Study of the total reaction cross section via QMD *

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Abstract: This paper presents a new empirical formula to calculate the average nucleon-nucleon (N-N) collision number for the total reaction cross sections (σ_R). Based on the initial average N-N collision number calculated by quantum molecular dynamics (QMD), quantum correction and Coulomb correction are taken into account within it. The average N-N collision number is calculated by this empirical formula. The total reaction cross sections are obtained within the framework of the Glauber theory. σ_R of ^{23}Al+^{12}C, ^{24}Al+^{12}C, ^{25}Al+^{12}C, ^{26}Al+^{12}C and ^{27}Al+^{12}C are calculated in the range of low energy. We also calculate the σ_R of ^{27}Al+^{12}C with different incident energies. The calculated σ_R are compared with the experimental data and the results of Glauber theory including the σ_R of both spherical nuclear and deformed nuclear. It is seen that the calculated σ_R are larger than σ_R of spherical nuclear and smaller than σ_R of deformed nuclear, whereas the results agree well with the experimental data in low-energy range.

Key words: total reaction cross section, average N-N collision number, impact parameter

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1 Introduction

With the development of a large-scale radioactive nuclear beam device, the study of nuclear physics has reached a new stage. In recent years, the total reaction cross section σ_R is one of the most fundamental quantities characterizing a nuclear reaction. It has been extensively studied both theoretically and experimentally. The knowledge that is taken from the total reaction cross section presents some nuclear characteristics, such as nuclear size, nuclear deformation, interaction potential between nucleons, the distribution of nucleons and so on.

Based on the individual nucleon-nucleon (N-N) collisions in the overlap volume of the colliding nuclei, the Glauber theory is a useful tool to calculate σ_R at high energies [1]. Using the Glauber theory, Urata calculated the reaction cross section for the deformed halo nucleus ^{31}Ne, and suggested that the ^{31}Ne had the ^{30}Ne+n structure [2]. The optical limit of the Glauber theory has been extensively used. For the reactions induced by stable nuclei, the results of the Glauber theory agree with the experimental data well. Considering the total cross section of the reaction medium effect and limited range interaction, Ismail calculated the total cross section of ^{12}C+^{238}U by using the optical limit of the Glauber theory [3]; the results agree with the experimental data excellently. Rashdan investigated the shape, deformation, and orientation dependence as well as in-medium effects for the reaction cross sections of ^{16,19}C+C systems within the optical limit of the Glauber theory [4]. Recently, taking advantage of the Glauber theory as well as Skyrme-Hartree-Fock densities, Horiuchi determined the nuclear density distribution of a wide range of nuclei self-consistently without assuming any spatial symmetry, and analyzed the total reaction cross sections for Ne, Mg, Si and S isotopes [5].

Application of the Glauber theory is successful at high energies, while it's not compatible with experimental data well in the low-energy range. As a nuclear transport theory, quantum molecular dynamics (QMD) has been extensively used in the intermediate-energy range [6]. Cozma applied QMD to investigate the in-medium N-N cross sections on the dynamics of heavy-ion collisions at intermediate energies. The proton and neutron elliptic flows show a monotonous dependence on the variable x that parameterizes the various asy-EOS employed, but in opposite directions for protons and neutrons respectively [7]. Based on QMD, Ma calculated the total reaction cross sections of ^{12}C+^{12}C, ^{12}C+^{27}Al, ^{14}N+^{64}Cu, ^{11}Li+^{12}C, ^{11}Be+^{27}Al, and found that σ_R are sensitive to the nuclear equation of state for E (incident energy) $<$100 MeV/nucleon and also sensitive to the in-medium nucleon-nucleon cross section in the whole researched energy range [8]. Within the framework of QMD model, it

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is easy to obtain the average N-N collision number and investigate the dynamics and quantum effects of the total reaction cross section. In the current research, based on the collision number calculated by QMD at impact parameter b=0 fm, we propose an empirical formula as a function of the impact parameter to calculate the average N-N collision number at any b. Combing the Glauber theory, the total cross reaction could be easily obtained.

2 The model

The Glauber theory is a useful tool to investigate the total reaction cross section in high-energy range (>1 GeV). Based on the high-energy approximation, the multiple scattering theory of Glauber assumes that the nucleons are frozen in their initial position during the process of reaction. According to it, the total cross section is given by [1]:

$$\sigma_{\mathrm{Rt}}(E_0)=\sigma_{\mathrm{R}}^{(N-A)}(E_0)=\sum_{n\geqslant 1}\sigma_n(E_0)=\int \mathrm{d}^2b(1-\mathrm{e}^{-\sigma T(b)}), \tag{1}$$

where $T(b)=\int_{-\infty}^{+\infty}\rho_{\mathrm{t}}(b,z)\mathrm{d}z=\int_{-\infty}^{+\infty}\rho_{\mathrm{t}}(\sqrt{b^2+z^2})\mathrm{d}z$, σ represents the N-N cross section. The Glauber theory, which is based on the individual nucleon-nucleon (N-N) collisions in the overlap volume of the colliding nuclei, is a useful tool to calculate σ_{R} at high energies, while its results and experimental data don't fit well in the low-energy range.

Taking into account both the quantum correction and Coulomb correction in low-energy range, according to the Glauber theory, the total cross section is given by [9–11]:

$$\sigma_{\mathrm{Rt}}(E_0) = \left\{\int \mathrm{d}^2b(1-\mathrm{e}^{-\sigma_{\mathrm{NN}}T(b)})\right\}\{(1+1/k_0R_{\mathrm{t}})^2 \times(1-V_{\mathrm{C}}(R_{\mathrm{t}})/E_0)\}, \tag{2}$$

where $k_0^2=2ME_0$, M is the mass of incident nuclear, E_0 represents the energy of incident nuclear, $1/k_0$ is the De Broglie wavelength of the incident nucleon, R_{t} is the radius of the target nuclear. $V_{\mathrm{c}}(R_{\mathrm{t}})$ is the Coulomb interaction potential between the projectile and the target.

Based on Formula (2), Guo developed a formula to calculate the total reaction cross sections [12]:

$$\sigma_{\mathrm{R}}(E) = \left\{\int \mathrm{d}^2b(1-\mathrm{e}^{-\sigma_{\mathrm{RT}}T_{\mathrm{eff}}(b)})\right\}\{(1+1/kR)^2 \times(1-V_{\mathrm{C}}(R)/(A_{\mathrm{p}}+A_{\mathrm{t}})E)\}, \tag{3}$$

where $T_{\mathrm{eff}}(b)=\int_{-\infty}^{+\infty}\rho_{\mathrm{eff}}(b,z)\mathrm{d}z=\int_{-\infty}^{+\infty}\rho_{\mathrm{eff}}(\sqrt{b^2+z^2})\mathrm{d}z$, ρ_{eff} is the effective density, and it is calculated by:

$$\rho_{\mathrm{eff}}(R)=\rho_{\mathrm{eff}}(\sqrt{b^2+z^2})=\frac{\int\rho_{\mathrm{t}}(r)\rho_{\mathrm{p}}(R-r)\mathrm{d}^3r}{\int\rho_{\mathrm{t}}(r)\mathrm{d}^3r}. \tag{4}$$

In Formula (4), ρ_{p} and ρ_{t} represent the density of projectile and target respectively.

The Glauber models present difficulties in investigating the dynamics and quantum effects on the total reaction cross sections. In order to solve this problem, Ma calculated the total reaction cross sections by using QMD [8]. In Formula (1), $\sigma\times T(b)$ is replaced by $N(b)$ (average N-N collision number). Within the framework of QMD model, N can be obtained as a function of the impact parameter by assuming a reasonable parameter of the N-N cross section σ_{NN}, and N at every impact parameter (b) could be calculated theoretically [13]. In fact, with a continuously changing b, it is a problem and cannot be achieved to calculate N at any b.

We propose that N is in accordance with the exponential decay and its attenuation coefficient is:

$$\lambda(b) = \exp\left\{-\frac{3.789\times b}{(r_1+r_2)\left[1+1.51\times\left(\frac{|N_1-Z_1|}{N_1+Z_1}+\frac{|N_2-Z_2|}{N_2+Z_2}\right)\right]}\right\}, \tag{5}$$

where N_1 and Z_1 represent the numbers of neutrons and protons for projectile, N_2 and Z_2 represent the numbers of neutrons and protons for target. r_1 and r_2 represent the root mean square radius (r_{rms}) of projectile and target. $\frac{|N_1-Z_1|}{N_1+Z_1}$ is the absolute value of relative neutron excess (δ). From Formula (5), it is clear that the attenuation coefficient ($\lambda(b)$) is large with large r_{rms} of projectile and target. $|\delta|$ has the same influence of $\lambda(b)$, which shows the fact that nucleons combine more unstably for nucleus with larger $|\delta|$.

Considering Formula (5), an empirical formula as a function of the impact parameter can be used to calculate N. It is:

$$N(b)=N(0)\lambda(b), \tag{6}$$

where $N(0)$ is the collision number at b=0 fm, and it is calculated by QMD [6]. In the initialization of the coordinate space, we distribute all the nucleons randomly in a sphere. The distribution of density of nucleus is the contribution of RMF (relativistic mean field), the parameter HS is used in RMF. Collision symmetric potential used in QMD is

$$U^{\mathrm{sym}}=32\left(\frac{\rho}{\rho_0}\right)^2\left[\frac{\rho_{\mathrm{n}}-\rho_{\mathrm{p}}}{\rho}\tau_{\mathrm{z}}+\frac{1}{2}\left(\frac{\rho_{\mathrm{n}}-\rho_{\mathrm{p}}}{\rho}\right)^2\right],$$

where ρ and ρ_0 are the nuclear density and its normal value. ρ_n and ρ_p represent the density of the neutron and proton. $\tau_z=1$ for neutron and $\tau_z=-1$ for proton.

Combining Formula (1) and (6), we can easily obtain σ_R by:

$$\sigma_R=2\pi\int b\mathrm{d}b\{1-\exp[-N(b)]\}. \quad (7)$$

Quantum correction and Coulomb correction are taken into account within the framework of QMD. In this paper, we use the above procedures to investigate σ_R of reactions. N (b) can be obtained from Formula (6).

3 Results

r_{rms} and δ of the incident nuclear and the target nuclear calculated by RMF are shown in Table 1, the parameter HS is used in RMF.

Table 1. The root mean square radius and the relative neutron excess for the reaction nucleus.

nuclear	^{23}Al	^{24}Al	^{25}Al	^{26}Al	^{27}Al	^{12}C
r_{rms}/fm	2.872	2.8762	2.8853	2.8975	2.9118	2.3562
δ	−0.1304	−0.0833	−0.04	0	0.037	0

Figure 1 shows the impact parameter and time dependence of collision number. Five curves are the contributions of ^{23}Al+^{12}C, ^{24}Al+^{12}C, ^{25}Al+^{12}C, ^{26}Al+^{12}C and ^{27}Al+^{12}C with the incident energies of 35.9, 32.8, 27.4, 24.7 and 22.0 MeV/u, respectively. Fig. 1(a) shows the N-N collision number distribution at different impact parameters b. All the lines in Fig. 1(a) are the contribution of Formula (2). From the Fig. 1(a), the N-N collision numbers for all reactions decrease with the increase of impact parameter, the number of N-N collisions for ^{27}Al+^{12}C decreases more rapidly than others. It is noticed that the N-N collision numbers have greater difference at the small impact parameter b, while as the impact parameter increases the difference becomes smaller. The results show that the collision numbers depend on the masses of the colliding systems at small impact parameters, and they depend on both the masses of the colliding systems and $|\delta|$ at large impact parameter. For ^{23}Al+^{12}C, ^{24}Al+^{12}C, ^{25}Al+^{12}C, ^{26}Al+^{12}C and ^{27}Al+^{12}C, the masses of the colliding systems increase, while the $|\delta|$ of nuclear reaction decreases in turn from ^{23}Al to ^{26}Al.

Figure 1(b) shows the time dependence of N-N collision number at b=0 fm. From Fig. 1(b), it is clearly shown that the larger-mass systems have more collision numbers. It is also seen from Fig. 1(b) that the larger mass the reaction system has, the later the peak appears, which suggests that isotopes have more neutrons, and their nucleons are combined more closely.

Figure 2 shows the relationship between the reaction cross sections and impact parameter b. Five curves represent the same as Fig. 1. It is clearly seen that σ_R in different colliding systems are almost the same at impact parameters $b \leqslant 4$ fm, while there is a difference between them at $b \geqslant 4$ fm. For the colliding system of ^{23}Al+^{12}C, the δ of ^{23}Al is larger than others and there is a bigger influence of Coulomb potential than other isotopes, its nucleons distribute in a wider range than others, σ_R of it is larger than others at $b \geqslant 7$ fm. It is noticed that the peaks of σ_R for all of the colliding systems appear at 5 fm $\leqslant b \leqslant$6 fm.

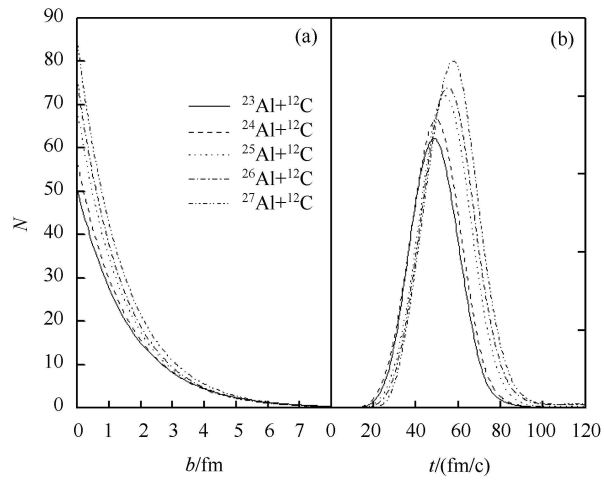

Fig. 1. (a) The number of collisions changed with impact parameter; (b) The time evolution of the average collision number N at impact parameter b=0 fm.

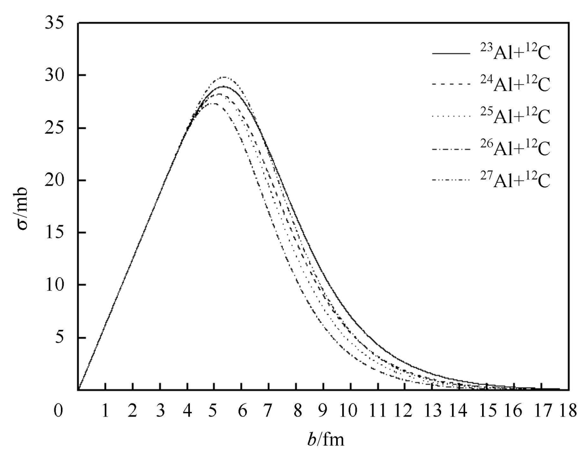

Fig. 2. The relationship between the reaction cross sections and impact parameter b.

Figure 3 shows the total reaction cross sections of the Al isotopes. The collision systems are the same as Fig. 1. It is clearly seen that the data calculated by the Glauber theory don't accord with the experimental data very well at those energies. σ_R are always underestimated by a large amount for the spherical nuclei induced reaction in Glauber calculation and overestimated for the deformed nuclei; the deformation effect makes σ_R larger and the difference between calculated data and experimental data smaller, and there are still big differences after taking the deformation effect into account. On the other hand, the σ_R calculated by Formula (6) agrees with the experimental data. It is noticed that σ_R of $^{25}Al+^{12}C$ and $^{26}Al+^{12}C$ are smaller than σ_R of $^{23}Al+^{12}C$ and $^{24}Al+^{12}C$, which shows that the nucleons in ^{25}Al and ^{26}Al are combined more closely than in ^{23}Al and ^{24}Al. It is seen that the results calculated by the Glauber theory don't reflect this phenomenon and the results calculated by Formula (6) do it easily in low-energy range. The experimental data are taken from Ref. [14].

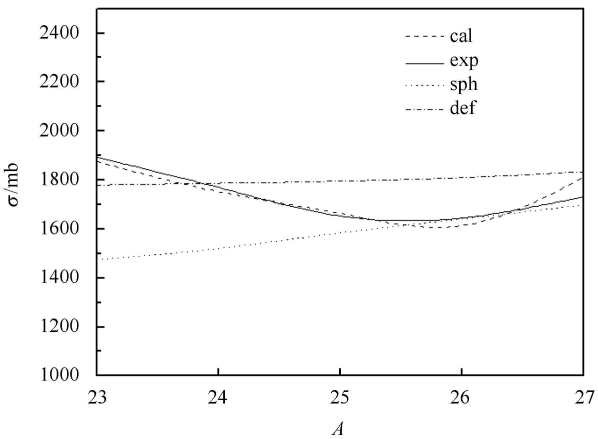

Fig. 3. Total reaction cross section of the Al isotopes.

Figure 4 shows the relationship between the total reaction cross section and the incident energy. The solid line is the contribution of experimental data, the dashed line represents the results of Formula (6), the dotted line and dashed dotted line are the contribution of the Glauber theory and they represent the results for the spherical nuclear and deformed nuclear reactions, respectively. σ_R is always underestimated for the spherical nuclei reaction and overestimated for the deformed nuclei reaction in the Glauber calculation. However, the σ_R calculated by Formula (6) are in agreement with experimental data, especially in low-energy range. The experimental data are taken from Ref. [15].

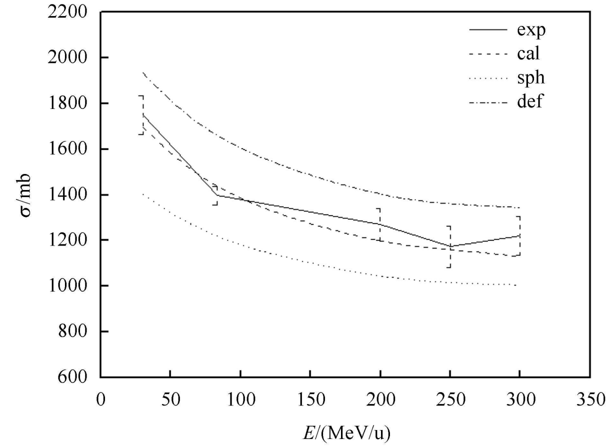

Fig. 4. The relationship between the total reaction cross section and the incident energy.

4 Conclusions

We propose an empirical formula as a function of the impact parameter to calculate the average N-N collision number. This formula includes an initial average N-N collision number, it is the average N-N collision number at b=0 fm and is obtained by the quantum molecular dynamics (QMD). Then we calculate the total reaction cross sections by Formula (7) and compare its results with the experimental data. In this paper, σ_R of $^{23}Al+^{12}C$, $^{24}Al+^{12}C$, $^{25}Al+^{12}C$, $^{26}Al+^{12}C$ and $^{27}Al+^{12}C$ are calculated, and they agree well with the experimental results. We also calculate the total reaction cross section of $^{27}Al+^{12}C$ with different incident energies and compare it with the experimental data. It is proved that the empirical formula is a good tool to investigate the total reaction cross section.

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