Theoretical investigation of the Gamow-Teller transition across N=40 shell*

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Abstract: The Gamow-Teller transitions for pf shell nuclei with proton number less than 40 and neutron number larger than 40 were believed to be blocked, due to the full filling of the neutron orbit. However, recent experimental research shows that the Gamow-Teller transitions for these kinds of nuclei are not blocked. In this paper, we systematically calculate the GT transition of pf shell nuclei ^{76}Se in different truncations, and the results are compared with experimental results. It is shown that, due to correlations, the believed blocked GT transition occurs, and the shell model calculations reproduce the experimental GT strength. In addition, the electron capture rates in a stellar environment are calculated and discussed.

Key words: Gamow-Teller transition, electron capture rate, shell model

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1 Introduction

The structure of a nucleus in a pre-supernova object plays an important role in the process of a supernova explosion and determines the subsequent evolution [1] of stars. As indicated by numerical simulations, the stellar evolution is mainly dependent on lepton fraction, which is controlled by the weak interaction (especially beta-decay and electron capture). The magnitude of the electron capture rate is the most important factor in the study of the mechanism of supernova explosion (both for Type-Ia and Type-II supernovae) [2–4]. A Type-II supernovae is related to the collapse of a massive star. If the iron core-mass exceeds the Chandrasekhar mass, the degenerate electron gas pressure can't resist the gravitational force, and then the core starts to collapse. On one hand, electron capture can reduce the lepton fraction and then the degenerate electron pressure. On the other hand, electron capture can produce a neutrino which takes away energy and hence accelerates the collapse. A Type-Ia supernovae is thought to be a thermonuclear explosion on an accreting white dwarf and its collapse is believed to be the general relativistic effect. However, the electron capture process is considered to be the reason for the high abundance of some iron isotopes in Type-Ia supernovae [4].

Since the electron capture process is important for the evolution of supernovae and stars, the study of stellar electron capture rates has been the focus of nuclear astrophysics. Based on the Independent Particle Model, Fuller, Fowler and Newman [3] investigated electron capture rates for iron group nuclei. Later, Langanke and Martinez Pinedo [5, 6] improved this work by using a large scale shell model, which was used to calculate the Gamow-Teller strength for nuclei with mass numbers A=45–64. According to the Independent Particle Model [7], GT_+ transitions for nuclei with proton numbers $Z<40$ and neutron numbers $N>40$ are forbidden. This is due to the fact that for these nuclei the neutron orbits are full, and consequently the GT_+ transitions, which change protons into neutrons within the same major shell, are blocked. However, recently, Grewe et al. measured the GT_+ distribution for ^{76}Se, one of the pf shell nuclei which have proton numbers $Z<40$ and neutron numbers $N>40$, by using the (d, ^{2}He) charge-exchange reaction [8]. The results showed that the

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GT transition strength of ^{76}Se, which was considered to be Pauli-blocked according to the Independent Particle Model, is not zero. This shows that, due to nuclear correlations, the GT_+ transitions for nuclei with $Z<40$ and $N>40$ are unblocked, which was pointed out in the Monte Carlo Shell model calculation [9]. In this paper, we will calculate the Gamow-Teller strength of nuclei ^{76}Se with the large scale shell model. The neutron occupation number across the $N=40$ gap and its effect on the Gamow-Teller strength will be investigated and discussed. Based on the calculated Gamow-Teller strength, we calculate the stellar electron capture rates in supernovae environments. The results are also compared with experimental data.

This paper is organized in the following way. The theoretical results and detailed discussions are given in Section 2. The last Section is a short summary.

2 Numerical results and discussions

The shell model is a very successful nuclear model and is widely used in nuclear physics and nuclear astrophysics [4, 5, 10]. In this paper, we will perform the shell-model calculations based on a ^{48}Ca core with the Antoine code [4]. The model spaces we used are full pf ($0f_{7/2}, 1p_{3/2}, 0f_{5/2}, 1p_{1/2}$) shell for protons and the ($1p_{3/2}, 0f_{5/2}, 1p_{1/2}, 0g_{9/2}$) orbits for neutrons. The single-particle energies and the associated effective interaction are based on those used in Refs. [11–13] and have been adopted in Ref. [14]. It should be noted that, in our model spaces, there is no problem with the so-called spurious center-of-mass excitations.

Before we proceed to discuss the Gamow-Teller transitions, we need to see how the model space and interaction can provide a reasonable description of the properties of ^{76}Se. Recently, Schiffer et al measured the different orbital neutron occupation numbers of ^{76}Se [15, 16]. The calculated neutron occupation numbers of different orbits are compared with the experimental data in Table 1 under truncation T=12. In Table 1, the first column is the neutron orbits. The second column is the calculated neutron occupancies from the shell model calculation. The third column contains data from recent experiments [15, 16].

In Table 1, one can see that the calculated neutron occupation number of $0g_{9/2}$ orbit is 5.86, which is very close to the experimental data 5.80±0.30. For $0f_{5/2}$ orbits, the calculated results are very close to the experimental data. But for $1p_{1/2}+1p_{3/2}$ orbits, the calculated result is a little smaller than the experiment. In general, our shell model calculations reproduce the experimental neutron occupation number of ^{76}Se.

Table 1. Shell model neutron occupation numbers are compared with experiment. The experimental data are taken from Refs. [15, 16].

orbit	SM	Exp.
$1p_{1/2}+1p_{3/2}$	3.67	4.41 ± 0.20
$0f_{5/2}$	4.42	3.83 ± 0.40
$0g_{9/2}$	5.86	5.80 ± 0.30

To see how the occupation numbers change with respect to the particles allowed to go into $0g_{9/2}$ orbit, we tabulate the calculated neutron occupation numbers as a function of truncations in Table 2. In Table 2, the first column is the truncation in shell model. Columns 2 to 5 are the shell model calculated neutron occupation numbers. In Table 2, one can see that the neutron occupation numbers in the pf shell decrease with the increase of the truncations. But the occupancies in $0g_{9/2}$ orbit increase with the increase in truncation levels. It is also found in Table 2 that the shell model calculations get the converged neutron occupation numbers for both pf orbits and the $0g_{9/2}$ orbit with truncation T=10 (10p-10h). Under T=12, the calculation becomes completely converged. Therefore, in the following, the shell model results are those obtained with truncation T=12.

Table 2. Neutron occupation numbers from a shell model in different truncations.

truncation	$1p_{3/2}$	$1p_{1/2}$	$0f_{5/2}$	$0g_{9/2}$
T=0	4.00	2.00	6.00	2.00
T=2	3.60	1.15	5.36	3.89
T=4	3.62	1.24	5.06	4.07
T=6	3.47	1.23	4.87	4.41
T=8	2.98	1.03	4.56	5.45
T=10	2.76	0.92	4.43	5.85
T=12	2.75	0.92	4.42	5.86

In Tables 1 and 2, another noticeable phenomenon is that the occupation number of $0g_{9/2}$ is 5.86 rather than 2, which is considered to be the number in the Independent Particle Model. It is also seen that the occupancies for neutron orbits of the pf shell are not full. This means that, due to the correlation, we get an additional 3.86 neutrons in the $0g_{9/2}$ orbit and consequently more neutron holes in the pf shell, which means the blocked GT transition can happen in the the large scale shell model.

We calculate the GT_+ strength distribution for the ground state of ^{76}Se. In Fig. 1, we plot the calculated GT_+ sum for the ground state of ^{76}Se in different excitation energy regions. In Fig. 1, the solid line represents the shell model calculation with T=12. The shell model results are quenched by a factor of $(0.74)^2$ [6]. The dotted-line curve represents experimental data from Ref. [8]. In the figure, one can see that the shell model calculated results are very close to the experimental data. The calculated results are within the error bar of the experiment for most energy regions. Therefore, one can see that our calculations reproduce the measured GT_+ strength of ^{76}Se. In the following, we will discuss the electron capture rates in a stellar environment due to the unblocked GT_+ transitions.

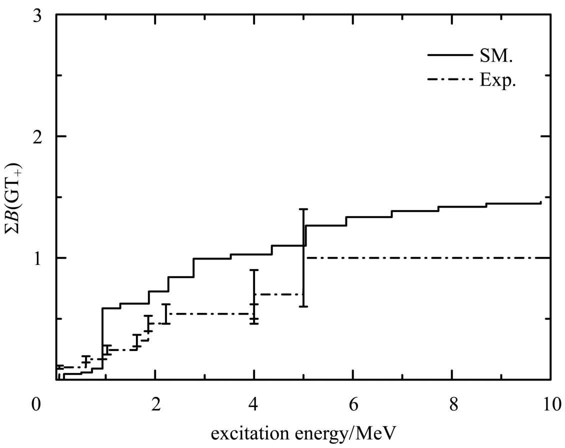

Fig. 1. Shell model calculated B(GT) for the ground state of ^{76}Se are compared with experimental data. The shell model results are quenched by a factor of 0.74.

Based on the calculated GT strength, we calculate the stellar electron capture rates. The detailed formulae of the rate calculation are in Ref. [6]. The rate is written as:

$$\lambda^{\rm ec}=\frac{\ln 2}{K}\sum_i\frac{(2J_i+1){\rm e}^{-Ei/(KT)}}{G(Z,A,T)}\sum_j B_{ij}\varPhi_{ij}^{\rm ec},\qquad(1)$$

where $G(Z,A,T)=\sum_i \exp(-E_i/(KT))$ is the partition function of the parent nucleus, B_{ij} is the reduced transition probability of the nuclear GT transitions. And $\varPhi_{ij}$ is the phase space integral:

$$\varPhi_{ij}^{\rm ec}=\int_{\omega_1}^{\infty}\omega p(Q_{ij}+\omega)^2F(Z,\omega)S_{\rm e}(\omega)\times(1-S_\nu(Q_{ij}+\omega)){\rm d}\omega,\qquad(2)$$

where $F(Z,\omega)$ is the Fermi function and Q_{ij}:

$$Q_{ij}=\frac{1}{m_{\rm e}c^2}(M_{\rm p}-M_{\rm d}+E_{\rm i}-E_{\rm j})\qquad(3)$$

where $M_{\rm p}$ and $E_{\rm i}$ are the nuclear mass and excitation energy of the parent nuclei. $M_{\rm d}$ and $E_{\rm j}$ are those of the daughter nuclei. $\omega_{\rm l}$ is the threshold total energy of electron capture. $S_{\rm e}$ and S_ν are the electron and neutron distribution functions, which have the same form.

$$S_{\rm e}=\frac{1}{\exp[(E_{\rm e}-\mu_{\rm e})/(KT)]+1}.\qquad(4)$$

Here $\mu_{\rm e}$ is the chemical potential:

$$\rho Y_{\rm e}=\frac{1}{\pi^2N_{\rm A}}\left(\frac{m_{\rm e}c}{\hbar}\right)^3\int_0^\infty(S_{\rm e}-S_{\rm p})p^2{\rm d}p,\qquad(5)$$

where $S_{\rm p}$ is the positron distribution function which has a similar form of electron but with $\mu_{\rm p}=-\mu_{\rm e}$. Based on these formulae, we calculate the electron capture rates of ^{76}Se under a stellar core collapse supernovae environment.

Figure 2 plots the electron capture rates obtained from the experimental and shell model calculations for the ^{76}Se ground state at different temperatures with densities equal to $\text{Log}_{10}(\rho Y_{\rm e})=9.6$. In Fig. 2, SM and Exp represent the results based on the shell model and experimental GT_+ strength. It is seen in Fig. 2 that, SM electron capture rates are a little larger than the experimental data. This is due to the fact that the GT_+ strength is a little larger than the experimental GT_+ strength distribution. However, one can see in this figure that, in general, the SM model results are very close to the rates obtained from the experimental GT_+ strength distribution. Therefore, one can say that the SM GT_+ strength and the electron capture rates are reliable. The results can be used for a future study of weak interaction rates in a star environment, especially in a core collapse supernovae environment.

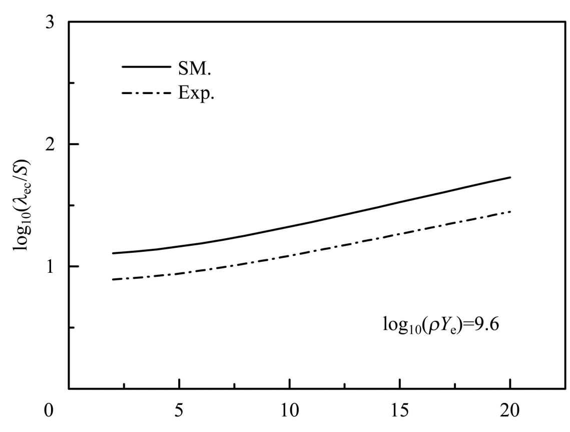

Fig. 2. Calculated electron capture rates based on the shell model B(GT) strength are compared with those based on the experimental data for different temperatures.

In a stellar environment, the neutron rich nuclei appear when the temperature is high. When the collapsing stellar core has finite temperatures of the order of $T = 1$ MeV ($T \sim$10 GK), neutron rich nuclei like ^{76}Se are abundantly present. In order to see the electron capture rates of nuclei in this stellar environment, we plot the electron capture rates under T=10 GK with different densities for both theoretical calculations and experimental data in Fig. 3. In Fig. 3, one can see that under such a stellar environment, the electron capture rates for both theoretical calculations and experiment increase with the densities. The electron capture rates obtained from theoretical calculation are a little larger than the experimental data. It is shown in Fig. 3 that, in general, theoretical results agree with experimental data; both the theoretical and experimental data suggest that the electron capture rates under T=10 GK vary from 10^{-1}–10^{4} when densities change from 10^{8}–10^{11}. It is obvious that this will affect the collapse of supernovae and the evolution of stars have the same environment. In the following work, we will discuss how this affects the energy production and release due to this unblocked transitions, and the core collapse of supernovae and the evolution of stars.

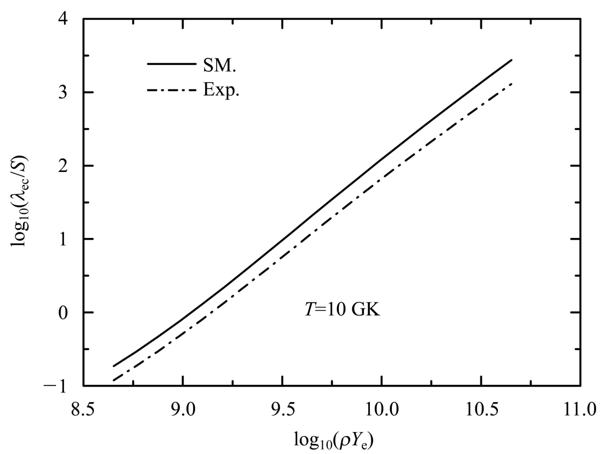

Fig. 3. Calculated electron capture rates from the shell model are compared with experimental data for T=10 GK.

3 Conclusions

Based on the large scale shell model with ^{48}Ca core interaction, we calculate the neutron occupation numbers for pf shell nuclei ^{76}Se. This calculation reproduces the experimental occupation number. We also discuss the evolution of the occupation numbers as a function of truncation levels. Furthermore, we calculate the the Gamow-Teller transitions across the N=40 shell gap for nuclei ^{76}Se and compare the results with experimental data. It shows that the calculation reproduces the experimental GT_+ strength. We then calculate the stellar electron capture rates in a collapsing supernovae environment. The calculations and discussions in this paper will be useful for future study for both nuclear physics and astrophysics.

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