Approximate solutions of Dirac equation with a ring-shaped Woods-Saxon potential by Nikiforov-Uvarov method

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Abstract: An approximate analytical solution of the Dirac equation is obtained for the ring-shaped Woods-Saxon potential within the framework of an exponential approximation to the centrifugal term. The radial and angular parts of the equation are solved by the Nikiforov-Uvarov method. The general results obtained in this work can be reduced to the standard forms already present in the literature.

Key words: Dirac equation, Woods-Saxon and ring-shaped potentials, Nikiforov-Uvarov method

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1 Introduction

It is well known that spin-1/2 fermions are analyzed within the framework of the Dirac equation. Nevertheless, not only for the Dirac equation, but also for any other field where the potential model is applied, no one can deterministically comment on the choice of the potential. Consequently, a lengthy list of potentials have been already investigated within the equation; Coulomb, harmonic oscillator, Hartmann, Woods-Saxon (WSP), Morse, Eckart, Pöschl-Teller, pseudoharmonic, Yukawa, etc. [1–21]. Among such cases, the WSP, due to its structure, stands as a meaningful term. On the other hand, the spherically symmetric potentials, despite their worthy advantages, are not enough for the study of ring-shaped molecules or deformed nuclei [22]. This point motivates consideration of angular-dependent terms. In our work, we decide to investigate a non-central potential, which contains the WSP modified with a ring-shaped term

$$V(r,\theta)=-\frac{V_0}{1+q\exp\left(\dfrac{r-R}{a}\right)}+\frac{\alpha+\beta\cos^2\theta}{r^2\sin^2\theta}. \tag{1}$$

The latter, due to its nature, has successfully accounted for some of the phenomena observed in quantum chemistry and nuclear physics and in particular the deformed cases including the deformed nuclei [23–31]. To go through the problem, we first review the Dirac equation in a few lines to have the prerequisites at hand. We next write the equation with scalar and vector angle-dependent potential and separate the corresponding partial differential equation in spherical coordinates. In Section 3 the Nikiforov-Uvarov (NU) [32–34] method is briefly introduced and Section 4 reports the approximate analytical solutions after a proper approximate scheme is used. The concluding remarks are presented in the last section.

2 Dirac equation with scalar and vector potentials

The Dirac equation with scalar potential $S(r)$ and vector potential $V(r)$ has the form [in units h=c=1] [28]

$$[\vec{\alpha}.\vec{p}+\beta(M+S(\vec{r}))-\mathrm{i}\beta\vec{\alpha}.\hat{r}U(r)]\psi(\vec{r})=[E-V(\vec{r})]\psi(\vec{r}), \tag{2}$$

where E, $\vec{p}=-\mathrm{i}\vec{\nabla}$ and M respectively denote the relativistic energy of the system, three-dimensional momentum operator and mass of the fermionic particle. α and β as usual are [28]

$$\alpha=\begin{pmatrix} 0 & \vec{\sigma}_i \\ \vec{\sigma}_i & 0 \end{pmatrix},\quad \beta=\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.\quad i=1,\ 2,\ 3, \tag{3}$$

where I is 2×2 unitary matrix and the spin matrices are

$$\sigma_1=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\quad \sigma_2=\begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix},\quad \sigma_3=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4}$$

In the Pauli-Dirac representation

$$\Psi(\vec{r})=\begin{pmatrix} \phi(\vec{r}) \\ \chi(\vec{r}) \end{pmatrix}, \tag{5}$$

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and substitution of Eqs. (3)–(5) into Eq. (2), gives

$$\sigma.p\chi(\vec{r}) = [E-V(\vec{r})-M-S(\vec{r})]\varphi(\vec{r}), \qquad (6a)$$
$$\sigma.p\varphi(\vec{r}) = [E-V(\vec{r})+M+S(\vec{r})]\chi(\vec{r}). \qquad (6b)$$

When the scalar potential $S(\vec{r})$ is equal to the vector potential $V(\vec{r})$, Eq. (6a) becomes

$$\sigma.p\chi(\vec{r}) = [E-M-2V(\vec{r})]\varphi(\vec{r}), \qquad (7a)$$
$$\chi(\vec{r}) = \frac{\sigma.p}{E+M}\varphi(\vec{r}). \qquad (7b)$$

Substituting Eq. (7b) into Eq. (7a), we find [28]

$$[p^2+2(E+M)V(\vec{r})]\varphi(\vec{r})=[E^2-M^2]\varphi(\vec{r}). \qquad (8)$$

Now, by inserting the potential into Eq. (8), we have

$$\left[-\nabla^2+2(E_{nlm}+M)\left\{-\frac{V_0}{1+q\exp\left(\frac{r-R}{a}\right)}+\frac{\alpha+\beta\cos^2\theta}{r^2\sin^2\theta}\right\}+M^2-E_{nlm}^2\right]\varphi_{nlm}(\vec{r})=0. \qquad (9)$$

For proceeding further, let us consider a solution of the form [28]

$$\varphi_{nlm}(\vec{r})=\frac{1}{\sqrt{2\pi}}\frac{R_{nlm}(r)}{r}\frac{H_l(\theta)}{(\sin\theta)^{1/2}}\mathrm{e}^{\mathrm{i}m\varphi}, \qquad (10)$$

which, after separation of variables in Eq. (9) results in

$$\frac{\mathrm{d}^2R_{nlm}(r)}{\mathrm{d}r^2}+\left[\frac{2V_0(E_{nlm}+M)}{1+q\exp\left(\frac{r-R}{a}\right)}+\frac{1/4-l^2}{r^2}-E_{nlm}^2+M^2\right]\times R_{nlm}(r)=0, \qquad (11a)$$

$$\frac{\mathrm{d}^2H_l(\theta)}{\mathrm{d}\theta^2}+\left[\frac{1/4-2\alpha(E_{nlm}+M)-2\beta(E_{nlm}+M)-m^2}{\sin^2\theta}+2\beta(E_{nlm}+M)+l^2\right]H_l(\theta)=0, \qquad (11b)$$

where $m=0,\pm1,\pm2,\cdots$.

3 Nikiforov-Uvarov method

The NU method, in its parametric version, solves a second order differential equation of the form [32–34]

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}s^2}+\frac{\alpha_1-\alpha_2 s}{s(1-\alpha_3 s)}\frac{\mathrm{d}}{\mathrm{d}s}+\frac{1}{[s(1-\alpha_3 s)]^2}[-\xi_1 s^2+\xi_2 s-\xi_3]\right\}\psi = 0. \qquad (12)$$

According to the NU method, the eigenfunctions and eigenenergies respectively are obtained via

$$\psi(s) = s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}}\times P_n^{\left(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-2\alpha_3 s), \qquad (13)$$

and

$$\alpha_2 n-(2n+1)\alpha_5+(2n+1)(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8})+n(n-1)\alpha_3+\alpha_7+2\alpha_3\alpha_8+2\sqrt{\alpha_8\alpha_9}=0, \qquad (14)$$

where

$$\begin{aligned}\alpha_4 &= \frac{1}{2}(1-\alpha_1),\alpha_5=\frac{1}{2}(\alpha_2-2\alpha_3),\alpha_6=\alpha_5^2+\xi_1,\\ \alpha_7 &= 2\alpha_4\alpha_5-\xi_2,\ \alpha_8=\alpha_4^2+\xi.\\ \alpha_9 &= \alpha_3\alpha_7+\alpha_3^2\alpha_8+\alpha_6,\alpha_{10}=\alpha_1+2\alpha_4+2\sqrt{\alpha_8},\\ \alpha_{11} &= \alpha_2-2\alpha_5+2(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}),\alpha_{12}=\alpha_4+\sqrt{\alpha_8},\\ \alpha_{13} &= \alpha_5-(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}).\end{aligned} \qquad (15)$$

In the rather more special case of $\alpha_3=0$,

$$\lim_{\alpha_3\to0}P_n^{\left(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-2\alpha_3 s) = L_n^{\alpha_{10}-1}(\alpha_{11}s), \qquad (16a)$$
$$\lim_{\alpha_3\to0}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}} = \mathrm{e}^{\alpha_{13}s}, \qquad (16b)$$

and, from Eq. (14), we find for the wave function

$$\psi=s^{\alpha_{12}}\mathrm{e}^{\alpha_{13}s}L_n^{\alpha_{10}-1}(\alpha_{11}s). \qquad (17)$$

4 Solution of equations

4.1 Solution of the radial equation

Let us first introduce the new variables $x=r–R$, $\alpha=\frac{1}{a}$ and consider the parameters

$$\eta_r = 2V_0(E_{nlm}+M),\quad \zeta_r=(1/4-l^2)/R^2,$$
$$\gamma_r = -M^2+E_{nlm}^2. \qquad (18)$$

As our resulting equation cannot be exactly solved, we have to proceed on an approximate basis. We introduce the acceptable replacement [35–37]

$$\frac{1/4-l^2}{r^2} = \frac{1/4-l^2}{R^2\left(1+\frac{x}{R}\right)^2}=\frac{1/4-l^2}{R^2}\left[D_0+\frac{D_1}{1+q\exp(\alpha x)}+\frac{D_2}{(1+q\exp(\alpha x))^2}\right], \qquad (19)$$

where

$$\begin{aligned}D_0 &= 1-\frac{(1+q)^2}{\alpha Rq^2}+\left(1-\frac{3}{\alpha R}\right)\\ D_1 &= \frac{(1+q)^2}{\alpha Rq^2}\left(-1+3q-\frac{6(1+q)}{\alpha R}\right)\\ D_2 &= \frac{(1+q)^3}{\alpha Rq^2}\left(\frac{(1-q)}{2}+\frac{3(1+q)}{\alpha R}\right),\end{aligned} \qquad (20)$$

to arrive at

$$\frac{\mathrm{d}^2R_{nlm}(x)}{\mathrm{d}x^2}+\left[\frac{\eta_r}{1+q\exp(\alpha x)}+\zeta_r\left[D_0+\frac{D_1}{1+q\exp(\alpha x)}+\frac{D_2}{(1+q\exp(\alpha x))^2}\right]+\gamma_r\right]R_{nlm}(x)=0. \tag{21}$$

The transformation $y=\frac{1}{1+q\exp(\alpha x)}$ brings Eq. (21) into the form

$$\frac{\mathrm{d}^2R_{nlm}(y)}{\mathrm{d}y^2}+\frac{1-2y}{y(1-y)}\frac{\mathrm{d}R_{nlm}(y)}{\mathrm{d}y}+\frac{1}{y^2(1-y)^2}\left[\frac{\zeta_rD_2}{\alpha^2}y^2+\left(\frac{\eta_r+\zeta_rD_1}{\alpha^2}\right)y+\frac{\zeta_rD_0+\gamma_r}{\alpha^2}\right]R_{nlm}(y)=0. \tag{22}$$

By comparing Eq. (22) with Eq. (12), we find

$$\begin{aligned}&\alpha_1=1,\alpha_2=2,\ \alpha_3=1,\ \xi_1=-\frac{\zeta_rD_2}{\alpha^2},\\ &\xi_2=\frac{\eta_r+\zeta_rD_1}{\alpha^2},\ \xi_3=\frac{-\zeta_rD_0-\gamma_r}{\alpha^2},\\ &\alpha_4=0,\ \alpha_5=0,\ \alpha_6=\xi_1,\ \alpha_7=-\xi_2,\\ &\alpha_8=\xi_3,\ \alpha_9=\xi_1-\xi_2+\xi_3,\ \alpha_{10}=1+2\sqrt{\xi_3},\\ &\alpha_{11}=2+2(\sqrt{\xi_1-\xi_2+\xi_3}+\sqrt{\xi_3}),\\ &\alpha_{12}=\sqrt{\xi_3},\ \alpha_{13}=-(\sqrt{\xi_1-\xi_2+\xi_3}+\sqrt{\xi_3}),\end{aligned} \tag{23}$$

which, from Eq. (14), determines the radial energy relation as follows

$$2n+(2n+1)\left(\sqrt{-\frac{\zeta_rD_2}{\alpha^2}-\left(\frac{\eta_r+\zeta_rD_1}{\alpha^2}\right)+\frac{-\zeta_rD_0-\gamma_r}{\alpha^2}}+\sqrt{\frac{-\zeta_rD_0-\gamma_r}{\alpha^2}}\right)+n(n-1)-\left(\frac{\eta_r+\zeta_rD_1}{\alpha^2}\right)+2\left(\frac{-\zeta_rD_0-\gamma_r}{\alpha^2}\right)$$
$$+2\sqrt{\left(\frac{-\zeta_rD_0-\gamma_r}{\alpha^2}\right)\left(-\frac{\zeta_rD_2}{\alpha^2}-\left(\frac{\eta_r+\zeta_rD_1}{\alpha^2}\right)+\frac{-\zeta_rD_0-\gamma_r}{\alpha^2}\right)}=0. \tag{24}$$

From Eq. (17), the radial eigenfunction is simply found to be

$$R_{nlm}(r)\ =\ N_n\left(\frac{1}{1+q\exp\left(\frac{r-R}{a}\right)}\right)^{\sqrt{\frac{-\zeta_rD_0-\gamma_r}{\alpha^2}}}\left(1-\frac{1}{1+q\exp\left(\frac{r-R}{a}\right)}\right)^{\sqrt{-\frac{\zeta_rD_2}{\alpha^2}-\left(\frac{\eta_r+\zeta_rD_1}{\alpha^2}\right)+\frac{-\zeta_rD_0-\gamma_r}{\alpha^2}}}$$
$$P_n^{\left(2\sqrt{\frac{-\zeta_rD_0-\gamma_r}{\alpha^2}},2\sqrt{-\frac{\zeta_rD_2}{\alpha^2}-\left(\frac{\eta_r+\zeta_rD_1}{\alpha^2}\right)+\frac{-\zeta_rD_0-\gamma_r}{\alpha^2}}\right)}\left(1-\frac{2}{1+q\exp\left(\frac{r-R}{a}\right)}\right). \tag{25}$$

4.2 Solution of the polar equation

In this case, we have

$$\frac{\mathrm{d}^2H_l(\theta)}{\mathrm{d}\theta^2}+\left[\frac{\eta_\theta}{\sin^2\theta}+\zeta_\theta\right]H_l(\theta)=0, \tag{26}$$

where

$$\begin{aligned}\eta_\theta\ &=\ 1/4-2\alpha(E_{nlm}+M)-2\beta(E_{nlm}+M)-m^2,\\ \zeta_\theta\ &=\ 2\beta(E_{nlm}+M)+l^2.\end{aligned} \tag{27}$$

The transformation $z=\sin^2\theta$, transforms Eq. (27) to

$$\frac{\mathrm{d}^2H_l(z)}{\mathrm{d}z^2}+\frac{\left(\frac{1}{2}-z\right)}{z(1-z)}\frac{\mathrm{d}H_l(z)}{\mathrm{d}z}+\frac{1}{(z(1-z))^2}\left(\frac{-\zeta_\theta z^2}{4}+\frac{(-\eta_\theta+\zeta_\theta)z}{4}+\frac{\eta_\theta}{4}\right)H_l(z)=0. \tag{28}$$

By comparing Eq. (28) with Eq. (12), we find

$$\begin{aligned}\alpha_1\ &=\ \frac{1}{2},\alpha_2=1,\alpha_3=1,\xi_1=\frac{\zeta_\theta}{4},\\ \xi_2\ &=\ \frac{(-\eta_\theta+\zeta_\theta)z}{4},\xi_3=-\frac{\eta_\theta}{4},\alpha_4=\frac{1}{4},\\ \alpha_5\ &=\ -\frac{1}{2},\alpha_6=\frac{1}{4}+\xi_1,\alpha_7=-\frac{1}{4}-\xi_2,\alpha_8=\frac{1}{16}+\xi_3,\\ \alpha_9\ &=\ \frac{1}{16}+\xi_1-\xi_2+\xi_3,\alpha_{10}=1+2\sqrt{\frac{1}{16}+\xi_3},\\ \alpha_{11}\ &=\ 2+2\left(\sqrt{\frac{1}{16}+\xi_1-\xi_2+\xi_3}+\sqrt{\frac{1}{16}+\xi_3}\right),\\ \alpha_{12}\ &=\ \frac{1}{4}+\sqrt{\frac{1}{16}+\xi_3},\end{aligned}$$

$$\alpha_{13} = -\frac{1}{2}-\left(\sqrt{\frac{1}{16}+\xi_1-\xi_2+\xi_3}+\sqrt{\frac{1}{16}+\xi_3}\right). \quad (29)$$

Thus, replacing Eq. (29) in Eq. (14), we find the corresponding energy relation as

$$n+\frac{(2n+1)}{2}+(2n+1)\left(\sqrt{\frac{1}{16}+\frac{\zeta_\theta}{4}-\frac{(-\eta_\theta+\zeta_\theta)}{4}-\frac{\eta_\theta}{4}}+\sqrt{\frac{1}{16}-\frac{\eta_\theta}{4}}\right)+n(n-1)-\frac{1}{4}-\frac{(-\eta_\theta+\zeta_\theta)}{4}+2\left(\frac{1}{16}-\frac{\eta_\theta}{4}\right)+2\sqrt{\left(\frac{1}{16}-\frac{\eta_\theta}{4}\right)\left(\frac{1}{16}+\frac{\zeta_\theta}{4}-\frac{(-\eta_\theta+\zeta_\theta)}{4}-\frac{\eta_\theta}{4}\right)}=0. \quad (30)$$

Substituting Eq. (29) into Eq. (13), we obtain the wave function as

$$H_l(\theta) = N_l(\sin^2\theta)^{\left(\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{\eta_\theta}{4}}\right)}(1-\sin^2\theta)^{1/2}\times P_n^{\left(2\sqrt{\frac{1}{16}-\frac{\eta_\theta}{4}},\frac{1}{2}\right)}(1-2\sin^2\theta). \quad (31)$$

According to the following equation

$$P_n^{(A,B)}(1-2x) = \frac{\Gamma(n+A+1)}{n!\Gamma(A+1)}\times{}_2F_1(-n,n+A+B+1;A+1;x). \quad (32)$$

Equation (31) can be rewritten in the following form

$$H_l(\theta) = N_l(\sin^2\theta)^{\left(\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{\eta_\theta}{4}}\right)}(1-\sin^2\theta)^{1/2}\times{}_2F_1\left(-n,n+2\sqrt{\frac{1}{16}-\frac{\eta_\theta}{4}}+\frac{3}{2};2\sqrt{\frac{1}{16}-\frac{\eta_\theta}{4}}+1;\sin^2\theta\right). \quad (33)$$

Therefore, $\varphi_{nlm}(r,\theta,\phi)$ is

$$\varphi_{nlm}(\vec{r}) = \frac{1}{\sqrt{2\pi}}\frac{R_{nlm}(r)}{r}\frac{H_l(\theta)}{(\sin\theta)^{1/2}}\mathrm{e}^{\mathrm{i}m\phi}$$
$$= N_{nlm}\frac{1}{r}\left[\left(\frac{1}{1+q\exp\left(\frac{r-R}{a}\right)}\right)^{\sqrt{\frac{-\zeta_r D_0-\gamma_r}{\alpha^2}}}\left(1-\frac{1}{1+q\exp\left(\frac{r-R}{a}\right)}\right)^{\sqrt{-\frac{\zeta_r D_2}{\alpha^2}-\left(\frac{\eta_r+\zeta_r D_1}{\alpha^2}\right)+\frac{-\zeta_r D_0-\gamma_r}{\alpha^2}}}\times P_n^{\left(2\sqrt{\frac{-\zeta_r D_0-\gamma_r}{\alpha^2}},2\sqrt{-\frac{\zeta_r D_2}{\alpha^2}-\left(\frac{\eta_r+\zeta_r D_1}{\alpha^2}\right)+\frac{-\zeta_r D_0-\gamma_r}{\alpha^2}}\right)}\left(1-\frac{2}{1+q\exp\left(\frac{r-R}{a}\right)}\right)\right]$$
$$\times\frac{1}{(\sin\theta)^{1/2}}\left[(\sin^2\theta)^{\left(\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{\eta_\theta}{4}}\right)}(1-\sin^2\theta)^{1/2}P_n^{\left(2\sqrt{\frac{1}{16}-\frac{\eta_\theta}{4}},\frac{1}{2}\right)}(1-2\sin^2\theta)\right]\frac{\exp(\mathrm{i}m\phi)}{\sqrt{2\pi}}, \quad (34)$$

and the total wave function, from Eq. (7b), is

$$\Psi_{nlm}(r,\theta,\phi) = N_{nlm}\left(\begin{array}{c}1\\ \frac{\sigma.p}{E_{nlm}+M}\end{array}\right)\frac{1}{r}\left[\left(\frac{1}{1+q\exp\left(\frac{r-R}{a}\right)}\right)^{\sqrt{\frac{-\zeta_r D_0-\gamma_r}{\alpha^2}}}\right.$$
$$\times\left(1-\frac{1}{1+q\exp\left(\frac{r-R}{a}\right)}\right)^{\sqrt{-\frac{\zeta_r D_2}{\alpha^2}-\left(\frac{\eta_r+\zeta_r D_1}{\alpha^2}\right)+\frac{-\zeta_r D_0-\gamma_r}{\alpha^2}}}P_n^{\left(2\sqrt{\frac{-\zeta_r D_0-\gamma_r}{\alpha^2}},2\sqrt{-\frac{\zeta_r D_2}{\alpha^2}-\left(\frac{\eta_r+\zeta_r D_1}{\alpha^2}\right)+\frac{-\zeta_r D_0-\gamma_r}{\alpha^2}}\right)}$$

$$\times\left(1-\frac{2}{1+q\exp\left(\dfrac{r-R}{a}\right)}\right)\Bigg]\frac{1}{(\sin\theta)^{1/2}}\left[(\sin^2\theta)^{\left(\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{\eta_\theta}{4}}\right)}(1-\sin^2\theta)^{1/2}\right.$$

$$\left.\times P_n^{\left(2\sqrt{\frac{1}{16}-\frac{\eta_\theta}{4}},\frac{1}{2}\right)}(1-2\sin^2\theta)\right]\frac{\exp(\mathrm{i}m\phi)}{\sqrt{2\pi}}. \tag{35}$$

Putting $\alpha=\beta=0$ yields the results of previously published papers [38].

5 Conclusion

We solved the radial, polar and azimuthal parts of the Dirac equation for the central Woods-Saxon potential modified with a physical ring-shaped potential by using the Nikiforov-Uvarov (NU) method. Our solution yields the results of the Woods-Saxon potential for special cases of the parameters and can be directly applied to the study of deformed nuclei.

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