I}}}{M^2_{\mathrm{z,z}}-M^2_{\eta_\mathrm{I}}}\Big],\\ g_2 &= \frac{2}{3}\times\frac{(M^2_{\mathrm{U_I}}-M^2_{\mathrm{x,x}})(M^2_{\mathrm{S_I}}-M^2_{\mathrm{x,x}})}{(M^2_{\eta_\mathrm{I}}-M^2_{\mathrm{x,x}})(M^2_{\mathrm{z,z}}-M^2_{\mathrm{x,x}})}\\ &\quad+\frac{3M^2_{\mathrm{x,x}}(M^2_{\mathrm{x,x}}-2M^2_{\eta_\mathrm{I}})+2M^4_{\mathrm{S_I}}+M^4_{\mathrm{U_I}}}{9(M^2_{\eta_\mathrm{I}}-M^2_{\mathrm{x,x}})^2},\\ g_3 &= \frac{2}{3}\times\frac{(M^2_{\mathrm{U_I}}-M^2_{\mathrm{z,z}})(M^2_{\mathrm{S_I}}-M^2_{\mathrm{z,z}})}{(M^2_{\eta_\mathrm{I}}-M^2_{\mathrm{z,z}})(M^2_{\mathrm{x,x}}-M^2_{\mathrm{z,z}})}\\ &\quad+\frac{3M^2_{\mathrm{z,z}}(M^2_{\mathrm{z,z}}-2M^2_{\eta_\mathrm{I}})+2M^4_{\mathrm{S_I}}+M^4_{\mathrm{U_I}}}{9(M^2_{\eta_\mathrm{I}}-M^2_{\mathrm{z,z}})^2},\\ g_4 &= \frac{(M^2_{\mathrm{U_I}}-M^2_{\mathrm{x,x}})(M^2_{\mathrm{S_I}}-M^2_{\mathrm{x,x}})}{3(M^2_{\eta_\mathrm{I}}-M^2_{\mathrm{x,x}})},\\ g_5 &= \frac{(M^2_{\mathrm{U_I}}-M^2_{\mathrm{z,z}})(M^2_{\mathrm{S_I}}-M^2_{\mathrm{z,z}})}{3(M^2_{\eta_\mathrm{I}}-M^2_{\mathrm{z,z}})}.\end{aligned}\tag{10}$$

The time Fourier transform of κ bubble (namely, $B^{\mathrm{M}\chi\mathrm{PT}}_{2+1,\kappa}(t)=\mathrm{F.T.}[B^{\mathrm{M}\chi\mathrm{PT}}_{2+1,\kappa}(p)]_{\boldsymbol{p}=\boldsymbol{0}}$) is then provided by

$$\begin{aligned}B^{\mathrm{M}\chi\mathrm{PT}}_{2+1,\kappa}(t) = \frac{\mu^2}{4L^3}\sum_{\boldsymbol{k}}\Big[&-g_1\frac{\mathrm{e}^{-(\omega_{\mathrm{xz}}+\omega_{\eta_\mathrm{I}})t}}{\omega_{\mathrm{xz}}\omega_{\eta_\mathrm{I}}}-g_2\frac{\mathrm{e}^{-(\omega_{\mathrm{xz}}+\omega_{\mathrm{xx}})t}}{\omega_{\mathrm{xx}}\omega_{\mathrm{xz}}}\\ &-g_3\frac{\mathrm{e}^{-(\omega_{\mathrm{xz}}+\omega_{\mathrm{zz}})t}}{\omega_{\mathrm{xz}}\omega_{\mathrm{zz}}}-g_4\frac{\mathrm{e}^{-(\omega_{\mathrm{xz}}+\omega_{\mathrm{xx}})t}}{2\omega_{\mathrm{xz}}\omega^3_{\mathrm{xx}}}(\omega_{\mathrm{xx}}t+1)\\ &-g_5\frac{\mathrm{e}^{-(\omega_{\mathrm{xz}}+\omega_{\mathrm{zz}})t}}{2\omega_{\mathrm{xz}}\omega^3_{\mathrm{zz}}}(\omega_{\mathrm{zz}}t+1)\\ &+\frac{\mathrm{e}^{-(\omega_{\mathrm{xs}}+\omega_{\mathrm{zs}})t}}{\omega_{\mathrm{xs}}\omega_{\mathrm{zs}}}+2\frac{\mathrm{e}^{-(\omega_{\mathrm{xu}}+\omega_{\mathrm{zu}})t}}{\omega_{\mathrm{xu}}\omega_{\mathrm{zu}}}\Big],\end{aligned}\tag{11}$$

where, for brevity, in this work we use the notation $\omega_\mathrm{i}\equiv\sqrt{\boldsymbol{k}^2+m^2_\mathrm{i}}$ from Ref. [29].

1) It is interesting and important to note that the corresponding bubble contribution to a_0 correlator is

$$B^{\mathrm{M}\chi\mathrm{PT}}_{2+1,a_0} = \mu^2\Big[\sum_{\mathrm{F=u,d,s}}\langle\Phi_{\mathrm{xF}}|\Phi_{\mathrm{Fx}}\rangle\langle\Phi_{\mathrm{Fy}}|\Phi_{\mathrm{yF}}\rangle+2\langle\Phi_{\mathrm{xx}}|\Phi_{\mathrm{yy}}\rangle\langle\Phi_{\mathrm{xy}}|\Phi_{\mathrm{yx}}\rangle+\sum_{\mathrm{v=x,y,z,\tilde{x},\tilde{y},\tilde{z}}}\langle\Phi_{\mathrm{xv}}|\Phi_{\mathrm{vx}}\rangle\langle\Phi_{\mathrm{vy}}|\Phi_{\mathrm{yv}}\rangle\Big],$$

which results in two extra terms to original Eq. (13) in Ref. [15], which are neatly canceled each other out in the final a_0 bubble. Consequently, it is nicely consistent with Sasa's result derived with 2 chiral valence quarks and 2+1 staggered sea quarks [15].

It is worth mentioning that no free parameters are presented in (11), which is solely predicted by MAχPT. The meson masses, coupling constant μ, mixed-meson splittings $a^2\Delta_{\rm mix}$ and taste-singlet breaking $a^2\Delta_{\rm I}$ are evaluated from lattice studies [14, 29]. Additionally, we notice that Eq. (11) gets unphysical contributions from KS intermediate states. Luckily, it never dominates the κ bubble at large t. Moreover, there are two double poles in its momentum-space propagator, which lead to the infrared-sensitive linear-in-t growth factors in the fourth and fifth terms of Eq. (11). We will observe that there is a desirable cancellation between two double poles.

Since full QCD is restored in MAχPT only at $a=0$, the KS contributions cannot be entirely removed for any selection of realistic simulation parameters. In the continuum limit ($a^2\Delta_I\to 0$, $a^2\Delta_{\rm Mix}\to 0$), the κ bubble reduces to a simple form

$$B_{\kappa}^{a=0}(t) = \frac{\mu^2}{4L^3}\sum_{\boldsymbol{k}}\left[\frac{3}{2}\frac{{\rm e}^{-(\omega_{{\rm U}_5}+\omega_{{\rm K}_5})t}}{\omega_{{\rm U}_5}\omega_{{\rm K}_5}}+\frac{1}{6}\frac{{\rm e}^{-(\omega_{{\rm K}_5}+\omega_{\eta_5})t}}{\omega_{{\rm K}_5}\omega_{\eta_5}}\right], \quad (12)$$

which is well consistent with that of SχPT [21, 22].

3.2 σ bubble

The bubble contribution to σ correlator is denoted in Ref. [17]. Applying the Wick contractions, we get [15]

$$\begin{aligned} B_{2+1,\sigma}^{\rm M\chi PT} = \mu^2\Big[&2\langle\Phi_{\rm xx}|\Phi_{\rm xx}\rangle\langle\Phi_{\rm xx}|\Phi_{\rm xx}\rangle \\ &+2\langle\Phi_{\rm xx}|\Phi_{\rm yy}\rangle\langle\Phi_{\rm xx}|\Phi_{\rm yy}\rangle \\ &+\sum_{\rm v=x,y,z,\tilde{x},\tilde{y},\tilde{z}}\langle\Phi_{xv}|\Phi_{\rm vx}\rangle\langle\Phi_{vx}|\Phi_{\rm xv}\rangle \\ &+\sum_{\rm F=u,d,s}\langle\Phi_{\rm xF}|\Phi_{\rm Fx}\rangle\langle\Phi_{\rm Fy}|\Phi_{\rm yF}\rangle\Big]. \end{aligned} \quad (13)$$

The bubble contribution is secured by plugging the relevant propagators into (13)

$$\begin{aligned} B_{2+1,\sigma}^{\rm M\chi PT}(p) = \mu^2\sum_{k}\Bigg\{&-\frac{4}{3}\frac{1}{(k+p)^2+M_{\rm x,x}^2} \\ &\times\frac{1}{(k^2+M_{\rm x,x}^2)^2}\frac{(k^2+M_{\rm U_I}^2)(k^2+M_{\rm S_I}^2)}{k^2+M_{\eta_{\rm I}}^2} \\ &+\frac{4}{9}\frac{1}{((k+p)^2+M_{\rm x,x}^2)^2} \\ &\times\frac{((k+p)^2+M_{\rm U_I}^2)((k+p)^2+M_{\rm S_I}^2)}{(k+p)^2+M_{\eta_{\rm I}}^2} \\ &\times\frac{1}{(k^2+M_{\rm x,x}^2)^2}\frac{(k^2+M_{\rm U_I}^2)(k^2+M_{\rm S_I}^2)}{k^2+M_{\eta_{\rm I}}^2} \\ &+2\frac{1}{(k+p)^2+M_{\rm x,x}^2}\frac{1}{(k^2+M_{\rm x,x}^2)^2} \\ &+2\frac{1}{(k+p)^2+M_{\rm x,u}^2}\frac{1}{k^2+M_{\rm x,u}^2} \\ &+\frac{1}{(k+p)^2+M_{\rm x,s}^2}\frac{1}{k^2+M_{\rm x,s}^2}\Bigg\}. \end{aligned} \quad (14)$$

It is convenient to use the partial fraction decomposition, expression (14) can then be simplified to a compact form

$$\begin{aligned} B_{2+1,\sigma}^{\rm M\chi PT}(p) = B^2\sum_{k}\Bigg\{&\frac{h_1}{(k+p)^2+M_{\rm x,x}^2}\frac{1}{k^2+M_{\rm x,x}^2} \\ &+\frac{h_2}{(k+p)^2+M_{\rm x,x}^2}\frac{1}{(k^2+M_{\rm x,x}^2)^2} \\ &+\frac{h_3}{(k+p)^2+M_{\rm x,x}^2}\frac{1}{k^2+M_{\eta_{\rm I}}^2} \\ &+\frac{h_4}{((k+p)^2+M_{\rm x,x}^2)^2}\frac{1}{(k^2+M_{\rm x,x}^2)^2} \\ &+\frac{h_5}{(k+p)^2+M_{\eta_{\rm I}}^2}\frac{1}{k^2+M_{\eta_{\rm I}}^2} \\ &+\frac{h_6}{((k+p)^2+M_{\rm x,x}^2)^2}\frac{1}{k^2+M_{\eta_{\rm I}}^2} \\ &+2\frac{1}{(k+p)^2+M_{\rm x,u}^2}\frac{1}{k^2+M_{\rm x,u}^2} \\ &+\frac{1}{(k+p)^2+M_{\rm x,s}^2}\frac{1}{k^2+M_{\rm x,s}^2}\Bigg\}, \end{aligned} \quad (15)$$

where

$$\begin{aligned} h_1 &= 2-\frac{4}{3}\frac{3M_{\rm x,x}^2(M_{\rm x,x}^2-2M_{\eta_{\rm I}}^2)+2M_{\rm S_I}^4+M_{\rm U_I}^4}{3(M_{\eta_{\rm I}}^2-M_{\rm x,x}^2)^2} \\ &\quad+\frac{4}{9}\left(\frac{3M_{\rm x,x}^2(M_{\rm x,x}^2-2M_{\eta_{\rm I}}^2)+2M_{\rm S_I}^4+M_{\rm U_I}^4}{3(M_{\eta_{\rm I}}^2-M_{\rm x,x}^2)^2}\right)^2, \\ h_2 &= \frac{(M_{\rm U_I}^2-M_{\rm x,x}^2)(M_{\rm S_I}^2-M_{\rm x,x}^2)}{M_{\eta_{\rm I}}^2-M_{\rm x,x}^2} \end{aligned}$$

$$\times\left(\frac{8}{9}\frac{3M_{\rm x,x}^2(M_{\rm x,x}^2-2M_{\eta_{\rm I}}^2)+2M_{\rm S_I}^4+M_{\rm U_I}^4}{3(M_{\eta_{\rm I}}^2-M_{\rm x,x}^2)^2}-\frac{4}{3}\right),$$

$$h_3 = \frac{(M_{\rm U_I}^2-M_{\eta_{\rm I}}^2)(M_{\rm S_I}^2-M_{\eta_{\rm I}}^2)}{(M_{\rm x,x}^2-M_{\eta_{\rm I}}^2)^2}$$

$$\times\left(\frac{8}{9}\frac{3M_{\rm x,x}^2(M_{\rm x,x}^2-2M_{\eta_{\rm I}}^2)+2M_{\rm S_I}^4+M_{\rm U_I}^4}{3(M_{\eta_{\rm I}}^2-M_{\rm x,x}^2)^2}-\frac{4}{3}\right),$$

$$h_4 = \frac{4}{9}\left(\frac{(M_{\rm U_I}^2-M_{\rm x,x}^2)(M_{\rm S_I}^2-M_{\rm x,x}^2)}{M_{\eta_{\rm I}}^2-M_{\rm x,x}^2}\right)^2,$$

$$h_5 = \frac{4}{9}\left(\frac{(M_{\rm U_I}^2-M_{\eta_{\rm I}}^2)(M_{\rm S_I}^2-M_{\eta_{\rm I}}^2)}{(M_{\rm x,x}^2-M_{\eta_{\rm I}}^2)^2}\right)^2,$$

$$h_6 = \frac{8}{9}\frac{(M_{\rm U_I}^2-M_{\rm x,x}^2)(M_{\rm S_I}^2-M_{\rm x,x}^2)}{M_{\eta_{\rm I}}^2-M_{\rm x,x}^2}$$

$$\times\frac{(M_{\rm U_I}^2-M_{\eta_{\rm I}}^2)(M_{\rm S_I}^2-M_{\eta_{\rm I}}^2)}{(M_{\rm x,x}^2-M_{\eta_{\rm I}}^2)^2}. \qquad (16)$$

The time Fourier transform of σ bubble is

$$B_{2+1,\sigma}^{\rm M\chi PT}(t) = \frac{\mu^2}{4L^3}\sum_{\boldsymbol{k}}\left[h_1\frac{{\rm e}^{-2\omega_{\rm xx}t}}{\omega_{\rm xx}^2}+h_2\frac{{\rm e}^{-2\omega_{\rm xx}t}}{2\omega_{\rm xx}^4}(\omega_{\rm xx}t+1)\right.$$

$$+h_3\frac{{\rm e}^{-(\omega_{\rm xx}+\omega_{\eta_{\rm I}})t}}{\omega_{\rm xx}\omega_{\eta_{\rm I}}}+h_4\frac{{\rm e}^{-2\omega_{\rm xx}t}}{4\omega_{\rm xx}^6}(\omega_{\rm xx}t+1)^2$$

$$+h_5\frac{{\rm e}^{-2\omega_{\eta_{\rm I}}t}}{\omega_{\eta_{\rm I}}^2}+h_6\frac{{\rm e}^{-(\omega_{\rm xx}+\omega_{\eta_{\rm I}})t}}{2\omega_{\rm xx}^3\omega_{\eta_{\rm I}}}(\omega_{\rm xx}t+1)$$

$$\left.+2\frac{{\rm e}^{-2\omega_{\rm xu}t}}{\omega_{\rm xu}^2}+\frac{{\rm e}^{-2\omega_{\rm xs}t}}{\omega_{\rm xs}^2}\right]. \qquad (17)$$

Once again, we note that expression (17) is solely predicted by MAχPT, and it also gets unphysical contributions from πη intermediate states, which luckily never dominate the σ correlator at large t. Moreover, there are two kind of double poles in its momentum-space propagator: the infrared-sensitive linear-in-t growth factors in the second and sixth terms of Eq. (17) arising from the double poles, and the strong infrared-sensitive quadratic-in-t^2 growth factor in the fourth term of Eq. (17), which stems from the multiplication of two double poles.

The unphysical πη intermediate states contribute to σ bubble since MAχPT is not unitary at $a\neq 0$. In the continuum limit ($a^2\Delta_I\to 0$, $a^2\Delta_{\rm Mix}\to 0$), the expression (17) reduces to a pretty simple form

$$B_{\sigma}^{a=0}(t) = \frac{\mu^2}{4L^3}\sum_{\boldsymbol{k}}\left[3\frac{{\rm e}^{-2\omega_{\rm U_5}t}}{\omega_{\rm U_5}^2}+\frac{{\rm e}^{-2\omega_{\rm K_5}t}}{\omega_{\rm K_5}^2}+\frac{1}{9}\frac{{\rm e}^{-2\omega_{\rm eta_5}t}}{\omega_{\eta_5}^2}\right], \qquad (18)$$

which is nicely consistent with that of SχPT [17, 19, 20].

4 Numerical illustration of bubbles

We plan on launching a series of lattice investigations of scalar mesons using DW fermions and the MILC 2+1 asqtad-improved staggered sea quarks. So far, the MILC lattice ensembles with two lattice spacings (the coarse and fine lattices), which are extensively studied. So, it is useful to exploit the MILC determined parameters [14] to acquire the preliminary numerical predictions for the bubble contributions to scalar mesons in MAχPT, which will then guide us in the ongoing lattice studies.

We illustrate these predictions only on two MILC lattice ensembles: one is a coarse ensemble ($a\approx 0.12$ fm, $am_{\rm u}/am_{\rm s}=0.005/0.05$), another one is a fine ensemble ($a\approx 0.09$ fm, $am_{\rm u}/am_{\rm s}=0.0062/0.031$), which are labeled "coarse" and "fine" lattice ensemble, respectively. We here just exhibit two of the most popular mass matchings of chiral valence quarks and staggered sea quarks. To help one quantitatively comprehend each term in the bubble contributions to scalar mesons, each is displayed in the corresponding figures, which indicate the whole bubble contribution with a black solid line.

4.1 Matching 1

The first selection is to fix the valence pion mass and kaon mass to be equal to the taste-pseudoscalar sea pion mass and kaon mass; to be specific, $M_{\rm x,x}=M_{\rm U_5}$, which is practiced in Ref. [34], and $M_{\rm x,z}=M_{\rm K_5}$. This tuning is attractive since the taste-pseudoscalar pion mass disappears in the chiral limit, even at $a\neq 0$.

Figure 1 shows κ bubble on the MILC coarse and fine ensembles (the top panel shows the result for coarse lattice, while bottom panel is that of fine lattice). In these figures, "πK" indicates the intermediate states with the valence pion and kaon, and likewise for "Kη" and "KS", while "πK Mixed" represents the intermediate states with mixed valence-sea pion and kaon, and likewise for "KS Mixed". "πK double pole" and "KS double pole" are the fourth and fifth terms in (11). The analogous notations are used to σ and a_0 bubbles.

From Fig. 1, we note that physical ηK states dominate the κ bubble until $t\approx 11$ for coarse lattice and $t\approx 27$ for fine lattice, and it quickly decreases afterwards. On the other hand, the "πK double pole" is pretty small at small t, but for enough large t it gradually dominates the κ bubble, whereas the "KS double pole" is negligible. It is important to note that there is a cancellation between the two double pole terms, which is a special feature of the κ bubble. This cancellation is good news for studying the κ meson since it decreases the unitarity-violation of κ bubble. Moreover, the πK state plays an important part in the κ bubble. It is important to note that the unphysical KS state never dominates the κ bubble.

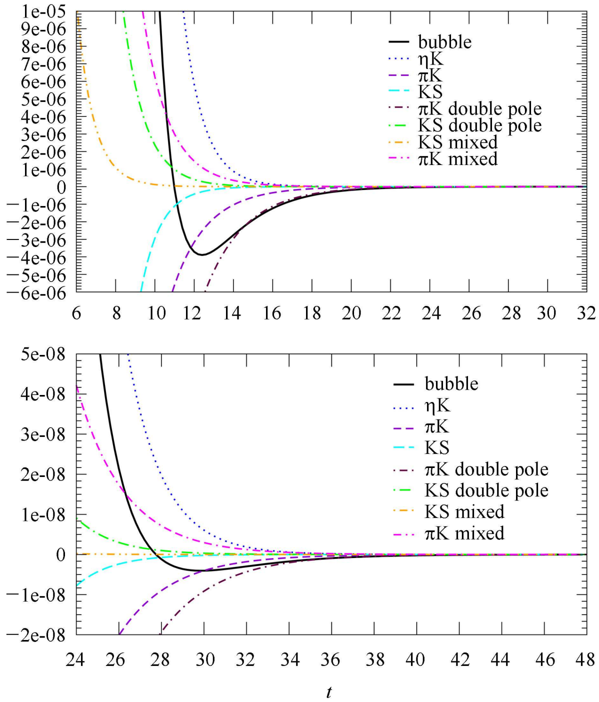

Fig. 1. κ bubble for a simulation with chiral fermions on a MILC coarse staggered configuration. The valence and sea quark masses are tuned by matching $M_{\mathrm{x,x}}=M_{\mathrm{U}_5}$ and $M_{\mathrm{x,z}}=M_{\mathrm{K}_5}$. The parameters $a^2\Delta_{\mathrm{I}}$ and $a^2\Delta_{\mathrm{Mix}}$ are taken from Refs. [14], [29], respectively.

Figure 2 shows a_0 bubble on the MILC coarse and fine ensembles (top panel for coarse lattice, bottom panel for fine lattice). We note that physical πη states dominate the a_0 bubble until $t\approx4$ for coarse lattice and $t\approx12$ for fine lattice, and it quickly decreases afterwards. On the other hand, the third term in Eq. (11) of Ref. [29] ("ππ double pole") is pretty small at small t, but for a large enough t it eventually dominates the a_0 bubble. We note that the physical KK state never dominates the a_0 bubble, while the unphysical ππ state plays a very important role in the a_0 bubble. This indicates that the reliable determination of a_0 meson mass is feasible only for appropriate quark masses and times [15].

Figure 3 shows the σ bubble on the MILC coarse and fine ensembles (the top panel for coarse lattice, and bottom panel for fine lattice). We note that physical ππ states (the second term in (17)) dominate the σ bubble until $t\approx3$ for coarse lattice and $t\approx8$ for fine lattice. On

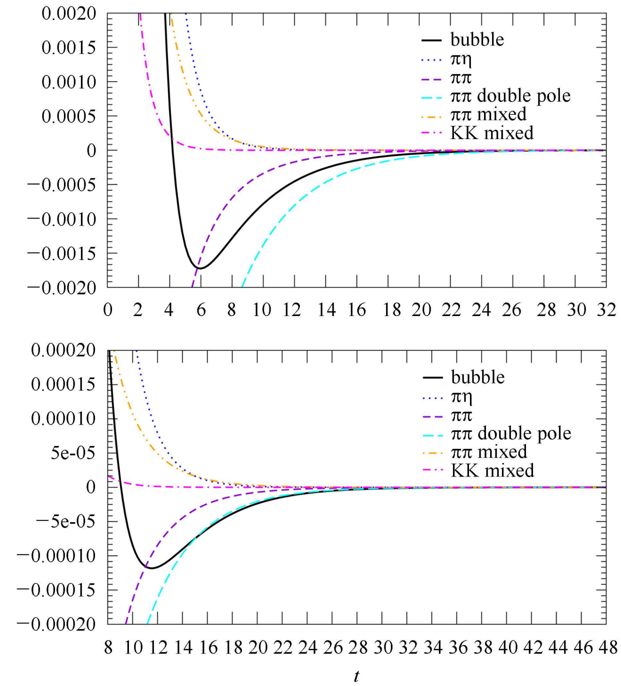

Fig. 2. a_0 bubble for a simulation with chiral fermions on a MILC coarse lattice. The valence and sea quark masses are tuned as $M_{\mathrm{x,x}}=M_{\mathrm{U}_5}$.

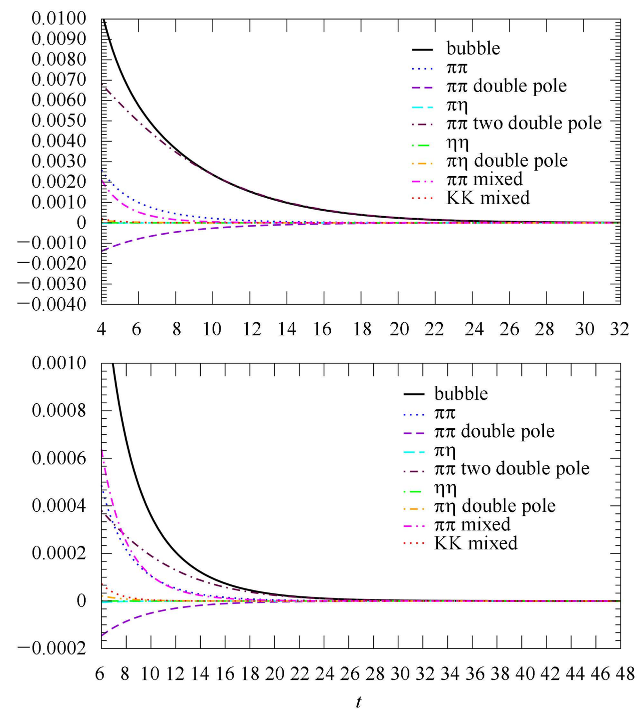

Fig. 3. σ bubble for a simulation with chiral fermions on a MILC coarse lattice. The valence and sea quark masses are tuned as $M_{\mathrm{x,x}}=M_{\mathrm{U}_5}$.

the other hand, the fourth term in (17) ("ππ two double pole") is pretty small at small t, but for a large enough t it eventually dominates the σ bubble[1)]. While the sixth term in (17) ("πη double pole") is negligible, it is obvious to note that there exists a cancellation among three term with double poles. Moreover, it is important to note that the unphysical πη state never dominates the σ bubble, and the σ bubble is positive for all of the times.

It is worth mentioning that all of the bubble contributions to scalar correlators are dominated by the double poles at large t for this tuning. Actually, the double pole is an unphysical effect that stems from selecting the valence-quark action, which is different from the sea-quark action [35]. We found that these unitarity violations are not likely to be fairly small for this tuning on MILC coarse lattice, while the degree of unitarity violation decreases with the finer lattice spacing, as expected.

4.2 Matching 2

The second choice is to fix the valence pion mass and valence kaon mass to be equal to the taste-singlet sea pion mass $\mathrm{U_I}$ and taste-singlet sea kaon mass, respectively; that is, $M_{\mathrm{x,x}}=M_{\mathrm{U_I}}$ and $M_{\mathrm{x,z}}=M_{\mathrm{K_I}}$. This tuning completely removes the double pole terms in both κ bubble, σ bubble, and $\mathrm{a_0}$ bubble. Moreover, these bubble contributions have turned out to be relatively small and usually positive. Nonetheless, it still does not entirely remove unphysical states. Additionally, in practice this tuning may not be recommendable since it would lead to a fairly heavy valence pion and kaon on the MILC coarse lattices.

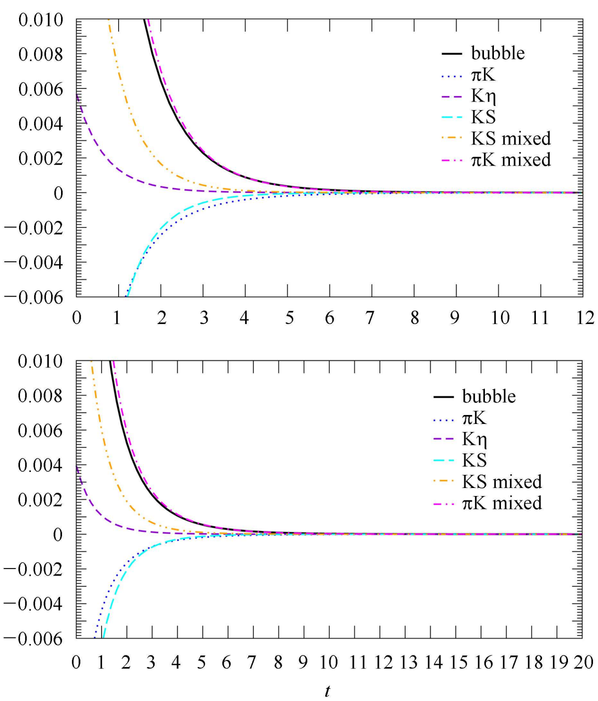

Fig. 4. κ bubble for a simulation with chiral fermions on MILC coarse lattice. The mass tunings are $M_{\mathrm{x,x}}=M_{\mathrm{U_I}}$ and $M_{\mathrm{x,z}}=M_{\mathrm{K_I}}$.

Figure 4 shows the κ bubble on a MILC coarse and fine ensemble. We note that physical πK states dominate the κ bubble at all of the times. Moreover, the unphysical KS state is small, while the physical Kη state makes a small contribution.

Figure 5 displays the σ bubble on a MILC coarse ensemble. We note that physical ππ states (the first term in (17)) dominate the σ bubble at all of the times. Moreover, physical ηη and KK states play a small role in the σ bubble.

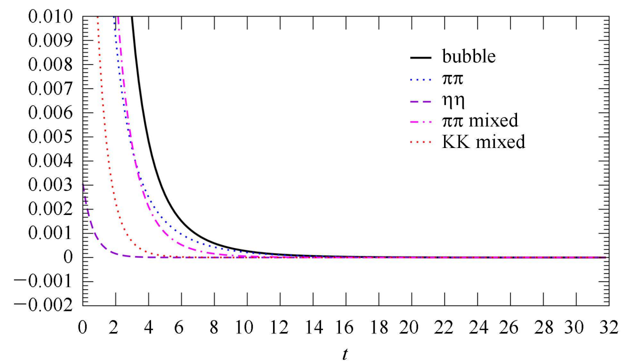

Fig. 5. σ bubble for the simulation with chiral fermions on 2+1 MILC fine lattice. The valence and sea quark masses are tuned as $M_{\mathrm{x,x}}=M_{\mathrm{U_I}}$.

5 Summary

In this work we extended Sasa's derivation on bubble contribution to $\mathrm{a_0}$ correlator in MAχPT [15] to those of κ and σ correlators. We found that these extensions are useful since κ and σ bubbles demonstrate many new features as compared with $\mathrm{a_0}$ bubble. For example, the κ and σ bubbles are dominated by the physical two-particle states at a large enough t, while the $\mathrm{a_0}$ bubble is dominated by unphysical two pions states. Moreover, we notice a strong infrared-sensitive quadratic-in-t^2 growth factor in the σ bubble due to the multiplication of the two double poles. In practice, special attention should be paid to monitor the size of this unitarity violation, otherwise it will lead to an "infrared-disaster" for the σ bubble for a given tuning.

MAχPT predicts the observed unitarity-violations pretty well. This is good news for one who employs the mixed actions to study scalar meson. Moreover, different

1) It overwhelmingly dominates the σ bubble as early as $t\approx6$ for coarse lattice, and dominates the σ bubble about $t\approx16$ for fine lattice. This means that if we do not choose the suitable simulation parameters, this term will be an "infrared-disaster" to the σ bubble.

mass tunings have remarkable effects on scalar correlators, our extended analytical expressions are helpful to aid researchers in selecting the appropriate simulation parameters for determining scalar meson masses.

Lattice studies of scalar mesons using DW valence quarks and staggered sea quarks contain the excellent traits of both fermion discretizations. The corresponding numerical results will be used as a cross-check with our lattice studies on scalar mesons with other methods [36]. Therefore, it will be interesting to measure the relevant lattice data to verify the MAχPT formulae for κ and σ bubbles. We will appeal for computational resources to pursue this challenging enterprise.

We appreciate K. F. Liu for helping us with some knowledge about the scalar meson and for bringing our attention to the domain-wall fermion. We benefit from the happy day spent studying scalar mesons with Carleton DeTar, Claude Bernard and Sasa Prelovsek during my Ph.D.

References

1 Beringer J et al. (Particle Data Group collaboration). Phys. Rev. D, 2012, **86**: 010001
2 Mathur N, Alexandru A, CHEN Y et al. Phys. Rev. D, 2007, **76**: 114505
3 Prelovsek S, Dawson C, Izubuchi T et al. Phys. Rev. D, 2004, **70**: 094503
4 McNeile C, Michael C. Phys. Rev. D, 2006, **74**: 014508
5 LIU K F. Prog. Theor. Phys. Suppl., 2007, **168**: 160
6 FU Zi-Wen, DeTar C. Chin. Phys. C (HEP & NP), 2011, **35**: 1079
7 Prelovsek S, Draper T, LANG C B et al. Phys. Rev. D, 2010, **82**: 094507
8 Alford M, Jaffe R. Nucl. Phys. B, 2000, **578**: 367
9 Loan M, LUO Z, Lam Y. Eur. Phys. J. C, 2008, **57**: 579
10 Alexandrou C, Daldrop J Q. Dalla Brida M et al. JHEP, 2013, **1304**: 137
11 Prelovsek S, Mohler D. Phys. Rev. D, 2009, **79**: 014503
12 Gribov V. Eur. Phys. J. C, 1999, **10**: 91
13 Gregory E, Irving A, McNeile C et al. PoS LAT2005, 2006, 027
14 Bernard C, Burch T, Orginos K et al. Phys. Rev. D, 2001, **64**: 054506; Aubin C, Bernard C, DeTar C et al. Phys. Rev. D, 2004, **70**: 094505
15 Prelovsek S. PoS LAT2005, 2006, 085; Prelovsek S. Phys. Rev. D, 2006, **73**: 014506
16 Bazavov A, Toussaint D, Bernard C et al. Rev. Mod. Phys., 2010, **82**: 1349; Bernard C, DeTar C, Di Pierro M et al. Phys. Rev. D, 2011, **83**: 034503
17 Bernard C, DeTar C, FU Z et al. Phys. Rev. D, 2007, **76**: 094504
18 FU Zi-Wen. Chin. Phys. Lett., 2011, **28**: 081202
19 FU Zi-wen, DeTar C. Chin. Phys. C (HEP & NP), 2011, **35**: 896
20 FU Z. 2006, UMI-32-34073
21 FU Zi-Wen. Chin. Phys. C (HEP & NP), 2012, **36**: 489
22 FU Z. Int. J. Mod. Phys. A, 2013, **28**: 1350059
23 Susskind L. Phys. Rev. D, 1977, **16**: 3031
24 Lee W, Sharpe S. Phys. Rev. D, 1999, **60**: 114503; Aubin C, Bernard C. Phys. Rev. D, 2003, **68**: 034014; Aubin C, Bernard C. Phys. Rev. D, 2003, **68**: 074011; Sharpe S, Van de Water R. Phys. Rev. D, 2005, **71**: 114505
25 Kaplan D. Phys. Lett. B, 1992, **288**: 342
26 Shamir Y. Nucl. Phys. B, 1993, **406**: 90
27 Bar O, Bernard C, Rupak G et al. Phys. Rev. D, 2005, **72**: 054502
28 Antonio D, Bowler K, Boyle P et al. Phys. Rev. D, 2008, **77**: 014509
29 Aubin C, Laiho J, Van de Water R. Phys. Rev. D, 2008, **77**: 114501
30 CHEN J, O'Connell D, Van de Water R et al. Phys. Rev. D, 2006, **73**: 074510
31 Tiburzi B. Phys. Rev. D, 2005, **72**: 094501
32 Aubin C, Laiho J, Van de Water R. Phys. Rev. D, 2007, **75**: 034502
33 CHEN J, O'Connell D, Walker-Loud A. Phys. Rev. D, 2007, **75**: 054501
34 Renner D, Schroersa W, Edwards R R et al. Nucl. Phys. Proc. Suppl., 2005, **140**: 255; Edwards R, Fleming G, Hägler P et al. PoS LAT2005, 2006, 056
35 Golterman M, Izubuchi T, Shamir Y. Phys. Rev. D, 2005, **71**: 114508
36 FU Z. JHEP, 2012, **1207**: 142; FU Z. JHEP, 2012, **1201**: 017; FU Z. Phys. Rev. D, 2012, **85**: 014506; FU Z. Phys. Rev. D, 2012, **85**: 074501; FU Z. Phys. Rev. D, 2013, **87**: 074501; FU Z, FU K. Phys. Rev. D, 2012, **86**: 094507; FU Zi-Wen. Commun. Theor. Phys., 2012, **57**: 78; FU Z. Eur. Phys. J. C, 2012, **72**: 2159