

Bulk viscosity of hot dense Quark matter in the PNJL model*

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Abstract: Starting from the Kubo formula and the QCD low energy theorem, we study the the bulk viscosity of hot dense quark matter in the PNJL model from the equation of state. We show that the bulk viscosity has a sharp peak near the chiral phase transition, and that the ratio of bulk viscosity over entropy rises dramatically in the vicinity of the phase transition. These results agree with those from the lattice and other model calculations. In addition, we show that the increase of chemical potential raises the bulk viscosity.

Key words: bulk viscosity, Kubo formula, PNJL model

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1 Introduction

The study of the transport properties of strong interacting matter has attracted considerable interest. It is particularly important for hydrodynamic simulations of heavy-ion collisions and for understanding the properties of compact stars [1–5]. Shear viscosity η characterizes how fast a system goes back to equilibrium under a shear mode perturbation. It is believed that the quark gluon plasma (QGP) found in the relativistic heavy-ion collider (RHIC) is strongly coupled, which is in contrast to the weak coupling picture that was expected earlier: this is the so-called sQGP. Lattice Monte Carlo simulation on sQGP has demonstrated that, although the ratio of the shear viscosity to the entropy density is rather small, it is still probably larger than the universal lower bound $1/4\pi$ that is obtained from AdS/CFT duality [6]. The experimental extracted value with viscous hydrodynamics combined with a microscopic transport model lies within in the range 1–2.5 times of the lower bound [5].

Bulk viscosity describes how fast a system goes back to equilibrium under a uniform expansion, which relates to the deviation from the conformal invariance of the system. It vanishes when the system has a conformal equation of state; therefore, the sharp peak of the bulk viscosity would strongly affect the physics of the QCD matter near critical temperatures, which is very important for the study of the QCD phase structure. Bulk viscosity also affects the elliptic flow near the QCD phase transition in relativistic heavy ion collisions [7, 8]. The study of bulk viscosity is also important for the physics of compact stars [1–4].

Recently, the lattice QCD calculation has shown that

the trace of energy-momentum tensor anomaly and the ratio of the bulk viscosity ζ over entropy density s either have a sharp peak or diverge near phase transition [9–12]. This sharp peak behavior of ζ has also been observed in many model calculations [13–16].

At present, most calculations are for zero baryon density [11, 17], except for a few papers that have tried to estimate the bulk viscosity with finite density [18, 19]. For example, in Ref. [19] the authors studied viscosity at finite μ with the Nambu-Jona-Lasinio (NJL) model. In Ref. [20] the authors studied the viscosity of strange quark matter at finite μ with a quasi particle model. Meanwhile, the bulk viscosity was studied in [18] with Dyson-Schwinger equations at finite μ but zero temperature. In this paper we promote the calculation of bulk viscosity to both finite temperature and finite baryon density in the PNJL model, incorporating both confinement and chiral symmetry.

2 Kubo formula in the QCD low energy theorem

The bulk viscosity of hot dense quark matter is related to the retarded Green's function of the trace of the energy-momentum tensor, which is calculated by the Kubo formula. By using low energy theorems at finite temperature and chemical potential, we can extract the bulk viscosity of hot dense quark matter from the small frequency ansatz.

From the Kubo formula, we can express the bulk viscosity at Lehmann representation [21]

$$\zeta = \frac{1}{9} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int_0^{\infty} dt \int d^3 \vec{r} \exp(i\omega t) \langle [\theta_{\mu}^{\mu}(x), \theta_{\mu}^{\mu}(0)] \rangle. \quad (1)$$

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where ω is the frequency, and θ_μ^μ is the trace of the energy-momentum tensor. Using Fourier transform and P-invariance, the formula is changed as

$$\begin{aligned}\zeta &= \frac{1}{9} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int_0^\infty dt \int d^3 \vec{r} \exp(i\omega t) iG^R(x) \\ &= \frac{1}{9} \lim_{\omega \rightarrow 0} \frac{1}{\omega} iG^R(\omega, \vec{0}) \\ &= -\frac{1}{9} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im}G^R(\omega, \vec{0}).\end{aligned}\quad (2)$$

In Lehmann representation, the Green function is related to spectral density $\rho(\omega, \vec{p}) = -\frac{1}{\pi} \text{Im}G^R(\omega, \vec{p})$. For Kramers-Kroning relation, we can obtain

$$\begin{aligned}G^R(\omega, \vec{p}) &= \frac{1}{\pi} \int_{-\infty}^\infty \frac{\text{Im}G^R(u, \vec{p})}{u - \omega - i\varepsilon} du \\ &= \int_{-\infty}^\infty \frac{\rho(u, \vec{p})}{\omega - u + i\varepsilon} du.\end{aligned}\quad (3)$$

The Euclidean Green's function is

$$G^E(\omega, \vec{p}) = -G^R(i\omega, \vec{p}), \quad \omega > 0.$$

Using formula (3), we have

$$G^E(0, \vec{0}) = 2 \int_0^\infty \frac{\rho(u, \vec{0})}{u} du.\quad (4)$$

For QCD, the trace of energy-momentum stress tensor reads

$$\theta_\mu^\mu = m_q \bar{q}q + \frac{\beta(g)}{2g} F_{\mu\nu}^a F^{a\mu\nu} \equiv \theta_F + \theta_G,\quad (5)$$

where g is the strong coupling constant, θ_F and θ_G are the contribution of quark fields and of the gluon field, respectively, and $\beta(g)$ is the QCD β -function that determines the running behavior of g . In Eq. (5) q are quark fields with two flavors (in this letter we will limit ourselves in two flavor case and set the current quark mass $m_u = m_d = m$).

From the QCD low-energy theorems at finite temperature T and μ [22], one can find

$$\left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - d\right) \langle \hat{\mathcal{O}} \rangle_T = \int d^4 x \langle T_t \{ \theta_G(x), \hat{\mathcal{O}}(0) \} \rangle,\quad (6)$$

where d is the canonical dimension of the operator $\hat{\mathcal{O}}$. Using the above equation, one has

$$\left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 4\right) \langle \theta_G \rangle_T = \int d^4 x \langle T_t \{ \theta_G(x), \theta_G(0) \} \rangle,\quad (7)$$

$$\left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 3\right) \langle \theta_F \rangle_T = \int d^4 x \langle T_t \{ \theta_G(x), \theta_F(0) \} \rangle.\quad (8)$$

From the above two relations one obtains

$$\begin{aligned}9\zeta\omega_0 &= \int d^4 x \langle T_t \{ \theta_\mu^\mu(x), \theta_\mu^\mu(0) \} \rangle \\ &= \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 4\right) \langle \theta_G \rangle_T + 2 \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 3\right) \langle \theta_F \rangle_T \\ &\quad + \int d^4 x \langle T_t \{ \theta_F(x), \theta_F(0) \} \rangle \\ &\approx \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 4\right) \langle \theta_\mu^\mu \rangle_T + \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 2\right) \langle \theta_F \rangle_T \\ &= f_1(T, \mu) (\varepsilon - 3P) + f_2(T, \mu) \langle \theta_F \rangle_T,\end{aligned}\quad (9)$$

where

$$\begin{aligned}f_1(T, \mu) &= \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 4\right) \\ f_2(T, \mu) &= \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 2\right),\end{aligned}\quad (10)$$

and ε is the energy density and P is the pressure density of QCD. Here, because the current quark mass m of u and d quark is very small, in deriving Eq. (9) we have neglected the term proportional to m^2 .

The low energy theorems adapt to long distance, low frequency and strong coupling QCD [23, 24]. Using non-perturbation theory, the Euclidean Green's function can be represented as

$$\begin{aligned}G^E(0, \vec{0}) &= \int d^4 x \langle T\theta(x), \theta(0) \rangle \\ &= f_1(T, \mu) \langle \theta \rangle_T + f_2(T, \mu) \langle \theta_F \rangle_T,\end{aligned}\quad (11)$$

where $\langle \theta \rangle_T$ is the trace of the energy-momentum tensor. Its average value in zero temperature is $\langle \theta \rangle_0 = -4|\varepsilon_v|$, ε_v is the vacuum energy density, including the quark condensates and the gluon condensates in our work. In the low energy theorems, the difference of energy density and the pressure corresponds to the non-zero vacuum expectation value of the energy-momentum tensor $\varepsilon - 3P = \langle \theta \rangle_T - \langle \theta \rangle_0$. Analogously, $\langle \theta_F \rangle_T = \langle m\bar{q}q \rangle_T + \langle m\bar{q}q \rangle_0$. Using the PCAC relations, we can express the vacuum expectation value $\langle m\bar{q}q \rangle_0$ through the Pion and Kaon masses and decay constants $\langle m\bar{q}q \rangle_0 = -M_\pi^2 f_\pi^2 - M_K^2 f_K^2$. Using these relations, by combining formula (4) and (5) we obtain [11]:

$$\begin{aligned}&2 \int_0^\infty \frac{\rho(u, \vec{0})}{u} du \\ &= \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 4\right) \langle \theta \rangle_T \\ &\quad + \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 2\right) \langle \theta_F \rangle_T \\ &= \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 4\right) (\varepsilon - 3P - 4|\varepsilon_v| + \langle m\bar{q}q \rangle_0) \\ &\quad + \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 2\right) (\langle m\bar{q}q \rangle_T + \langle m\bar{q}q \rangle_0).\end{aligned}\quad (12)$$

This formula does not include the perturbative contribution as long as we consider the strong coupling situation. So we can use the following ansatz in the small frequency region [11]

$$\frac{\rho(\omega, \vec{0})}{\omega} = \frac{9\zeta\omega^2}{\pi(\omega_0^2 + \omega^2)}.$$

Where ζ is the bulk viscosity and ω_0 is a scale at which the perturbation theory becomes valid, $\omega_0 \sim T$. Using this ansatz and the formula (6), we extract the bulk viscosity:

$$\begin{aligned} \zeta = & \frac{1}{9\omega_0} \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 4 \right) (\varepsilon - 3P - 4|\varepsilon_v| + \langle m\bar{q}q \rangle_0) \\ & + \frac{1}{9\omega_0} \left(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 2 \right) (\langle m\bar{q}q \rangle_T + \langle m\bar{q}q \rangle_0). \end{aligned} \quad (13)$$

3 Parameters of PNJL model

The NJL model is based on an effective Lagrangian of relativistic fermions, which interact through local current-current couplings. It can be used to illustrate both the transmutation of originally light quarks into massive quasi-particles, as well as the spontaneously broken chiral symmetry. However, the quark confinement is missing in the NJL model. The de-confinement phase transition is characterized by spontaneous breaking of the Z(3) center symmetry of QCD. The corresponding order parameter is the Polyakov loop (p-loop). So, the PNJL model introduces both the chiral condensate $\langle \bar{\Psi}\Psi \rangle$ and the p-loop Φ coupling to the quarks to solve the problem of the NJL model [25, 26]. The PNJL model is an effective method to deal with the non-perturbative QCD. Consequently, the bulk viscosity extracted from the formula in the low energy theorems can be calculated in this model. The Lagrangian of two-flavor PNJL model at finite chemical potential is given by [26]

$$\begin{aligned} \mathcal{L}_{\text{PNJL}} = & \bar{q}(i\gamma^\mu D_\mu - \hat{m})q + g \left[(\bar{q}q)^2 + (\bar{q}\gamma_5 \vec{\tau} q)^2 \right] \\ & - \mathcal{U}(\Phi(A), \bar{\Phi}(A), T). \end{aligned} \quad (14)$$

Where $D^\mu = \partial^\mu - iA^\mu$, $A^\mu = \delta_{\mu 0} A^0$. The effective potential \mathcal{U} is expressed in terms of the traced p-loop $\Phi = \frac{\text{Tr}_c L}{N_C}$ and its conjugate $\bar{\Phi} = \frac{\text{Tr}_c L^\dagger}{N_C}$, where $L = \exp\left(\frac{iA_4}{T}\right)$, A_4 is the gauge field.

$$\begin{aligned} \frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} = & -\frac{b_2(T)}{2} \bar{\Phi}\Phi - \frac{b_3}{6} (\bar{\Phi}^3 + \Phi^3) + \frac{b_4}{4} (\bar{\Phi}\Phi)^2; \\ b_2(T) = & a_0 + a_1 \frac{T_0}{T} + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3. \end{aligned}$$

The parameters in the effective potential are chosen in Table 1 [25]:

Table 1. The parameters in the effective potential.

a_0	a_1	a_2	a_3	b_3	b_4
6.75	-1.95	2.625	-7.44	0.75	7.5

With the definition of the chiral condensate $\sigma = \langle \bar{q}q \rangle$ and the constituent quark mass $M = m - 2g\sigma$, the grand potential density is given by

$$\begin{aligned} \Omega(\Phi, \bar{\Phi}, M, T, \mu) = & \mathcal{U}(\Phi, \bar{\Phi}, T) + g \langle \bar{q}q \rangle^2 \\ & - 2N_C N_f \int \frac{d^3 p}{(2\pi)^3} E_p \\ & + 2N_f T \int \frac{d^3 p}{(2\pi)^3} [\ln N_\Phi^+(E_p) \\ & + \ln N_\Phi^-(E_p)]. \end{aligned} \quad (15)$$

Where

$$\begin{aligned} \frac{1}{N_\Phi^+(E_p)} = & 1 + 3(\Phi + \bar{\Phi} \exp(-\beta E_p^+)) \exp(-\beta E_p^+) \\ & + \exp(-3\beta E_p^+), \\ \frac{1}{N_\Phi^-(E_p)} = & 1 + 3(\bar{\Phi} + \Phi \exp(-\beta E_p^-)) \exp(-\beta E_p^-) \\ & + \exp(-3\beta E_p^-), \end{aligned}$$

$E_p = \sqrt{p^2 + M^2}$ is the quasi-particle energy for the quarks. $E_p^\pm = E_p \mp \mu$, μ is the quark chemical potential. Here we consider the isospin symmetry. Now, we introduce the mean-field approach by minimizing Ω with respect to σ , Φ and $\bar{\Phi}$, the mean-field equations are given by

$$\begin{aligned} \sigma = & -6N_f \int \frac{d^3 p}{(2\pi)^3} E_p \frac{M}{E_p} [\theta(\Lambda^2 - p^2) \\ & - M_\Phi^+(E_p) N_\Phi^+(E_p) - M_\Phi^-(E_p) N_\Phi^-(E_p)]; \end{aligned} \quad (16)$$

$$\begin{aligned} 0 = & \frac{T^4}{2} [-b_2(T)\bar{\Phi} - b_3\bar{\Phi}^2 + b_4\bar{\Phi}\bar{\Phi}^2] \\ & - 12T \int \frac{d^3 p}{(2\pi)^3} [\exp(-2\beta E_p^+) N_\Phi^+(E_p) \\ & + \exp(-\beta E_p^-) N_\Phi^-(E_p)]; \end{aligned} \quad (17)$$

$$\begin{aligned} 0 = & \frac{T^4}{2} [-b_2(T)\Phi - b_3\Phi^2 + b_4\Phi\Phi^2] \\ & - 12T \int \frac{d^3 p}{(2\pi)^3} [\exp(-\beta E_p^+) N_\Phi^+(E_p) \\ & + \exp(-2\beta E_p^-) N_\Phi^-(E_p)]. \end{aligned} \quad (18)$$

The limit of integration is $0 \sim \Lambda$, which is a global cutoff [25]. Where

$$\begin{aligned} M_\Phi^+(E_p) = & (\Phi + 2\bar{\Phi} \exp(-\beta E_p^+)) \exp(-\beta E_p^+) \\ & + \exp(-3\beta E_p^+), \\ M_\Phi^-(E_p) = & (\bar{\Phi} + 2\Phi \exp(-\beta E_p^-)) \exp(-\beta E_p^-) \\ & + \exp(-3\beta E_p^-). \end{aligned}$$

By numerically solving the three coupled equations above, we can obtain a series of σ , Φ , $\bar{\Phi}$ at different

temperatures and chemical potentials. The thermodynamic quantities (such as pressure, quark number density, entropy and energy density) can be computed with the thermodynamic relations:

$$P = -\frac{\Omega}{V}; \rho_q = -\left(\frac{\partial\Omega}{\partial\mu}\right)_T;$$

$$S = -\left(\frac{\partial P}{\partial T}\right)_\mu; \varepsilon = TS + \mu\rho_q - P.$$

To this end, we can calculate the bulk viscosity from Eq. (13).

In this work we consider two-flavor quark matter. For numerical calculations, we choose the parameters as follows [25]: the global cutoff $\Lambda = 0.651$ GeV, the quark current mass $m = 0.0055$ GeV, the coupling constant $g = 5.04$ GeV. We also choose $T_0 = 0.27$ GeV, the zero temperature quark condensation $|\sigma_0| = 0.251^3$ GeV and $\omega_0 = 1$ GeV. The vacuum energy density $|\epsilon_v|^{1/4} = 0.25$ GeV.

The temperature dependences of the order parameters for chiral phase transition and de-confinement phase transition σ/σ_0 , $\bar{\Phi}$, Φ are plotted in Fig. 1, which shows that the chiral phase transition temperature is about 0.24 GeV with a quark chemical potential $\mu = 0.2$ GeV. This phase transition is a cross over. While the deconfinement phase transition might happen at higher temperature, the Polyakov loops are not exact order parameters for the deconfinement phase transition of QCD with quarks included.

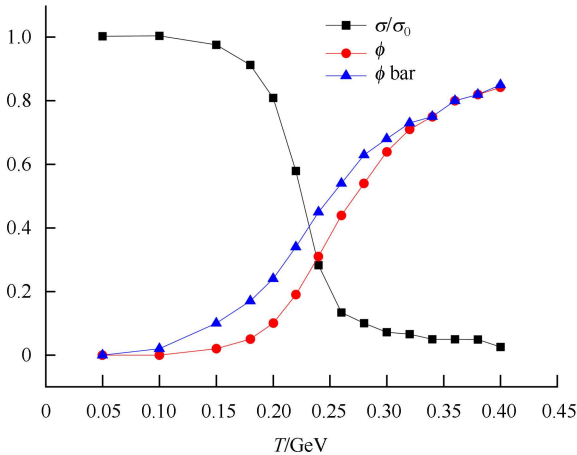


Fig. 1. Scaled chiral condensate and Polyakov loops as functions of temperature at $\mu = 0.2$ GeV.

4 Numerical results for bulk viscosity

The numerical results for bulk viscosity are depicted in Fig. 2 at different quark chemical potentials. One can see that the bulk viscosity has a sharp peak around the chiral phase transition temperature, just as it does in the results of Masashi Mizutani [17]. This indicates that the

finite quark chemical potential increases the bulk viscosity with the same temperature.

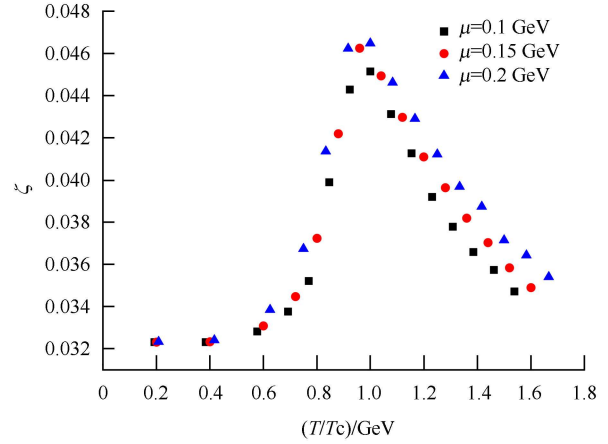


Fig. 2. Bulk viscosity in unit $(\text{GeV})^{-3}$ at different chemical potentials, the increasing chemical potential raises the bulk viscosity.

We have also computed the specific bulk viscosity, and the ratio of the bulk viscosity and entropy density, at finite temperature and density, as shown in Fig. 3. We show that this ratio starts to increase rapidly and blows up around the critical temperature. This result is in agreement with the results obtained in Ref. [11], where the bulk viscosity is obtained by combining low energy theorems with lattice results for QCD with a physical strange quark mass and almost physical light quark masses at zero quark chemical potential. The behavior of bulk viscosity is also qualitatively similar to the one observed for the case of $SU(3)$ gluodynamics in Ref. [27]. The finite quark chemical potential decreases the specific bulk viscosity through increases in the bulk viscosity, which happens because the finite chemical potential enhances the entropy density more rapidly than the bulk viscosity.

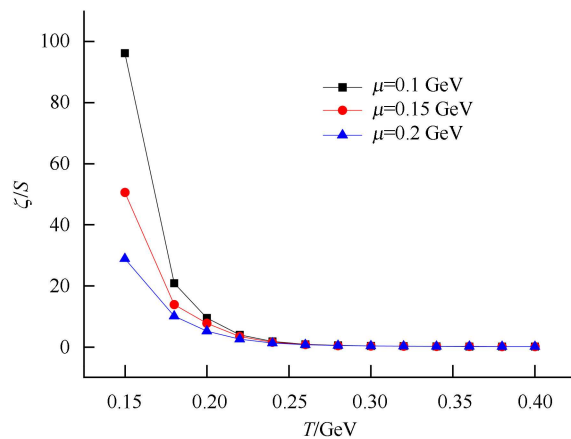


Fig. 3. The ratio of bulk viscosity to entropy at different chemical potential.

5 Summary

In summary, we have studied the bulk viscosity of hot quark matter at finite temperature and density within PNJL model by making use of the the Kubo formula and the QCD low energy theorems. We show that the bulk viscosity has a sharp peak near the chiral phase transition, and that the ratio of bulk viscosity and the entropy density rises dramatically in the vicinity of the chiral phase transition. These results agree qualitatively with those from the lattice and other model calculations [11, 16, 27]. In addition, we show that the increase of chemical potential raises the bulk viscosity but decreases the ratio of the bulk viscosity and entropy density. Although the bulk viscosity is small away from the critical

temperature, close to the critical temperature the rapid growth of the trace anomaly causes a dramatic increase of bulk viscosity. Therefore, we cannot neglect the bulk viscosity correction to the ideal hydrodynamical behavior in the vicinity of the deconfinement and chiral symmetry restoration phase transitions.

In this paper we have worked within the effective QCD model PNJL model. The advantage of the phenomenological model is that it is possible to provide equations of state (EOS), even at nonzero chemical potential. The EOS from the PNJL model can reproduce recent progress in lattice QCD with small chemical potential with good agreement [28].

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