A preliminary quadrupole asymmetry study of a β=0.12 superconducting single spoke cavity *

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Abstract: An Accelerator Driven System (ADS) has been launched in China for nuclear waste transmutation. For the application of high intensity proton beam acceleration, the quadrupole asymmetry effect needs to be carefully evaluated for cavities. Single spoke cavities are the main accelerating structures in the low energy front-end. The single spoke cavity has small transverse electromagnetic field asymmetry, which may lead to transverse RF defocusing asymmetry and beam envelope asymmetry. A superconducting single spoke resonator (PKU-2 Spoke) of β=0.12 and f=325 MHz with a racetrack-shaped inner conductor has been designed at Peking university. The study of its RF field quadrupole asymmetry and its effect on transverse momentum change has been performed. The quadrupole asymmetry study has also been performed on a β=0.12 and f=325 MHz ring-shaped single spoke cavity. Our results show that the quadrupole asymmetry is very small for both the racetrack-shaped and the ring-shaped single spoke cavity.

Key words: spoke cavity, quadrupole asymmetry, transverse momentum change

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1 Introduction

The Chinese Accelerator Driven System (ADS) is a project launched for nuclear waste transmutation. Protons are accelerated from the radio frequency quadrupole (RFQ) to 1.5 GeV in a superconducting linac. The superconducting linac includes two parts: a low energy part and a high energy part. In the low energy part, protons are first accelerated from RFQ to 10 MeV through single spoke cavities or half wave resonator (HWR) cavities, and then are accelerated to 178 MeV through two families of single spoke cavities with β=0.21 and β=0.40, respectively. In the high energy part, protons are accelerated from 178 MeV to 1.5 GeV through two families of elliptical cavities with β=0.63 and β=0.82, respectively [1].

Great efforts have been made to the development of the superconducting linac, especially to the low energy front-end. The low energy front-end of a superconducting linac is the most important part because it influences the performance and reliability of the rest of the linac. For example, failure of a low-β cavity to achieve its design gradient may mean that the particles will not be captured by the following section [2]. The single spoke cavity has been considered as a good choice for acceleration of low-velocity ions. The single spoke cavity is topologically equivalent to the coaxial transmission line, terminated by a short at both ends. A quasi-TEM electromagnetic wave bunches up and down along the inner conductor. Since the diameter of the single spoke cavity is on the order of $\lambda/2$, so the frequency is half that of an elliptical cavity for the same transverse dimension [3]. So the chief advantage of the coaxial transmission line topology is its manageable volume at low frequency. Also, it is mechanically robust, geometrically stable and has fewer welds. But as can be seen from Fig. 1, the high-voltage region of both the racetrack-shaped and the ring-shaped single spoke cavity, which is in the middle of the inner conductor, has a fairly complicated geometry with no simple symmetry. It can be seen from Fig. 1 that, the inner conductor is symmetric in the x-y plane, which can prevent dipole steering. But the inner conductor results in geometrically quadrupole asymmetry, which may cause transverse electromagnetic asymmetry in the x and y direction. As a consequence, when an axisymmetric beam travels through the cavity, it will experience a different defocusing strength in the x and y direction.

Because solenoids are used in the low energy front-end to provide symmetric transverse focusing, so the

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asymmetric defocusing arising from the cavity may have an impact on the transverse beam envelope. In this paper, we will study the quadrupole asymmetry of both the β=0.12, f=325 MHz racetrack-shaped and ring-shaped single spoke cavity, and their impact on the transverse momentum change.

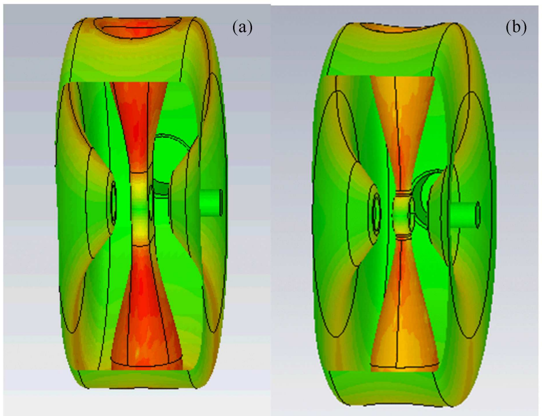

Fig. 1. CST microwave studio pictures of the surface field in the open cavities are shown for reference. (a) racetrack-shaped spoke cavity (b) ring-shaped spoke cavity.

2 Field asymmetry

The initial design of the β=0.12, f =325 MHz racetrack-shaped PKU-2 single spoke cavity for Chinese ADS can be found in Ref. [4]. It has a racetrack-shaped inner conductor with a circular (Φ=35 mm) beam pipe area. It has been recently observed that there may be other geometries in which the quadrupole asymmetry effect can be minimized, especially the ring-shaped single spoke cavity, which exhibits more symmetry about the beamline. So we built a β=0.12, f=325 MHz ring-shaped single spoke cavity which has not been fully optimized yet for high shunt-impedance and low surface field. However, it already exhibits promising RF characteristics. Table 1 lists the cavities' relevant RF parameters.

To study the quadrupole asymmetry of the racetrack-shaped and ring-shaped single spoke cavity, we first estimate the paraxial transverse electric and magnetic field. The results are shown in Figs. 2 and 3.

Figures 2 and 3 show that, for both the racetrack-shaped and the ring-shaped single spoke cavity, there is almost no deviation between E_x and E_y with different radial offset. However, because the electric field lines

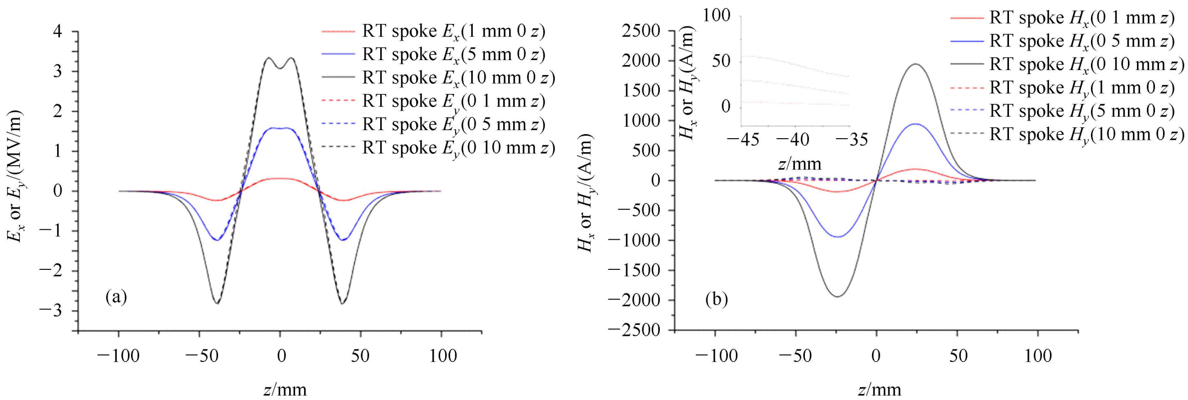

Fig. 2. Transverse electric field (a) and magnetic field (b) of the racetrack-shaped single spoke cavity in the x and y direction with different radial offsets.

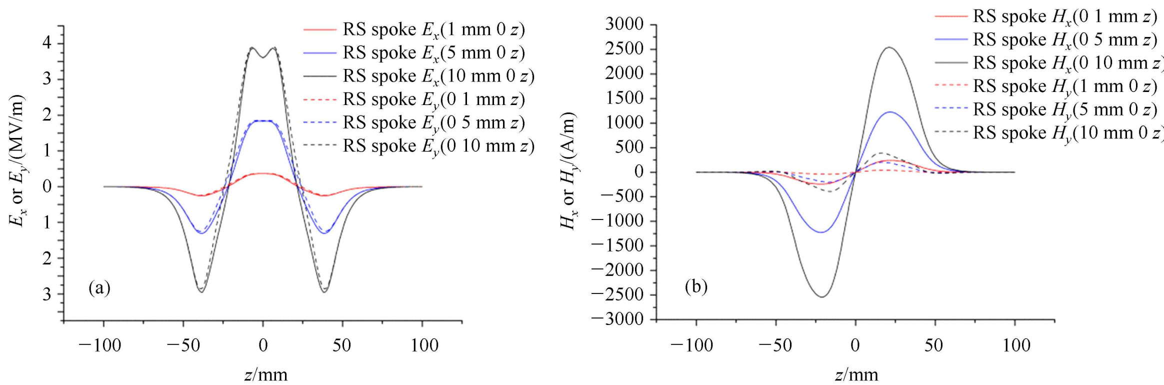

Fig. 3. Transverse electric field (a) and magnetic field (b) of the ring-shaped single spoke cavity in the x and y direction with different radial offsets.

Table 1. RF parameters for the β=0.12, f= 325 MHz single spoke cavities.

parameter	racetrack-shaped	ring-shaped
frequency/MHz	325	325
β_0	0.12	0.12
$(R/Q)/\Omega$	142	165
$(R/Q)(QR_s)/\Omega^2$	8502	10004
E_p/E_{acc}	4.87	6.21
(B_p/E_{acc})/(mT/(MV/m))	6.23	6.68
(B_p/E_p)/(mT/(MV/m))	1.3	1.1
V_{acc}/MV	0.54	0.58
energy content/J	1	1

extend perpendicularly from the surface of the inner conductor while the magnetic field lines surround it, H_y is much smaller than H_x.

3 Transverse momentum change deviation

The results above show that for both the racetrack-shaped and the ring-shaped single spoke cavity, the transverse electric field deviation between the two transverse planes is very small. While the transverse magnetic field deviation is clearly noticeable. But the transverse effect that a passes through charged particle suffers is cumulative. So next we will calculate the charged particle's transverse momentum change when it passes through the single spoke cavity.

Because the single spoke cavity has two accelerating gaps, the field profiles of the longitudinal electric field $E_z(z)$ and the transverse magnetic field ($H_x(z)$ or $H_y(z)$) are odd with respect to the center of the cavity, while the profile of the transverse electric field is even. Also the electromagnetic field is a standing wave field in the cavity, so the phase separation between the electric field and magnetic field is $\pi/2$. So the transverse momentum change of a passing through a charged particle in the x direction is:

$$\Delta p_x=q\int\left[E_x(z)\sin(\omega t+\phi)-\mu_0\beta cH_y(z)\sin\left(\omega t+\phi+\frac{\pi}{2}\right)\right]\mathrm{d}t, \tag{1}$$

here q is the charge of the particle, $E_x(z)$, $E_y(z)$, $H_x(z)$ and $H_y(z)$ are the profiles of the transverse electromagnetic field, ω is the angular frequency of the electromagnetic field, and ϕ is the phase of the field when the particle reaches the reference position (z=0).

Assume that the energy gain is sufficiently small so that the velocity of the particle does not change in the cavity. We can deduce that:

$$\Delta p_x = \frac{q}{\beta c}\sin\phi\left[\int E_x(z)\cos\frac{\omega z}{\beta c}\mathrm{d}z -\int\mu_0\beta cH_y(z)\cos\left(\frac{\omega z}{\beta c}+\frac{\pi}{2}\right)\mathrm{d}z\right]. \tag{2}$$

Similarly, the transverse momentum change of a passing through a charged particle in the y direction is:

$$\Delta p_y = \frac{q}{\beta c}\sin\phi\left[\int E_y(z)\cos\frac{\omega z}{\beta c}\mathrm{d}z +\int\mu_0\beta cH_x(z)\cos\left(\frac{\omega z}{\beta c}+\frac{\pi}{2}\right)\mathrm{d}z\right]. \tag{3}$$

The terms in the brackets of Eqs. (2) and (3) have the same dimensions as the accelerating voltage. So to get a better understanding of the beam dynamics, we define transverse voltages:

$$V_{x,y}=\Delta p_{x,y}\times\frac{\beta c}{q}. \tag{4}$$

Here, we let ϕ equal to $\pi/2$, in which the transverse momentum change is largest.

We present in Fig. 4(a) the plot of the transverse voltages normalized in V_{acc} as a function of the transverse offset from the design beamline. It is clear that defocusing occurs in both the x and y direction, and the defocusing strength is in direct proportion to the transverse offset, which is characteristic of the quadrupole field. Moreover, the slopes of all the fitted lines for both the racetrack-shaped and ring-shaped single spoke cavity are almost identical, which means that the quadrupole asymmetry of the two kinds of single spoke cavity are both very small.

Because both Δp_x and Δp_y is directly proportional to sinϕ, we define a function that describes the quadrupole component of the accelerating field [5]:

$$Q(\beta)=\frac{\Delta p_x-\Delta p_y}{\Delta p_x+\Delta p_y}\times 2. \tag{5}$$

We can see that $Q(\beta)$ is independent of ϕ.

Figure 5 shows the $Q(\beta)$-β curve for both the racetrack-shaped and ring-shaped single spoke cavity in given range of β. It can be seen that, for both the racetrack-shaped and the ring-shaped single spoke cavity, their Q value is within the range of −0.1–0.1. Especially for the racetrack-shaped single spoke cavity, its Q value is smaller than 0.06 in the complete range of β. By contrast, the Q value of the ring-shaped single spoke cavity changes more noticeably. Also, the Q value of the ring-shaped single spoke cavity is higher than that of the racetrack-shaped single spoke cavity at both ends of the β range. But the $Q(\beta)$-β curve of the ring-shaped single spoke cavity crosses the zero point near the cavity's geometry β. It means that it may be able to compensate the asymmetry for a wide range of β, and that will be an advantage of the ring-shaped single spoke cavity over the racetrack-shaped single spoke cavity.

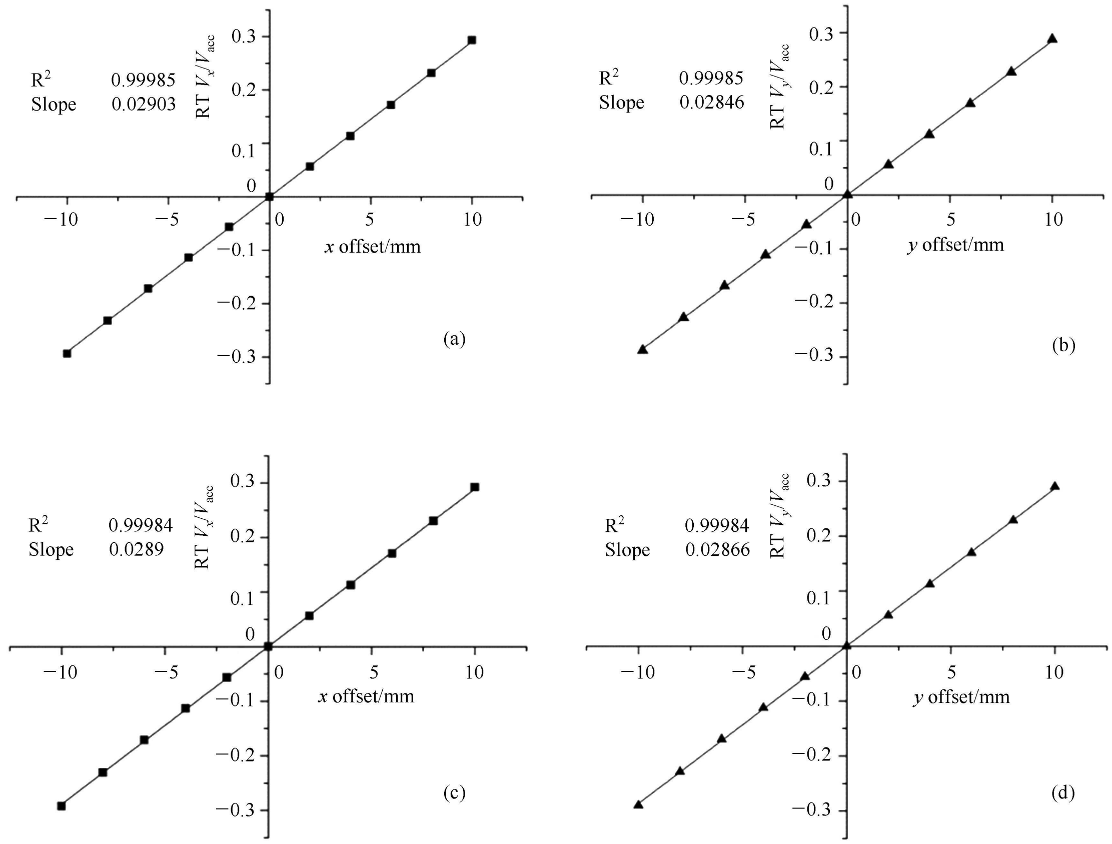

Fig. 4. Normalized transverse voltages as a function of the transverse offset, for both the racetrack-shaped and ring-shaped single spoke cavity. (a) $V_x/V_{\rm acc}$ for the racetrack-shaped spoke (b) $V_y/V_{\rm acc}$ for the racetrack-shaped spoke (c) $V_x/V_{\rm acc}$ for the ring-shaped spoke (d) $V_y/V_{\rm acc}$ for the ring-shaped spoke.

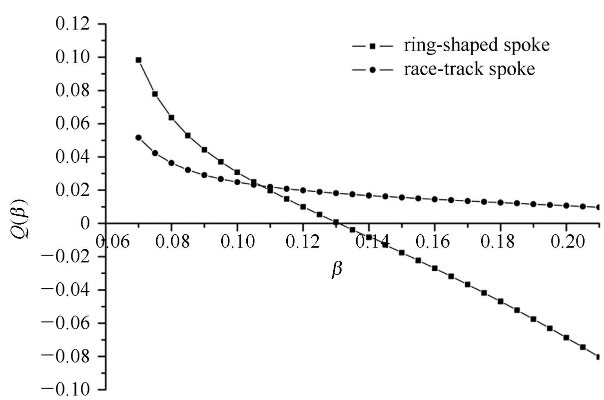

Fig. 5. $Q(\beta)$-β curve for both the racetrack-shaped and ring-shaped single spoke cavity in a given range of β.

Apart from the electromagnetic quadrupole asymmetry, the high-voltage region of the ring-shaped single spoke cavity has a more complicated geometry structure, which calls for higher machining precision. By contrast, the racetrack-shaped single spoke cavity has not only simple structure but is also mechanically robust and geometrically stable.

4 Analysis

For a particle carrying a single basic unit of electrical charge, the Lorentz force is expressed by:

$$\vec{F}=e(\vec{E}+\vec{v}\times\vec{B}), \quad (6)$$

Here e is the basic unit of electrical charge. The Lorentz force has two components originating from either an electrical field $\vec{E}$ or a magnetic field $\vec{B}$. For the single spoke cavity, the paraxial area is located in its high-voltage region where the magnetic field is weakest. Also the cavity's $\beta_{\rm optimal}$=0.12, and the magnetic field force is reduced proportional to the particle velocity $\beta=v/c$. For example, in the x direction, the maximum electric field force is 40 times larger than the maximum magnetic field force with the radial offset=1 mm. As stated above, the almost negligible transverse electric field deviation results in the very small Q value of the β=0.12 single spoke cavity. While the clearly observable transverse magnetic field deviation almost has no effect on the quadrupole asymmetry for such a low β single spoke cavity. With the increasing of the single spoke cavity's β value, the quadrupole asymmetry may become more evident.

5 Conclusion

We studied the RF field quadrupole asymmetry and its effect on transverse momentum change of both the β=0.12, f=325 MHz racetrack-shaped and ring-shaped single spoke cavity. The results show that for both the racetrack-shaped and ring-shaped single spoke cavity, their quadrupole asymmetry is very small. Although the $Q(\beta)$-β curve of the ring-shaped single spoke cavity crosses the zero point near the cavity's geometry β, which means that it may be able to compensate asymmetry for a wide range of β, the racetrack-shaped single spoke cavity has not only a simple structure but is also mechanically robust and geometrically stable. The study of the effect that the single spoke cavity's asymmetry field may have on the beam envelope is being performed.

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