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Semileptonic decays $B_c \to (\eta_c,\ J/\Psi) l\nu$ in the perturbative QCD approach*

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Abstract: In this paper we study the semileptonic decays of $B_c^- \to (\eta_c,\ J/\psi) l^- \bar{\nu}_l$. We first evaluate the $B_c \to (\eta_c, J/\Psi)$ transition form factors $F_0(q^2)$, $F_+(q^2)$, $V(q^2)$ and $A_{0,1,2}(q^2)$ by employing the pQCD factorization approach, and then we calculate the branching ratios for all considered semileptonic decays. Based on the numerical results and the phenomenological analysis, we find that: (a) the pQCD predictions for the values of the $B_c \to \eta_c$ and $B_c \to J/\Psi$ transition form factors agree well with those obtained by using other methods; (b) the pQCD predictions for the branching ratios of the considered decays are $Br\left(B_c^- \to \eta_c e^- \bar{\nu}_e(\mu^- \bar{\nu}_\mu)\right) = (4.41^{+1.22}_{-1.09}) \times 10^{-3}$, $Br\left(B_c^- \to \eta_c \tau^- \bar{\nu}_\tau\right) = (1.37^{+0.37}_{-0.34}) \times 10^{-3}$, $Br(B_c^- \to J/\Psi e^- \bar{\nu}_e(\mu^- \bar{\nu}_\mu)) = (10.03^{+1.33}_{-1.18}) \times 10^{-3}$, and $Br\left(B_c^- \to J/\Psi \tau^- \bar{\nu}_\tau\right) = (2.92^{+0.40}_{-0.34}) \times 10^{-3}$; and (c) we also define and calculate two ratios of the branching ratios R_{η_c} and $R_{J/\Psi}$, which will be tested by LHCb and the forthcoming Super-B experiments.

Key words: pQCD, B meson, semileptonic decay

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1 Introduction

The charmed B_c meson was found by the CDF Collaboration at Tevatron [1] fifteen years ago. It is a pseudoscalar ground state of two heavy quarks b and c, which can decay individually. Being below the B-D threshold, B_c meson can only decay through weak interactions, it is an ideal system to study weak decays of heavy quarks. Due to the different decay rate of the two heavy quarks, the B_c meson decays are rather different from those of B or B_s meson. Although the phase space in $c \to s$ transition is smaller than that in $b \to c$ decays, the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{cs}| \sim 1$ is much larger than $|V_{cb}| \sim 0.04$. Thus the c-quark decays provide the dominant contribution (about 70%) to the decay width of B_c meson [2]. At LHC experiments, around 5×10^{10} B_c events per year are expected [2, 3], which provide a very good platform to study various B_c meson decay modes.

In fact, the B_c decays have been studied intensively by many authors [4–13]. In Ref. [4], for example, Dhir and Verma presented a detailed analysis of the B_c decays in the Bauer-Stech-Wirbel (BSW) framework, while the authors of the Refs. [5–7] studied the B_c meson decays in the non-relativistic or relativistic quark model. In the perturbative QCD (pQCD) approach, furthermore, various B_c decay modes have also been studied for example in Refs. [11–13].

In this paper, we will study the semileptonic decays of $B_c \to (\eta_c,\ J/\Psi) l\bar{\nu}_l$ (here l stands for leptons e, μ, and τ) in the pQCD factorization approach [14, 15]. The lowest order diagrams for $B_c \to (\eta_c,\ J/\Psi) l\bar{\nu}_l$ are displayed in Fig. 1, where B stands for B_c meson and M for η_c or J/Ψ meson, and the leptonic pairs come from the b-quark's weak decay. In this work, we first calculate the q^2-dependent form factors for $B_c \to (\eta_c,\ J/\Psi)$ transitions, and then we give the branching ratios of the considered semileptonic decay modes.

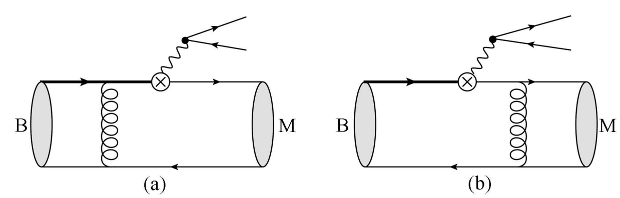

Fig. 1. The typical Feynman diagrams for the semileptonic decays $B_c \to (\eta_c,\ J/\Psi) l\bar{\nu}$, where B stands for B_c meson and M for η_c or J/Ψ meson.

The structure of this paper is as follows: after this introduction, we collect the distribution amplitudes of the B_c meson and the η_c, J/Ψ mesons in Section 2. In Section 3, based on the k_T factorization theorem, we calculate and present the expressions for the $B_c \to (\eta_c,\ J/\Psi)$

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transition form factors in the large recoil regions. The numerical results and relevant discussions are given in Section 4, and a short summary will also be included in this section.

2 Kinematics and the wave functions

In the $\mathrm{B_c}$ meson rest frame, where the $m_{\mathrm{B_c}}(m)$ stands for the mass of the $\mathrm{B_c}$(η_c or J/Ψ) meson, and $r=m/m_{\mathrm{B_c}}$, the momenta of $\mathrm{B_c}$ and η_c(J/Ψ) mesons are defined in the same way as in Ref. [16]

$$p_1=\frac{m_{\mathrm{B_c}}}{\sqrt{2}}(1,1,0_\perp),\quad p_2=\frac{m_{\mathrm{B_c}}}{\sqrt{2}}(r\eta^+,r\eta^-,0_\perp),\tag{1}$$

where $\eta^+=\eta+\sqrt{\eta^2-1}$ and $\eta^-=\eta-\sqrt{\eta^2-1}$ with the definition of the η of the form

$$\eta=\frac{m_{\mathrm{B_c}}}{2m}\left[1+\frac{m^2-q^2}{m_{\mathrm{B_c}}^2}\right],\tag{2}$$

where $q^2=(p_1-p_2)^2$ is the invariant mass of the lepton pairs. The momenta of the spectator quarks in $\mathrm{B_c}$ and η_c(J/Ψ) mesons are parameterized as

$$k_1 = \left(0,x_1\frac{m_{\mathrm{B_c}}}{\sqrt{2}},k_{1\perp}\right),$$
$$k_2 = \left(x_2\frac{m_{\mathrm{B_c}}}{\sqrt{2}}r\eta^+,x_2\frac{m_{\mathrm{B_c}}}{\sqrt{2}}r\eta^-,k_{2\perp}\right).\tag{3}$$

For the J/Ψ meson, we define its polarization vector ϵ as

$$\epsilon_{\mathrm{L}}=\frac{1}{\sqrt{2}}(\eta^+,-\eta^-,0_\perp),\quad \epsilon_{\mathrm{T}}=(0,0,1),\tag{4}$$

where ϵ_{L} and ϵ_{T} denotes the longitudinal and transverse polarization of the J/Ψ meson.

In the calculations, one can ignore the k_{T} contributions of $\mathrm{B_c}$ meson. Furthermore, one can assume that the b and c quark in $\mathrm{B_c}$ meson are on the mass shell approximately and treat its wave function as a δ function. In this work, we use the same distribution amplitude for $\mathrm{B_c}$ meson as those used in Refs. [11, 12]

$$\Phi_{\mathrm{B_c}}(x)=\frac{\mathrm{i}}{\sqrt{2N_c}}[(\not{p}+m_{\mathrm{B_c}})\gamma_5\phi_{\mathrm{B_c}}(x)]_{\alpha\beta}\tag{5}$$

with

$$\phi_{\mathrm{B_c}}(x)=\frac{f_{\mathrm{B_c}}}{2\sqrt{2N_c}}\delta(x-m_c/m_{\mathrm{B_c}}),\tag{6}$$

where m_c is the mass of c-quark.

For pseudoscalar meson η_c, the wave function is the form of

$$\Phi_{\eta_c}(x)=\frac{\mathrm{i}}{\sqrt{2N_c}}\gamma_5[\not{p}\phi^{\mathrm{v}}(x)+m_{\eta_c}\phi^{\mathrm{s}}(x)].\tag{7}$$

The twist-2 and twist-3 asymptotic distribution amplitudes, $\phi^{\mathrm{v}}(x)$ and $\phi^{\mathrm{s}}(x)$, can be written as [17, 18]

$$\phi^{\mathrm{v}}(x) = 9.58\frac{f_{\eta_c}}{2\sqrt{2N_c}}x(1-x)\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7},$$
$$\phi^{\mathrm{s}}(x) = 1.97\frac{f_{\eta_c}}{2\sqrt{2N_c}}\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7}.\tag{8}$$

As for the vector J/Ψ meson, we take the wave function as follows,

$$\Phi^{\mathrm{L}}_{\mathrm{J/\Psi}}(x) = \frac{1}{\sqrt{2N_c}}\{m_{\mathrm{J/\Psi}}\not{\epsilon}_{\mathrm{L}}\phi^{\mathrm{L}}(x)+\not{\epsilon}_{\mathrm{L}}\not{p}\phi^{\mathrm{t}}(x)\};$$
$$\Phi^{\mathrm{T}}_{\mathrm{J/\Psi}}(x) = \frac{1}{\sqrt{2N_c}}\{m_{\mathrm{J/\Psi}}\not{\epsilon}_{\mathrm{T}}\phi^{\mathrm{V}}(x)+\not{\epsilon}_{\mathrm{T}}\not{p}\phi^{\mathrm{T}}(x)\}.\tag{9}$$

The asymptotic distribution amplitudes of J/Ψ meson read as [17]

$$\phi^{\mathrm{L}}(x) = \phi^{\mathrm{T}}(x)=9.58\frac{f_{\mathrm{J/\Psi}}}{2\sqrt{2N_c}}x(1-x)\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7},$$
$$\phi^{\mathrm{t}}(x) = 10.94\frac{f_{\mathrm{J/\Psi}}}{2\sqrt{2N_c}}(1-2x)^2\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7},$$
$$\phi^{\mathrm{V}}(x) = 1.67\frac{f_{\mathrm{J/\Psi}}}{2\sqrt{2N_c}}[1+(2x-1)^2]\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7}.\tag{10}$$

Here, ϕ^{L} and ϕ^{T} denote for the twist-2 distribution amplitudes, and ϕ^{t} and ϕ^{V} for the twist-3 distribution amplitudes.

3 Form factors and semileptonic decays

As is well-known, the form factors of $\mathrm{B_{(s)}}\to$P,V,S[1)] transitions have been calculated by many authors in the pQCD factorization approach and other methods [19, 20], and the pQCD predictions are generally consistent with those from other methods.

The $\mathrm{B_c}\to\eta_c$ form factors induced by vector currents are defined in Refs. [16, 21, 22]

$$\begin{aligned}&\langle\eta_c(p_2)|\bar{c}(0)\gamma_\mu b(0)|B_c(p_1)\rangle\\ &= \left[(p_1+p_2)_\mu-\frac{m_{\mathrm{B_c}}^2-m^2}{q^2}q_\mu\right]F_+(q^2)\\ &+\frac{m_{\mathrm{B_c}}^2-m^2}{q^2}q_\mu F_0(q^2),\end{aligned}\tag{11}$$

where $q=p_1-p_2$ is the momentum transfer to the lepton pairs, and m is the mass of η_c meson. In order to cancel the poles at $q^2=0$, $F_+(0)$ should be equal to $F_0(0)$. For

1) Here P, V, S denote the pseudoscalar, vector and scalar meson respectively.

the sake of the calculation, it is convenient to define the auxiliary form factors $f_1(q^2)$ and $f_2(q^2)$

$$\langle\eta_c(p_2)|\bar{c}(0)\gamma_\mu b(0)|B_c(p_1)\rangle = f_1(q^2)p_{1\mu}+f_2(q^2)p_{2\mu}. \quad (12)$$

In terms of $f_1(q^2)$ and $f_2(q^2)$ the form factor $F_+(q^2)$ and $F_0(q^2)$ read

$$\begin{aligned} F_+(q^2) &= \frac{1}{2}[f_1(q^2)+f_2(q^2)], \\ F_0(q^2) &= \frac{1}{2}f_1(q^2)\left[1+\frac{q^2}{m_{B_c}^2-m^2}\right] \\ &\quad +\frac{1}{2}f_2(q^2)\left[1-\frac{q^2}{m_{B_c}^2-m^2}\right]. \end{aligned} \quad (13)$$

The form factors F_+ and F_0 (or f_1 and f_2) of the $B_c\to\eta_c$ transition are dominated by a single gluon exchange in the leading-order and in the large recoil regions. In the hard-scattering kernel, the transverse momentum is retained to regulate the endpoint singularity. The factorization formula for the $B\to\eta_c$ form factors in pQCD approach is written as [19]

$$\begin{aligned} &\langle\eta_c(p_2)|\bar{c}(0)\gamma_\mu b(0)|B_c(p_1)\rangle \\ &= g_s^2 C_F N_c \int dx_1 dx_2 d^2k_{1T} d^2k_{2T} \frac{dz^+ d^2z_T}{(2\pi)^3}\frac{dy^- d^2y_T}{(2\pi)^3} \\ &\quad \times e^{-ik_2\cdot y}\langle\eta_c(p_2)|\bar{c}'_\gamma(y)c_\beta(0)|0\rangle e^{ik_1\cdot z} \\ &\quad \times\langle 0|\bar{c}_\alpha(0)b'_\delta(z)|B_c(p_1)\rangle T^{\gamma\beta;\alpha\delta}_{H\mu}. \end{aligned} \quad (14)$$

In the transverse configuration b-space and by including the Sudakov form factors and the threshold resummation effects, we obtain the $B\to\eta_c$ form factors $f_1(q^2)$ and $f_2(q^2)$ as follows,

$$\begin{aligned} f_1(q^2) &= 8\pi m_{B_c}^2 r C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\times\Big\{2[\phi^s(x_2)-rx_2\phi^v(x_2)]\cdot h_1(x_1,x_2,b_1,b_2)\cdot\alpha_s(t_1)\exp[-S_{ab}(t_1)] \\ &\quad +\left[4r_c\phi^s(x_2)-2r\phi^v(x_2)+\frac{x_1\eta^+(\eta^+\phi^v(x_2)-2\phi^s(x_2))}{\sqrt{\eta^2-1}}\right]\times h_2(x_1,x_2,b_1,b_2)\cdot\alpha_s(t_2)\exp[-S_{ab}(t_2)]\Big\}, \end{aligned} \quad (15)$$

$$\begin{aligned} f_2(q^2) &= 8\pi m_{B_c}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\times\Big\{[2\phi^v(x_2)-4rx_2(\phi^s(x_2)-\eta\phi^v(x_2))] \\ &\quad \times h_1(x_1,x_2,b_1,b_2)\cdot\alpha_s(t_1)\exp[-S_{ab}(t_1)]+\left[4r\phi^s(x_2)-2r_c\phi^v(x_2)-\frac{x_1(\eta^+\phi^v(x_2)-2\phi^s(x_2))}{\sqrt{\eta^2-1}}\right] \\ &\quad \times h_2(x_1,x_2,b_1,b_2)\cdot\alpha_s(t_2)\exp[-S_{ab}(t_2)]\Big\}, \end{aligned} \quad (16)$$

where $C_F=4/3$ is a color factor, r is the same as in Eqs. (1,3), while $r_c=m_c/m_{B_c}$, and m_c is the mass of c-quark. The functions h_1 and h_2, the scales t_1, t_2 and the Sudakov factors S_{ab} are given in Appendix A of this paper.

For the charged current $B_c\to\eta_c l\bar{\nu}_l$, the quark level transitions are the $b\to cl\bar{\nu}_l$ transition with the effective Hamiltonian [23]

$$\mathcal{H}_{eff}(b\to cl\bar{\nu}_l) = \frac{G_F}{\sqrt{2}}V_{cb}\bar{c}\gamma_\mu(1-\gamma_5)b\cdot\bar{l}\gamma^\mu(1-\gamma_5)\nu_l, \quad (17)$$

where $G_F=1.16637\times10^{-5}$ GeV^{-2} is the Fermi-coupling constant. With the two form factors $F_+(q^2)$ and $F_0(q^2)$, we can write down the differential decay width of the decay mode $B_c\to\eta_c l\bar{\nu}_l$ as [9, 24]

$$\begin{aligned} \frac{d\Gamma(B_c\to\eta_c l\bar{\nu}_l)}{dq^2} &= \frac{G_F^2|V_{cb}|^2}{192\pi^3 m_{B_c}^3}\frac{q^2-m_l^2}{(q^2)^2}\sqrt{\frac{(q^2-m_l^2)^2}{q^2}}\sqrt{\frac{(m_{B_c}^2-m^2-q^2)^2}{4q^2}-m^2} \\ &\quad \times\Big\{(m_l^2+2q^2)\left[q^2-(m_{B_c}-m)^2\right]\left[q^2-(m_{B_c}+m)^2\right]F_+^2(q^2)+3m_l^2\left(m_{B_c}^2-m^2\right)^2F_0^2(q^2)\Big\}, \end{aligned} \quad (18)$$

where m_l and m is the mass of the lepton and η_c respectively. If the produced lepton is $e^\pm$ or $\mu^\pm$, the corresponding mass terms of the lepton could be ignored.

The form factors $V(q^2)$ and $A_{0,1,2}(q^2)$ for $B_c\to J/\Psi$ transition are defined in the same way as in Refs. [16, 21, 22] and are written explicitly as,

$$\begin{aligned} V(q^2) =\ & 4\pi m_{\mathrm{B_c}}^2 C_{\mathrm{F}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 \phi_{\mathrm{B_c}}(x_1)\cdot(1+r) \\ & \times\left\{ 2\left[\phi^{\mathrm{T}}(x_2)-rx_2\phi^{\mathrm{V}}(x_2)\right]\cdot h_1(x_1,x_2,b_1,b_2)\cdot\alpha_{\mathrm{s}}(t_1)\exp[-S_{\mathrm{ab}}(t_1)] \right. \\ & \left. +\left[\left(2r+\frac{x_1}{\sqrt{\eta^2-1}}\right)\phi^{\mathrm{V}}(x_2)\right]\cdot h_2(x_1,x_2,b_1,b_2)\cdot\alpha_{\mathrm{s}}(t_2)\exp[-S_{\mathrm{ab}}(t_2)]\right\}, \end{aligned} \tag{19}$$

$$\begin{aligned} A_0(q^2) =\ & 8\pi m_{\mathrm{B_c}}^2 C_{\mathrm{F}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 \phi_{\mathrm{B_c}}(x_1)\times\left\{ \left[(1-r^2x_2+2rx_2\eta)\phi^{\mathrm{L}}(x_2)+r(1-2x_2)\phi^{\mathrm{t}}(x_2)\right]\right. \\ & \times h_1(x_1,x_2,b_1,b_2)\cdot\alpha_{\mathrm{s}}(t_1)\exp[-S_{\mathrm{ab}}(t_1)]+\left[\left(r^2+r_{\mathrm{c}}+\frac{x_1}{2}-rx_1\eta+\frac{x_1(\eta+r(1-2\eta^2))}{2\sqrt{\eta^2-1}}\right)\phi^{\mathrm{L}}(x_2)\right] \\ & \left.\times h_2(x_1,x_2,b_1,b_2)\cdot\alpha_{\mathrm{s}}(t_2)\exp[-S_{\mathrm{ab}}(t_2)]\right\}, \end{aligned} \tag{20}$$

$$\begin{aligned} A_1(q^2) =\ & 8\pi m_{\mathrm{B_c}}^2 C_{\mathrm{F}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 \phi_{\mathrm{B_c}}(x_1)\cdot\frac{r}{1+r}\times\left\{ \left[2(1+rx_2\eta)\phi^{\mathrm{V}}(x_2)-2(2rx_2-\eta)\phi^{\mathrm{T}}(x_2)\right]\right. \\ & \left.\times h_1(x_1,x_2,b_1,b_2)\cdot\alpha_{\mathrm{s}}(t_1)\exp[-S_{\mathrm{ab}}(t_1)]+\left[(2r_{\mathrm{c}}-x_1+2r\eta)\phi^{\mathrm{V}}(x_2)\right]\cdot h_2(x_1,x_2,b_1,b_2)\cdot\alpha_{\mathrm{s}}(t_2)\exp[-S_{\mathrm{ab}}(t_2)]\right\}, \end{aligned} \tag{21}$$

$$\begin{aligned} A_2(q^2) =\ & \frac{(1+r)^2(\eta-r)}{2r(\eta^2-1)}\cdot A_1(q^2)-8\pi m_{\mathrm{B_c}}^2 C_{\mathrm{F}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 \phi_{\mathrm{B_c}}(x_1)\cdot\frac{1+r}{\eta^2-1} \\ & \times\left\{ \left[(\eta(1-r^2x_2)-r(1+x_2-2x_2\eta^2))\phi^{\mathrm{L}}(x_2)+(1+2r^2x_2-r\eta(1+2x_2))\phi^{\mathrm{t}}(x_2)\right]\right. \\ & \times h_1(x_1,x_2,b_1,b_2)\cdot\alpha_{\mathrm{s}}(t_1)\exp[-S_{\mathrm{ab}}(t_1)] \\ & +\left[x_1\left(r\eta-\frac{1}{2}\right)\sqrt{\eta^2-1}+\left(r_{\mathrm{c}}+r^2-\frac{x_1}{2}\right)\eta+r\left(1-r_{\mathrm{c}}-\frac{x_1}{2}+x_1\eta^2\right)\right]\phi^{\mathrm{L}}(x_2) \\ & \left.\times h_2(x_1,x_2,b_1,b_2)\cdot\alpha_{\mathrm{s}}(t_2)\exp[-S_{\mathrm{ab}}(t_2)]\right\}, \end{aligned} \tag{22}$$

where $r=m_{\mathrm{J/\Psi}}/m_{\mathrm{B_c}}$, C_{F} and r_{c} are the same as in Eqs. (15, 16). The expressions of the hard function h_1 and h_2, hard scales t_1 and t_2, and Sudakov function S_{ab} are all given in Appendix A. One should note that the pQCD predictions for the form factors $f_{1,2}(q^2)$, $V(q^2)$ and $A_{0,1,2}(q^2)$ as given in Eqs. (15, 16, 19–22) are all leading order results. The NLO contributions to the form factors of $\mathrm{B}\to(\pi,\mathrm{K})$ transitions as given in Refs. [25, 26] do not apply here because of the large mass of c-quark and (η_{c}, $\mathrm{J/\Psi}$) mesons.

The effective Hamiltonian for the decay modes $\mathrm{B_c}\to\mathrm{J/\Psi}\mathrm{l}\bar{\nu}_{\mathrm{l}}$ is the same as $\mathrm{B_c}\to\eta_{\mathrm{c}}\mathrm{l}\bar{\nu}_{\mathrm{l}}$, but corresponding differential decay widths are different. For $\mathrm{B_c}\to\mathrm{J/\Psi}\mathrm{l}\bar{\nu}_{\mathrm{l}}$, we have [9, 27, 28]

$$\begin{aligned} \frac{\mathrm{d}\Gamma_{\mathrm{L}}(\mathrm{B_c}\to\mathrm{J/\Psi}\mathrm{l}\bar{\nu}_{\mathrm{l}})}{\mathrm{d}q^2} =\ & \frac{G_{\mathrm{F}}^2|V_{\mathrm{cb}}|^2}{192\pi^3 m_{\mathrm{B_c}}^3}\frac{q^2-m_{\mathrm{l}}^2}{(q^2)^2}\sqrt{\frac{(q^2-m_{\mathrm{l}}^2)^2}{q^2}}\sqrt{\frac{\left(m_{\mathrm{B_c}}^2-m^2-q^2\right)^2}{4q^2}-m^2} \\ & \times\left\{3m_{\mathrm{l}}^2\lambda(q^2)A_0^2(q^2)+\frac{m_{\mathrm{l}}^2+2q^2}{4m^2}\times\left[(m_{\mathrm{B_c}}^2-m^2-q^2)(m_{\mathrm{B_c}}+m)A_1(q^2)-\frac{\lambda(q^2)}{m_{\mathrm{B_c}}+m}A_2(q^2)\right]^2\right\}, \end{aligned} \tag{23}$$

$$\begin{aligned} \frac{\mathrm{d}\Gamma_{\pm}(\mathrm{B_c}\to\mathrm{J/\Psi}\mathrm{l}\bar{\nu}_{\mathrm{l}})}{\mathrm{d}q^2} =\ & \frac{G_{\mathrm{F}}^2|V_{\mathrm{cb}}|^2}{192\pi^3 m_{\mathrm{B_c}}^3}\frac{q^2-m_{\mathrm{l}}^2}{q^2}\sqrt{\frac{(q^2-m_{\mathrm{l}}^2)^2}{q^2}}\sqrt{\frac{\left(m_{\mathrm{B_c}}^2-m^2-q^2\right)^2}{4q^2}-m^2} \\ & \times\left\{(m_{\mathrm{l}}^2+2q^2)\lambda(q^2)\left[\frac{V(q^2)}{m_{\mathrm{B_c}}+m}\mp\frac{(m_{\mathrm{B_c}}+m)A_1(q^2)}{\sqrt{\lambda(q^2)}}\right]^2\right\}, \end{aligned} \tag{24}$$

where $m=m_{J/\psi}$, and $\lambda(q^2)=(m_{B_c}^2+m^2-q^2)^2-4m_{B_c}^2m^2$ is the phase-space factor. The combined transverse and total differential decay widths are defined as

$$\frac{d\Gamma_T}{dq^2}=\frac{d\Gamma_+}{dq^2}+\frac{d\Gamma_-}{dq^2},\quad \frac{d\Gamma}{dq^2}=\frac{d\Gamma_L}{dq^2}+\frac{d\Gamma_T}{dq^2}. \tag{25}$$

4 Numerical results and discussions

In the numerical calculations we use the following input parameters (here masses and decay constants in unit GeV) [11, 29]

$$\begin{aligned}
\Lambda_{\overline{MS}}^{(f=4)} &= 0.287, \quad m_{\eta_c}=2.981, \quad m_{J/\Psi}=3.097,\\
m_{B_c} &= 6.277, \quad m_c=1.275\pm0.025, \quad m_\tau=1.777,\\
f_{B_c} &= 0.489, \quad \tau_{B_c}=(0.45\pm0.04)\ \text{ps},\\
|V_{cb}| &= (41.2^{+1.1}_{-0.5})\times10^{-3}, \quad f_{\eta_c}=(0.420\pm0.050),\\
f_{J/\Psi} &= (0.405\pm0.014).
\end{aligned} \tag{26}$$

By using the expressions in Eqs. (15, 16, 19–22) and the definitions in Eq. (13) we calculate the values of the form factors $F_0(q^2)$, $F_+(q^2)$, $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$ and $A_2(q^2)$ for given value of q^2 in the region of $0\leqslant q^2\leqslant(m_{B_c}-m)^2$. But one should note that the pQCD predictions for the considered form factors are reliable only for small values of q^2. For the form factors in the larger q^2 region, one has to make an extrapolation for them from the lower q^2 region to larger q^2 region. In this work we make the extrapolation by using the formula in Refs. [9, 30]

$$F(q^2)=F(0)\cdot\exp(a\cdot q^2+b\cdot(q^2)^2), \tag{27}$$

where F stands for the form factors $F_{0,+}$, V and $A_{0,1,2}$, and a and b are the parameters to be determined by the fitting procedure.

The numerical values of the form factors $F_{0,+}$, V and $A_{0,1,2}$ at $q^2=0$ and their fitted parameters are listed in Table 1. The first error of the pQCD predictions for the form factors in Table 1 comes from the uncertainty of the decay constants of η_c and/or J/Ψ mesons, and the second error comes from the uncertainty of $m_c=1.275\pm0.025$ [29]. For the parameters (a, b), their errors from the decay constants f_{η_c}, $f_{J/\Psi}$ or from $m_c=1.275\pm0.025$ are negligibly small and not shown here explicitly. As a comparison, we also present some results obtained by other authors based on different methods in Table 2. One should note that the definition of the $B_c\to J/\Psi$ transition form factors in this paper are different from those in Ref. [6] (ISK) and [5] (HNV). From Table 2, we find that our

Table 1. The pQCD predictions for form factors $F_{0,+},V,A_{0,1,2}$ at $q^2=0$ and the parametrization constants a and b for $B_c\to\eta_c$ and $B_c\to J/\Psi$ transitions.

	$F(0)$	a	b
$F_0^{B_c\to\eta_c}$	0.48±0.06±0.01	0.037	0.0007
$F_+^{B_c\to\eta_c}$	0.48±0.06±0.01	0.055	0.0014
$V^{B_c\to J/\Psi}$	0.42±0.01±0.01	0.065	0.0015
$A_0^{B_c\to J/\Psi}$	0.59±0.02±0.01	0.047	0.0017
$A_1^{B_c\to J/\Psi}$	0.46±0.02±0.01	0.038	0.0015
$A_2^{B_c\to J/\Psi}$	0.64±0.02±0.01	0.064	0.0041

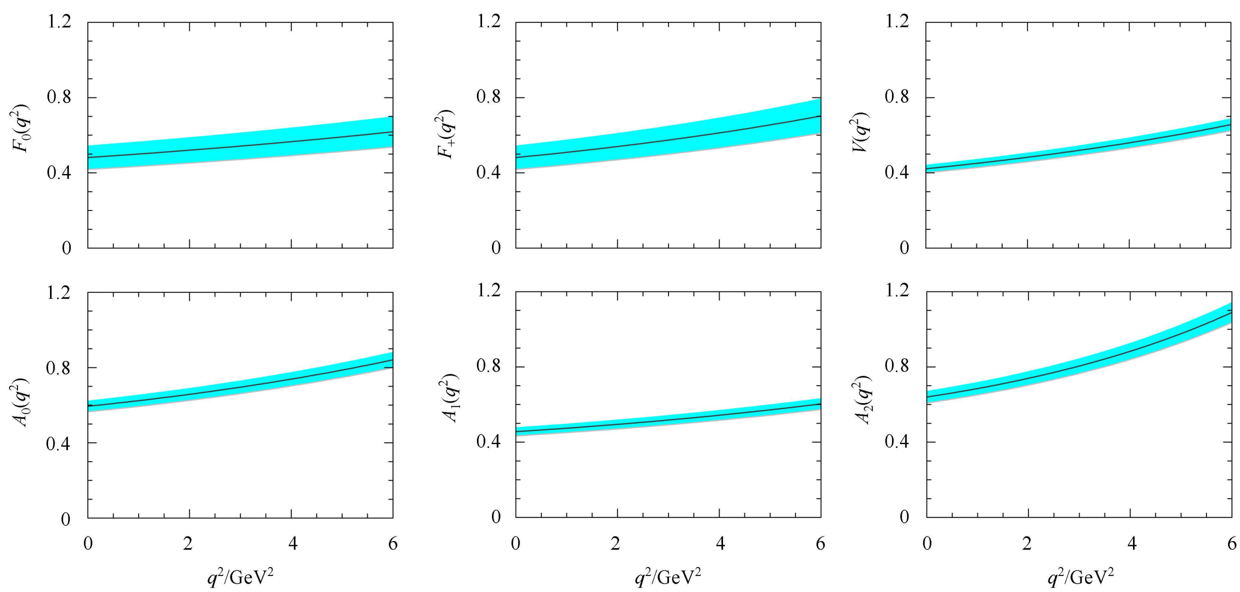

Fig. 2. The pQCD predictions for the q^2-dependence of the form factors F_0, F_+, V, A_0, A_1 and A_2. The solid lines stand for the central values and the bands show the errors of the corresponding form factors.

Table 2. $B_c \to \eta_c, J/\Psi$ transition form factors at $q^2=0$ evaluated in this paper and in other literature.

	pQCD	WSL[9]	EFG[7]	ISK[6]	HNV[5]	DV[4]
$F_0^{B_c\to\eta_c}=F_+^{B_c\to\eta_c}$	0.48	0.61	0.47	0.61	0.49	0.58
$V^{B_c\to J/\Psi}$	0.42	0.74	0.49	0.83	0.61	0.91
$A_0^{B_c\to J/\Psi}$	0.59	0.53	0.40	0.57	0.45	0.58
$A_1^{B_c\to J/\Psi}$	0.46	0.50	0.50	0.56	0.49	0.63
$A_2^{B_c\to J/\Psi}$	0.64	0.44	0.73	0.54	0.56	0.74

results agree well with the results in other literature. In Fig. 2, we show the pQCD predictions for the q^2-dependence of those form factors, where the solid lines stand for the central values, and the bands show the theoretical errors of the corresponding form factors.

By using the relevant formulas and the input parameters as defined or given in previous sections, it is straightforward to calculate the branching ratios for all the considered decays. By making the numerical integration over the physical range of q^2, we find the pQCD predictions for the branching ratios of considered decay modes:

$$\begin{aligned}
&Br(B_c^-\to\eta_c e^-\bar{\nu}_e(\mu^-\bar{\nu}_\mu))\\
&=\left(4.41^{+1.11}_{-0.99}(f_{\eta_c})\pm0.39(\tau_{B_c})^{+0.24}_{-0.11}(V_{cb})^{+0.22}_{-0.21}(m_c)\right)\times10^{-3},\\
&Br(B_c^-\to\eta_c\tau^-\bar{\nu}_\tau)\\
&=\left(1.37^{+0.34}_{-0.31}(f_{\eta_c})\pm0.12(\tau_{B_c})^{+0.07}_{-0.03}(V_{cb})^{+0.07}_{-0.06}(m_c)\right)\times10^{-3},\\
&Br(B_c^-\to J/\Psi e^-\bar{\nu}_e(\mu^-\bar{\nu}_\mu))\\
&=\left(10.03^{+0.71}_{-0.68}(f_{J/\Psi})\right.\\
&\quad\left.\pm0.89(\tau_{B_c})^{+0.54}_{-0.24}(V_{cb})^{+0.41}_{-0.27}(m_c)\right)\times10^{-3},\\
&Br(B_c^-\to J/\Psi\tau^-\bar{\nu}_\tau)\\
&=\left(2.92^{+0.21}_{-0.20}(f_{J/\Psi})\right.\\
&\quad\left.\pm0.26(\tau_{B_c})^{+0.16}_{-0.07}(V_{cb})^{+0.12}_{-0.08}(m_c)\right)\times10^{-3},
\end{aligned} \tag{28}$$

where the major theoretical errors come from the uncertainties of the input parameters f_{η_c}, $f_{J/\Psi}$, $|V_{cb}|$, τ_{B_c} and m_c as given explicitly in Eq. (26).

From the pQCD predictions for the form factors $F_{0,+}, V, A_{0,1,2}$ as given in Table 1 and the pQCD predictions for the branching ratios of $B_c\to(\eta_c, J/\Psi)l\nu$ as given in Eq. (28), we find the following points:

1) The form factor $F_0(0)$ equals to $F_+(0)$ by definition, but they have different q^2-dependence. The error bands of $F_0(q^2)$ and $F_+(q^2)$ in Fig. 2 are larger than that of $V(q^2)$ and $A_{0,1,2}(q^2)$. The reason is that the uncertainty of the decay constant f_{η_c} in $B_c\to\eta_c$ transition is larger than the one of $f_{J/\Psi}$ in $B_c\to J/\Psi$ transition.

2) The pQCD predictions for the form factors as listed in Table 2 agree well with those obtained by using other methods.

3) The pQCD predictions for the branching ratios of the four decay modes $B_c\to(\eta_c, J/\Psi)l\nu$ are at the order of 10^{-3}. Because of its large mass of τ lepton, the decays involving a τ in the final state has a smaller decay rate than those with light e^- or μ^-. Since the ratio of the branching ratios has smaller theoretical error than the decay rates themselves, we here define two ratios R_{η_c} and $R_{J/\Psi}$, the pQCD predictions for them are the following

$$R_{\eta_c} = \frac{Br(B_c^-\to\eta_c l^-\bar{\nu}_l)}{Br(B_c^-\to\eta_c\tau^-\bar{\nu}_\tau)}\approx3.2,\quad \text{for } l=(e,\mu), \tag{29}$$

$$R_{J/\Psi} = \frac{Br(B_c^-\to J/\Psi l^-\bar{\nu}_l)}{Br(B_c^-\to J/\Psi\tau^-\bar{\nu}_\tau)}\approx3.4,\quad \text{for } l=(e,\mu). \tag{30}$$

These relations will be tested by LHCb and the forthcoming Super-B experiments.

In short we calculated the branching ratios of the semileptonic decays $B_c^-\to(\eta_c, J/\psi)l^-\bar{\nu}_l$ in the pQCD factorization approach. We first calculated the relevant form factors by employing the pQCD factorization approach, and then evaluated the branching ratios for all considered semileptonic B_c decays. Based on the numerical results and the phenomenological analysis, we find that

1) For $B_c\to(\eta_c, J/\Psi)$ transitions, the LO pQCD predictions for the form factors $F_{0,+}(0), V(0)$ and $A_{0,1,2}(0)$ agree with those derived by using other different methods.

2) The pQCD predictions for the branching ratios of the considered decay modes are:

$$\begin{aligned}
Br(B_c^-\to\eta_c e^-\bar{\nu}_e(\mu^-\bar{\nu}_\mu)) &= (4.41^{+1.22}_{-1.09})\times10^{-3},\\
Br(B_c^-\to\eta_c\tau^-\bar{\nu}_\tau) &= (1.37^{+0.37}_{-0.34})\times10^{-3},\\
Br(B_c^-\to J/\Psi e^-\bar{\nu}_e(\mu^-\bar{\nu}_\mu)) &= (10.03^{+1.33}_{-1.18})\times10^{-3},\\
Br(B_c^-\to J/\Psi\tau^-\bar{\nu}_\tau) &= (2.92^{+0.40}_{-0.34})\times10^{-3},
\end{aligned} \tag{31}$$

where the individual theoretical errors in Eq. (28) have been added in quadrature.

3) We also defined two ratios of the branching ratios R_{η_c} and $R_{J/\Psi}$ and presented the corresponding pQCD predictions, which will be tested by LHCb and the forthcoming Super-B experiments.

Appendix A

Relevant functions

In this appendix, we present the functions seen in the previous sections. The threshold resummation factor $S_{\rm t}(x)$ is adopted from Ref. [19]:

$$S_{\rm t}=\frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \tag{A1}$$

and we here set the parameter c=0.3. The hard functions h_1 and h_2 come from the Fourier transform and can be written as

$$h_1(x_1,x_2,b_1,b_2) = K_0(\beta_1 b_1)[\theta(b_1-b_2)I_0(\alpha_1 b_2)K_0(\alpha_1 b_1) +\theta(b_2-b_1)I_0(\alpha_1 b_1)K_0(\alpha_1 b_2)]S_{\rm t}(x_2), \tag{A2}$$

$$h_2(x_1,x_2,b_1,b_2) =K_0(\beta_2 b_2)[\theta(b_1-b_2)I_0(\alpha_2 b_2)K_0(\alpha_2 b_1) +\theta(b_2-b_1)I_0(\alpha_2 b_1)K_0(\alpha_2 b_2)]S_{\rm t}(x_1), \tag{A3}$$

with $\alpha_1=m_{\rm B_c}\sqrt{x_2 r\eta}$, $\beta_1=m_{\rm B_c}\sqrt{x_1x_2r\eta^+}$, $\alpha_2=m_{\rm B_c}\sqrt{x_1r\eta^+}$ and $\beta_2=\beta_1$. The functions K_0 and I_0 are modified Bessel functions.

The factor $\exp(-S_{\rm ab}(t))$ contains the Sudakov logarithmic corrections and the renormalization group evolution effects of both the wave functions and the hard scattering amplitude with $S_{\rm ab}(t)$=$S_{\rm B_c}(t)$+$S_{\rm M}(t)$, where

$$S_{\rm B_c}(t) = s\left(x_1\frac{m_{\rm B_c}}{\sqrt{2}},b_1\right)+2\int_{1/b_1}^{t}\frac{{\rm d}\bar{\mu}}{\bar{\mu}}\gamma_{\rm q}(\alpha_{\rm s}(\bar{\mu})), \tag{A4}$$

$$S_{\rm M}(t) = s\left(x_2\frac{m_{\rm B_c}}{\sqrt{2}},b_2\right)+s\left((1-x_2)\frac{m_{\rm B_c}}{\sqrt{2}},b_2\right) +2\int_{1/b_2}^{t}\frac{{\rm d}\bar{\mu}}{\bar{\mu}}\gamma_{\rm q}(\alpha_{\rm s}(\bar{\mu})), \tag{A5}$$

with the quark anomalous dimension $\gamma_{\rm q}=-\alpha_{\rm s}/\pi$. The explicit expressions of the functions $s(Q,b)$ can be found for example in Appendix A of Ref. [15]. The hard scales t_i in Eqs. (16, 22) are chosen as the largest scale of the virtuality of the internal particles in the hard b-quark decay diagram,

$$t_1 = \max\{m_{\rm B_c}\sqrt{x_2r\eta},1/b_1,1/b_2\},$$

$$t_2 = \max\{m_{\rm B_c}\sqrt{x_1r\eta^+},1/b_1,1/b_2\}. \tag{A6}$$

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