Spatial anisotropy: a method to study anisotropic flow in heavy ion collisions*

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Abstract: We introduce a method to study anisotropic flow parameter v_n as a collective probe to Quark Gluon Plasma in relativistic heavy ion collisions. The emphasis is put on the use of the Fourier expansion of initial spatial azimuthal distributions of participant nucleons in the overlapped region. The coefficients ε_n of Fourier expansion are called the spatial anisotropy parameter for the n-th harmonic. We propose that collective dynamics can be studied by v_n/ε_n. In this paper, we will discuss in particular the second ($n=2$) and the fourth ($n=4$) harmonics.

Key words: Fourier expansion, spatial anisotropy parameter, collective dynamics, thermalization

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1 Introduction

The main goal of heavy ion collision experiments is to search for de-confined nuclear matter Quark Gluon Plasma (QGP) in the laboratory [1]. Recent results at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory indicate that the dense and rapidly thermalizing matter has been formed [2, 3]. Collective anisotropic flow as one of the bulk properties of QGP can provide collective dynamics information, such as degree of freedom and thermalization at the early stage [4–6]. This is based on the following arguments [7–9]: in noncentral heavy-ion collisions, the overlapped area has a spatial azimuthal anisotropy. Rescatterings among the system's constituents convert the initial coordinate-space anisotropy to a momentum-space anisotropy. The spatial anisotropy decreases as the system evolves so that the momentum-space anisotropy is sensitive to the early phase of the evolution, when the spatial anisotropy is large.

In experiments, anisotropic flow is studied by the Fourier expansion of momentum space azimuthal angle (ϕ) distributions of hadrons with respect to the reaction plane [5],

$$E\frac{\mathrm{d}^3N}{\mathrm{d}^3p}=\frac{1}{2\pi}\frac{\mathrm{d}^2N}{p_{\mathrm{T}}\mathrm{d}p_{\mathrm{T}}\mathrm{d}y}\left(1+\sum_{n=1}^{\infty}2v_n\cos[n(\phi-\Psi_{\mathrm{r}})]\right), \tag{1}$$

where Ψ_{r} is the azimuthal angle of the reaction plane. The ϕ distribution in momentum space reflects the global picture of anisotropic flow. By expanding in Fourier series, the global anisotropic flow is described by the sum of anisotropic flow for each harmonic. The Fourier coefficients v_n are measured and used to characterize the n-th harmonic azimuthal anisotropy of hadron production. The second and fourth harmonic coefficient v_2 and v_4 are called elliptic flow and deformed flow by the typical shape of its azimuthal anisotropy.

Recently, two important properties have been observed for hadrons' v_2 in 200 GeV Au+Au collisions at RHIC: (i) at low p_{T}, v_2 has a mass ordering for identified hadrons; (ii) at intermediate p_{T}, v_2 of mesons and baryons follows constituent quark number scaling behavior [2, 10–14]. The results indicate that partonic

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collectivity has been reached at RHIC [2, 15, 16]. In the scenario of thermalization, the ratio of the eccentricity to v_2 (v_2/ε) is a constant by hydrodynamic prediction [17]. However, v_2/ε increases with the number of participants in Au+Au and Cu+Cu collisions, implying incomplete thermalization at RHIC [14, 18]. Note that the conclusion is based on v_2 and its centrality dependence only. The thermalization can possibly happen to be reached in most central Au+Au collisions. Different harmonic anisotropic flows are interrelated since they develop under the global collective flow dynamics. The study of different harmonic anisotropic flow can shed more light on collective dynamics and thermalization than only the study of v_2. In this paper, we will introduce a method to study harmonic anisotropy flow, and will especially focus on two large components v_2 and v_4, using the data available at RHIC.

2 Methods

In this section, we introduce a method to analyze all harmonics of anisotropic flow for associated study. It is known that anisotropic flow originates from initial spatial anisotropy. Anisotropic flow is well described by Fourier expansion in momentum space [5]. Each harmonic component denotes a certain type of anisotropic flow. In order to remove the initial geometry effect for each harmonic component, we can write the initial spatial azimuthal distribution of participants in the reaction zone in Fourier expansion in the so -called out-plane perpendicular to the reaction plane,

$$\frac{\mathrm{d}N}{\mathrm{d}\phi_{\mathrm{s}}} \propto 1+\sum_{n=1}^{\infty} 2\varepsilon_n \cos[n(\phi_{\mathrm{s}}-\Psi_{\mathrm{rs}})], \tag{2}$$

$$\varepsilon_n = \langle \cos[n(\phi_{\mathrm{s}}-\Psi_{\mathrm{rs}})]\rangle, \tag{3}$$

where ϕ_{s} denotes the azimuthal angle of participant nucleon, Ψ_{rs} denotes the azimuthal angle of out-plane. The coefficient ε_n is called spatial anisotropy parameter and can be written in the form of Eq. (3), which is similar to $v_n = \langle\cos[n(\phi - \Psi_{\mathrm{r}})]\rangle$. This definition makes long principal axis of spatial anisotropy along the direction of out-plane, thus ε_n is typically positive.

To learn the transverse collective dynamics from some or all harmonics of anisotropic flow, we suggest to analyze them together. The ratio v_n/ε_n to some extent reflects the strength of collective expansion for the n-th harmonic component. Due to the different sensitivity for each harmonic component to the system evolution, the associated study of v_n/ε_n for some or all of them can shed light on the information such as collective dynamics and thermalization, which are not achievable by only studying one component.

In this paper, we will investigate v_2/ε_2 and v_4/ε_4 as a function of collision centrality with available data for v_2 and v_4.

3 Spatial anisotropy calculation

There have been no direct experimental measurements of initial spatial anisotropy until now. The Monte Carlo Glauber model [19, 20] is used to calculate the spatial anisotropy parameter. The nucleon distribution in nucleus is sampled according to the Wood-saxon distribution [19]. The spatial anisotropy parameter is determined by the participants of collisions. Assuming spatial anisotropy has perfect reflection symmetry regarding the reaction plane, ε_2 and ε_4 are calculated in the reaction plane frame, where the x direction is along the impact parameter. We call them standard spatial anisotropy [21] $\varepsilon_{2\mathrm{std}}$ and $\varepsilon_{4\mathrm{std}}$,

$$\varepsilon_{2\mathrm{std}} = \frac{\langle y^2-x^2\rangle}{\langle y^2+x^2\rangle}, \tag{4}$$

$$\varepsilon_{4\mathrm{std}} = \frac{\langle (y^2-x^2)^2-4x^2y^2\rangle}{\langle (x^2+y^2)^2\rangle}, \tag{5}$$

where the average is over coordinates x and y of participants. For small overlapped regions, fluctuations in nucleon positions may cause the major axis to rotate. Due to the effect, spatial anisotropy calculated in the reaction plane frame is not as large as possible. In order to avoid this case, a new frame, the participant frame, is defined. In the participant frame, the x and y axis are along the short and long major axis of the overlapped region. The values of spatial anisotropy calculated in the participant frame are maximized. The spatial anisotropy parameter calculated in the participant frame is called the participant spatial anisotropy [21, 22]. In order to compare with v_2 and v_4 measured in the second harmonic event plane[5, 23], we choose the frame to make ε_2 maximal. $\varepsilon_{2\mathrm{part}}$ and $\varepsilon_{4\mathrm{part}}$ are given by

$$\varepsilon_{2\mathrm{part}} = \frac{\langle y'^2-x'^2\rangle}{\langle y'^2+x'^2\rangle}, \tag{6}$$

$$\varepsilon_{4\mathrm{part}} = \frac{\langle (y'^2-x'^2)^2-4x'^2y'^2\rangle}{\langle (x'^2+y'^2)^2\rangle}, \tag{7}$$

where $x' = (x-\bar{x})\cos\varphi + (y-\bar{y})\sin\varphi$, $y' = (y-\bar{y})\cos\varphi - (x-\bar{x})\sin\varphi$. $\bar{x}, \bar{y}$ are the coordinates of center-of-mass of participants in the overlapped region. φ is the rotational angle of x, y axis relative to x', y' axis.

4 Results and discussions

Figure 1 shows the initial spatial anisotropy parameter ε_2 (circles) and ε_4 (squares) as a function of the number of participants for 200 GeV Au+Au collisions. For each one, both values calculated in the reaction plane frame and participant plane frame are shown by open and solid symbols, respectively. We can see the spatial anisotropy in the participant frame is larger than that in the reaction plane frame for a given harmonic. This conforms to the expectation that spatial anisotropy is maximized in the participant frame with nucleon fluctuations modifying the major axis directions of overlapped reaction region. Standard spatial anisotropy first increases and then decreases with $N_{\rm part}$ below and above $N_{\rm part} = 25$, while participant one monotonically increases with $N_{\rm part}$. In the central and mid-central collisions, the value of the participant anisotropy is close to that of the standard one, while in the peripheral collisions participant one is much larger than the standard one. In most peripheral collisions, the fluctuations of participants can largely reduce the magnitude of the spatial anisotropy. It can be seen that the standard spatial anisotropy drops down in the region $N_{\rm part} < 25$. So the centrality dependence of participant anisotropy indicates that it is more reasonable to describe the initial spatial anisotropy, which leads to the anisotropic collective expansion. In the following, we will focus on the results from $\varepsilon_{\rm 2part}$ and $\varepsilon_{\rm 4part}$.

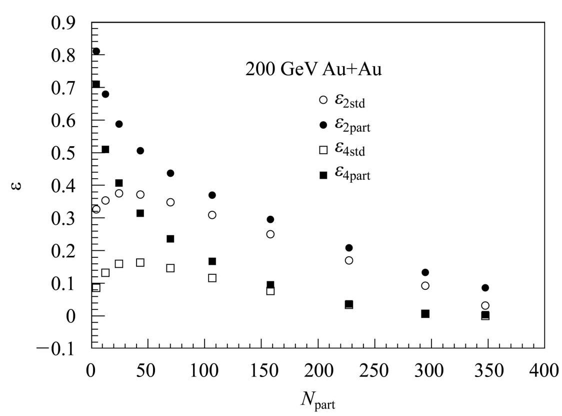

Fig. 1. Initial spatial anisotropy parameter ε_2 (circles) and ε_4 (squares) as a function of the number of participants $N_{\rm part}$ for 200 GeV Au+Au collisions. The values calculated in the reaction plane frame and participant frame are shown by open and solid symbols, respectively.

In heavy ion collisions, the fire ball has a density gradient from inside to outside. Re-scatterings among constituent particles will convert the density gradient to the pressure gradient [2]. $\varepsilon_{\rm 2part}$ and $\varepsilon_{\rm 4part}$ characterize the size of spatial azimuthal anisotropy for the second and fourth harmonic with elliptic-shape and deformed-shape, respectively. Because of re-scatterings, the elliptic-shape and deformed-shape spatial anisotropy will induce the elliptic-shape and deformed-shape pressure gradient anisotropy. Hence, the elliptic-shape and deformed-shape momentum space anisotropy are converted from the corresponding shape of spatial anisotropy. Fig. 1 shows that $\varepsilon_{\rm 4part}$ has sizable values. $\varepsilon_{\rm 4part}$ is smaller than $\varepsilon_{\rm 2part}$ for a given $N_{\rm part}$, which is consistent with smaller v_4 than v_2 [23]. As a function of $N_{\rm part}$, $\varepsilon_{\rm 2part}$ and $\varepsilon_{\rm 4part}$ get larger in more peripheral collisions. Centrality dependence of $\varepsilon_{\rm 2part}$ and $\varepsilon_{\rm 4part}$ are consistent with that of v_2 and v_4.

To study the collective dynamics from centrality dependence of anisotropic flow, the anisotropic flow is divided by spatial anisotropy for each harmonic to remove the geometric effect. The ratio $v_n/\varepsilon_{n\rm part}$ to some extent reflects the strength of expansion for the n-th harmonic. The anisotropic flow of each harmonic is one part of global anisotropic flow in the system evolution. Ideally, the strength of expansion for each harmonic reflects that of the global anisotropic flow. Thus $v_n/\varepsilon_{\rm npart}$ is expected to be identical if re-scatterings are enough to make the flow fully develop. Fig. 2 shows the charged hadron $v_2/\varepsilon_{\rm 2part}$ and $v_4/\varepsilon_{\rm 4part}$ as a function of $N_{\rm part}$ for 200 GeV Au+Au collisions. v_2 and v_4 data are from Ref. [23]. The $v_2/\varepsilon_{\rm 2part}$ and $v_4/\varepsilon_{\rm 4part}$ increase with $N_{\rm part}$. $v_4/\varepsilon_{\rm 4part}$ increases faster than $v_2/\varepsilon_{\rm 2part}$. Conversion from spatial anisotropy to momentum space anisotropy for the fourth harmonic is more sensitive to the centrality than the second harmonic. The more the central collision, the larger the density. So it is more sensitive to the variation in density, which is sensitive to the variation in number of re-scatterings though the fourth harmonic flow has a smaller component than the second harmonic flow. The comparison of $v_2/\varepsilon_{\rm 2part}$ and $v_4/\varepsilon_{\rm 4part}$ can tell us whether and how much the flow will fully develop depending on the number of re-scatterings. In the region of $N_{\rm part} < 250$, $v_4/\varepsilon_{\rm 4part}$ is smaller than $v_2/\varepsilon_{\rm 2part}$. This indicates that the value of density is still small so larger component flow develops to a larger extent than a smaller one. In the region of $N_{\rm part} > 250$, the value of $v_4/\varepsilon_{\rm 4part}$ is close to that of $v_2/\varepsilon_{\rm 2part}$. These two values are consistent within two sigma based on statistical uncertainty. If the flow fully develops in this region, the number of re-scatterings is enough and the thermalization is possibly achieved.

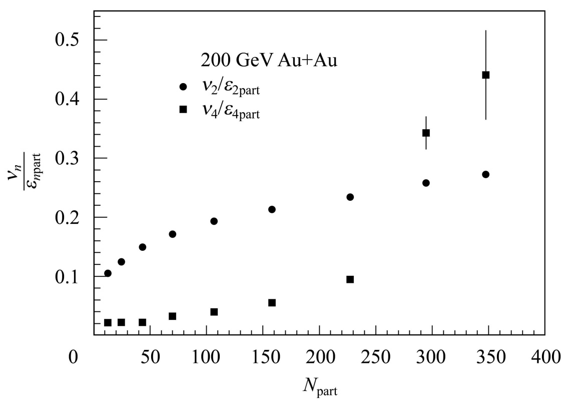

Fig. 2. Charged hadron anisotropic flow scaled by spatial anisotropy as a function of N_{part} for 200 GeV Au + Au collisions. The second harmonic ratios $v_2/\varepsilon_{2\text{part}}$ and the fourth harmonic ratios $v_4/\varepsilon_{4\text{part}}$ are presented by circles and squares, respectively.

Recently, the thermalization issue has been hotly discussed at RHIC [14, 18, 24]. In the case of thermalization, $v_2/\varepsilon_{2\text{part}}$ is a constant [24]. Hydrodynamic calculations with assumptions of local thermalization show low sensitivity to the collision centrality [14, 17]. However, as found in Ref. [18], $v_2/\varepsilon_{2\text{part}}$ in 200 GeV Cu+Cu and Au+Au collisions increases with N_{part}, indicating incomplete thermalization at RHIC, which is obtained due to only the centrality dependence of $v_2/\varepsilon_{2\text{part}}$. This conclusion depends on only the centrality dependence of $v_2/\varepsilon_{2\text{part}}$. In most central Au+Au collisions, the study of consistency of $v_2/\varepsilon_{2\text{part}}$ and $v_4/\varepsilon_{4\text{part}}$ could get information about the collective dynamics, such as thermalization, which cannot be achieved by $v_2/\varepsilon_{2\text{part}}$ only. In order to reach a conclusion, both theoretical and experimental work should be done. For experiments, statistical and systematic uncertainties should be reduced. For theory, work with models with assumptions of thermalization should be provided.

5 Summary

Anisotropic flow is well studied at RHIC. We propose a method to systematically study the anisotropic flow to obtain the information of collective dynamics in relativistic heavy ion collisions. In this method, spatial azimuthal anisotropy is presented as a Fourier series of azimuthal distributions of participant nucleons with respect to the out-plane. In order to remove the initial geometric effect, v_n is divided by ε_n. Collective dynamics can be researched by analyzing v_n/ε_n together. Based on the Monte Carlo data generated with the Glauber model, ε_2 and ε_4 are shown as a function of the number of participant nucleons N_{part}. The variables are calculated in the reaction plane frame and participant frame. Variables in the participant frame ($\varepsilon_{\text{part}}$) should be used to take the effect of fluctuations of nucleon positions into account. $v_2/\varepsilon_{2\text{part}}$ is larger than $v_4/\varepsilon_{4\text{part}}$ in the region of $N_{\text{part}} < 250$. And $v_2/\varepsilon_{2\text{part}}$ and $v_4/\varepsilon_{4\text{part}}$ are close in $N_{\text{part}} > 250$. This shows some clues to thermalization in most central Au + Au collisions. To be conclusive, experimental measurements with reduced statistical and systematical uncertainties should be done and theoretical/model calculations can provide insight.

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