Hamiltonian analysis of a Green-Schwarz sigma model on a supercoset target with $\mathbb{Z}_{4m}$ grading*

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Abstract: We perform a Hamiltonian analysis of the Green-Schwarz sigma model on a supercoset target with $\mathbb{Z}_{4m}$ grading. The fundamental Poisson brackets between the spatial component of the flat currents depending on a continuous parameter, which can be thought of as a first step in the complete calculation of the algebra of the transition matrices, are obtained. When $m=1$, our results are reduced to the results of the type IIB Green-Schwarz superstring on $AdS_5\times S^5$ background obtained by Das, Melikyan and Sato.

Key words: Green-Schwarz sigma model, supercoset target, Hamiltonian analysis, transition matrix

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1 Introduction

The motivation to study non-linear sigma models on supermanifolds is its appearance in string theory and condensed matter physics. String theory on $AdS_d\times S^d$ $(d=2,3,5)$, $AdS_p(p=2,4,6)$, $AdS_5\times S^1$ and $AdS_4\times CP^3$ background are described by superspace sigma models with $\mathbb{Z}_4$ grading [1–10] and the $\mathbb{Z}_4$ grading is a key element for demonstrating the classical integrability of the underlying models. It is shown [11–13] that similar constructions hold for sigma models on coset spaces G/H with general $\mathbb{Z}_{2n}$ grading. In their construction, the kinetic terms of the action contain both the target space bosons and fermions. This choice of the action is called the "hybrid action" and the sigma model is called the "hybrid sigma model". It is proved in Ref. [14] that the flat currents of hybrid sigma models [11] satisfy the equations of motion and the Virasoro constraint. This means that one can generate a series of classical solutions from an existing one. The other choice of action is the "Green-Schwarz action" which has a kinetic term only for the bosons. The action and the flat currents of a Green-Schwarz type sigma model with $\mathbb{Z}_{4m}$ grading were investigated in Ref. [15]. This type of sigma model can be used to describe the Green-Schwarz superstring. When $m=1$, the model given in Ref. [15] is reduced to the well-known model given by Metsaev and Tseytlin.

The Green-Schwarz sigma models with $\mathbb{Z}_{4m}$ grading are models generalized from the Green-Schwarz sigma models with $\mathbb{Z}_4$ grading. The classical integrability of the superstring on $AdS_5\times S^5$ has led to a lot of interesting studies recently. Bena, Polchinski and Roiban [16] constructed a one-parameter family of flat currents of a Green-Schwarz superstring in $AdS_5\times S^5$, which would naturally lead to a hierarchy of classical conserved non-local charges. Chen et al [6] extended the Bena, Polchinski and Roiban's results to the superstring on $AdS_3\times S^3$ and $AdS_5\times S^1$ background. Das, Melikyan and Sato [17] investigated the Hamiltonian analysis and the Poisson algebra of the

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transition matrices of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ string sigma model. Later they [18] calculated the algebra of the flat currents for the model in the light-cone gauge with fixed κ symmetry. Mikhailov and Schafer-Nameki [19] studied the product of transfer matrices in the near flat space expansion of the $\mathrm{AdS}_5\times\mathrm{S}^5$ string theory in the pure spinor formalism and Magro [20] presented the exchange algebra of the model both in pure spinor formalism and Green-Schwarz formalism. The Green-Schwarz sigma models whose targets are quotients of supergroups G by subgroups H defined by gradings of order greater than 4 may have applications in condensed matter physics, string theory and other domains in physics. Thus it is important to continue the analysis of these sigma models.

In this paper, we carry out the Hamiltonian analysis of the Green-Schwarz sigma model on a supercoset target with $\mathbb{Z}_{4m}$ grading given in Ref. [15]. We also present the Poisson brackets between the spatial components of the flat currents with a spectral parameter.

The structure of this paper is as follows: in Section 2 we introduce some notation and write down the sigma model action of the model. In Section 3 we formulate the Hamiltonian formalism of the model and calculate the Poisson bracket of the left-invariant currents and the Poisson bracket of the constraints. We determine the total Hamiltonian of the model and compute the algebra between the spatial component of the flat currents with a spectral parameter.

2 Formulation of the sigma model on supercoset targets

Let $\mathfrak{g}$ be a Lie superalgebra admitting a $\mathbb{Z}_{4m}$ grading. That is, $\mathfrak{g}$ may be written as a direct sum $\mathfrak{g}=\sum\limits_{k=0}^{4m-1}\mathfrak{g}^{(k)}$ of vector subspaces where $\mathfrak{g}^{(0)}=\mathfrak{h}$ and this decomposition satisfies the algebra $\left[\mathfrak{g}^{(r)},\mathfrak{g}^{(s)}\right]\subset\mathfrak{g}^{(p)}$ with $p=r+s \mod 4m$. The generators of $\mathfrak{g}^{(2s)}, s=0,1,2,\ldots,2m-1$ are even while those of $\mathfrak{g}^{(2s+1)}, s=0,1,2,\ldots,2m$ are odd. The supertrace is compatible with the $\mathbb{Z}_{4m}$ grading, which means that if $X_i\in\mathfrak{g}_{(i)}$, $Y_j\in\mathfrak{g}_{(j)}$ then $\mathrm{Str}X_iY_j=0$ unless $i+j=0 \mod 4m$, and if $\mathrm{Str}(X_iY)=0$ for any X_i then $Y=0$. For simplicity, in this paper, we will employ an index free tensor notation defined as $A_{\underline{1}}=A\otimes 1, B_{\underline{2}}=1\otimes B$, and $(A\otimes B)_{ij,km}=A_{ik}B_{jm}$. The quadratic Casimir is defined by

$$\begin{aligned}C_{\underline{12}} &= \eta^{AB}T_A\otimes T_B=\eta^{i_0j_0}T_{i_0}\otimes T_{j_0}\\ &\quad+\sum_{r=1}^{4m-1}\eta^{i_rj_r}T_{i_r}\otimes T_{j_r}=C_{\underline{12}}^{(00)}+\sum_{r=1}^{4m-1}C_{\underline{12}}^{(r4m-r)}.\end{aligned}\quad(1)$$

Here $\eta^{AB}=\mathrm{Str}(T_AT_B)$, we denote generically T_A as the generator of the Lie algebra $\mathfrak{g}$ and for each grading $T_{i_0}\in\mathfrak{g}^{(0)}, i_0=1,2,\cdots,\ \dim\mathfrak{g}^{(0)}$, $T_{i_r}\in\mathfrak{g}^{(r)}, i_r=1,2,\cdots,\ \dim\mathfrak{g}^{(r)}$, $r=1,2,\cdots,4m-1$. For any $M^{(k)}\in\mathfrak{g}^{(k)}$, the quadratic Casimir possesses the property

$$[C_{\underline{12}}^{(i4m-i)},M_{\underline{2}}^{(i+j)}]=-[C_{\underline{12}}^{(4m-j\,j)},M_{\underline{1}}^{(i+j)}].\quad(2)$$

Let $g(x^\mu)$ be a field valued in an even faithful matrix representation of a supergroup G, where $x^0=\tau, x^1=\sigma$ are world-sheet coordinates. The current 1 form $J=-g^{-1}\mathrm{d}g\in\mathfrak{g}$ can be decomposed as $J=\sum\limits_{k=0}^{4m-1}J^{(k)}$, here $J^{(k)}\in\mathfrak{g}^{(k)}, k=0,1,2,\cdots,\ 4m-1$. The current J satisfies a zero curvature condition

$$\mathrm{d}J-J\wedge J=0\,.\quad(3)$$

The action of the non-linear sigma model is [15]

$$\begin{aligned}S &= \frac{1}{2}\int\mathrm{Str}(J^{(2m)}\wedge *J^{(2m)})\\ &\quad+\frac{1}{2}\int\sum_{r=1}^{2m-1}\mathrm{Str}(q_rJ^{(r)}\wedge J^{(4m-r)}),\end{aligned}\quad(4)$$

where $q_r=-\dfrac{r}{m}$. The star $*$ denotes Hodge dual in the world-sheet. The one parameter family of flat currents is constructed in Ref. [15].

$$J(z)=J^{(0)}+\sum_{i=1}^{4m-1}e(i,z)J^{(i)}+\beta(z)*J^{(2m)},\quad(5)$$

where

$$e(i,z) = \begin{cases} z^i,\ i<2m,\\ z^{i-4m},\ i>2m\end{cases},$$

$$e(2m,z) = \frac{1}{2}(z^{2m}+z^{-2m}),$$

$$\beta(z) = \frac{1}{2}(z^{2m}-z^{-2m}),$$

z is the spectral parameter. As is shown in Ref. [15], the vanishing curvature condition for the current $J(z)$ leads to all the equations of motion of the model and the zero curvature condition (3).

3 The Hamiltonian analysis

In this section we are going to perform the Hamiltonian analysis of the action (4). Our analysis is based on the approach introduced in Ref. [21]. We treat the space component J_1 of the current J as the only independent dynamical variable and regard the time component J_0 of the current J as a function of it through the relation (3), namely,

$$J_0=\nabla_1^{-1}(\partial_0 J_1)\,,\quad(6)$$

where

$$\nabla_1=\partial_1-[J_1,\cdot],\ \partial_0=\frac{\partial}{\partial\tau},\ \partial_1=\frac{\partial}{\partial\sigma},\ \nabla_1^{-1}$$

is the inverse operator of ∇_1. We define the conjugate momentum as the variation of the action (4) with respect to $\partial_0 J_1$ and we get

$$\Pi = \sum_{i=0}^{4m-1}\Pi^{(i)}=-\nabla_1^{-1}\left(J_0^{(2m)}-\frac{1}{2}\sum_{r=1}^{2m-1}q_rJ_1^{(r)}+\frac{1}{2}\sum_{r=1}^{2m-1}q_rJ_1^{(4m-r)}\right), \tag{7}$$

where we have used the fact that

$$\int\mathrm{d}^2x\mathrm{Str}\,[(\nabla_1^{-1}G)F]=-\int\mathrm{d}^2x\mathrm{Str}\,[G(\nabla_1^{-1}F)].$$

The basic canonical Poisson bracket of the canonical variables can be written as

$$\left\{J_{1\underline{1}}^{(0)}(\sigma),\Pi_{\underline{2}}^{(0)}(\sigma')\right\} = C_{\underline{12}}^{(00)}\delta(\sigma-\sigma'), \tag{8}$$

$$\left\{J_{1\underline{1}}^{(i)}(\sigma),\Pi_{\underline{2}}^{(4m-i)}(\sigma')\right\} = C_{\underline{12}}^{(i\,4m-i)}\delta(\sigma-\sigma'),\quad i = 1,2,\cdots,\ 4m-1. \tag{9}$$

Decomposing the canonical momentum (7) into the appropriate subspaces, we obtain

$$J_0^{(2m)} = -(\nabla_1\Pi)^{(2m)}, \tag{10}$$

$$\varphi^{(0)} = -(\nabla_1\Pi)^{(0)}, \tag{11}$$

$$\varphi^{(r)} = -(\nabla_1\Pi)^{(r)}+\frac{1}{2}q_rJ_1^{(r)} \approx 0,r=1,2,\cdots,2m-1, \tag{12}$$

$$\varphi^{(4m-r)} = -(\nabla_1\Pi)^{(4m-r)}-\frac{1}{2}q_rJ_1^{(4m-r)} \approx 0,r=1,2,\cdots,2m-1. \tag{13}$$

Since $J_0^{(2m)}$ contains time derivatives, Eq. (10) can be used to express velocities in terms of canonical momenta. While none of the relations (11)–(13) depends on time derivatives, each of them corresponds to primary constraints of the theory. In the conformal gauge, the Virasoro constraints take the forms

$$\varphi^{(4m)} = \frac{1}{2}\mathrm{Str}(J_0^{(2m)}J_0^{(2m)}+J_1^{(2m)}J_1^{(2m)})\approx 0, \tag{14}$$

$$\varphi^{(4m+1)} = \mathrm{Str}(J_0^{(2m)}J_1^{(2m)})\approx 0. \tag{15}$$

The Poisson brackets of the currents are

$$\left\{J_{1\underline{1}}^{(0)}(\sigma),J_{1\underline{2}}^{(0)}(\sigma')\right\}=0,$$

$$\left\{J_{1\underline{1}}^{(i)}(\sigma),J_{1\underline{2}}^{(4m-i)}(\sigma')\right\}=0,\quad i=1,2,\cdots,\ 4m-1, \tag{16}$$

$$\left\{J_{0\underline{1}}^{(2m)}(\sigma),J_{1\underline{2}}^{(0)}(\sigma')\right\}=-[C_{\underline{12}}^{(2m\,2m)},J_{\underline{12}}^{(2m)}(\sigma')]\times\delta(\sigma-\sigma'), \tag{17}$$

$$\left\{J_{0\underline{1}}^{(2m)}(\sigma),J_{1\underline{2}}^{(r)}(\sigma')\right\}=-[C_{\underline{12}}^{(2m\,2m)},J_{\underline{12}}^{(2m+r)}(\sigma')]\times\delta(\sigma-\sigma'), \tag{18}$$

$$\left\{J_{0\underline{1}}^{(2m)}(\sigma),J_{1\underline{2}}^{(2m)}(\sigma')\right\}=-[C_{\underline{12}}^{(2m\,2m)},J_{\underline{12}}^{(0)}]\delta(\sigma-\sigma')+C_{\underline{12}}^{(2m\,2m)}\partial_1\delta(\sigma-\sigma'), \tag{19}$$

$$\left\{J_{0\underline{1}}^{(2m)}(\sigma),J_{1\underline{2}}^{(4m-r)}(\sigma')\right\}=-[C_{\underline{12}}^{(2m\,2m)},J_{\underline{12}}^{(2m-r)}(\sigma')]\times\delta(\sigma-\sigma'). \tag{20}$$

$$\left\{J_{0\underline{1}}^{(2m)}(\sigma),J_{0\underline{2}}^{(2m)}(\sigma')\right\}=[C_{\underline{12}}^{(2m\,2m)},(\nabla_1\Pi)^{(0)}(\sigma')]\times\delta(\sigma-\sigma'). \tag{21}$$

where $r=1,2,\cdots,\ 2m-1$. We note that there is only one non-ultra local term in the Poisson brackets (19).

The total Hamiltonian density is given as

$$H_{\mathrm{T}} = \frac{1}{2}\mathrm{Str}(J_0^{(2m)}J_0^{(2m)}+J_1^{(2m)}J_1^{(2m)})+\mathrm{Str}\lambda^{(0)}\varphi^{(0)}+\sum_{r=1}^{2m-1}\lambda^{(4m-r)}\varphi^{(r)}+\sum_{r=1}^{2m-1}\lambda^{(r)}\varphi^{(4m-r)}, \tag{22}$$

where $\lambda^{(0)}$, $\lambda^{(r)}$ and $\lambda^{(4m-r)}$ are the Lagrange multipliers to be determined. Imposing the consistency conditions $\mathrm{d}\varphi^r/\mathrm{d}\tau=0,\ \mathrm{d}\varphi^{4m-r}/\mathrm{d}\tau=0$, we arrive at the result

$$\lambda^{(r)} = J_1^{(r)}\quad r=1,2,\ldots,2m-1, \tag{23}$$

$$\lambda^{(4m-r)} = -J_1^{(4m-r)}\quad r=1,2,\ldots,2m-1, \tag{24}$$

which determinate $\lambda^{(r)}$ and $\lambda^{(4m-r)}$. The condition $\mathrm{d}\varphi^{(0)}/\mathrm{d}\tau = 0$ imposes no further constraints. φ_{4m}, φ_{4m+1} can also be checked to be conserved under the hamiltonian flow. The Lagrange multiplier λ_0 is undetermined due to the fact that φ_0 is the generator of a gauge symmetry of the theory.

With the help of Eqs. (8)–(9), (11)–(13) we obtain Poisson brackets between the constraints

$$\left\{\varphi^{(0)}_{\underline{1}}(\sigma),\varphi^{(0)}_{\underline{2}}(\sigma')\right\} = -[C^{(00)}_{\underline{12}},\varphi^{(0)}_{\underline{2}}(\sigma')]\delta(\sigma-\sigma')\approx 0, \tag{25}$$

$$\left\{\varphi^{(0)}_{\underline{1}}(\sigma),\varphi^{(r)}_{\underline{2}}(\sigma')\right\} = -[C^{(00)}_{\underline{12}},\varphi^{(r)}_{\underline{2}}(\sigma')]\delta(\sigma-\sigma')\approx 0, \tag{26}$$

$$\left\{\varphi^{(0)}_{\underline{1}}(\sigma),\varphi^{(4m-r)}_{\underline{2}}(\sigma')\right\} = -[C^{(00)}_{\underline{12}},\varphi^{(4m-r)}_{\underline{2}}(\sigma')]\delta(\sigma-\sigma')\approx 0, \tag{27}$$

$$\left\{\varphi^{(r)}_{\underline{1}}(\sigma),\varphi^{(s)}_{\underline{2}}(\sigma')\right\} = -\left[C^{(r\,4m-r)}_{\underline{12}},\frac{1}{2}q_{r+s}J^{(r+s)}_{1\underline{2}}(\sigma')-(\nabla_1\Pi)^{(r+s)}_{2}(\sigma')\right]\delta(\sigma-\sigma'), \tag{28}$$

$$\left\{\varphi^{(r)}_{\underline{1}}(\sigma),\varphi^{(4m-s)}_{\underline{2}}(\sigma')\right\} = -[C^{(r\,4m-r)}_{\underline{12}},\varphi^{(r-s)}_{\underline{2}}(\sigma')]\delta(\sigma-\sigma')\approx 0,\quad r>s, \tag{29}$$

$$\left\{\varphi^{(r)}_{\underline{1}}(\sigma),\varphi^{(4m-r)}_{\underline{2}}(\sigma')\right\} = -[C^{(r\,4m-r)}_{\underline{12}},\varphi^{(0)}_{\underline{2}}(\sigma')]\delta(\sigma-\sigma')\approx 0, \tag{30}$$

$$\left\{\varphi^{(r)}_{\underline{1}}(\sigma),\varphi^{(4m-s)}_{\underline{2}}(\sigma')\right\} = -[C^{(r\,4m-r)}_{\underline{12}},\varphi^{(4m+r-s)}_{\underline{2}}(\sigma')]\delta(\sigma-\sigma')\approx 0,\quad r<s, \tag{31}$$

$$\left\{\varphi^{(4m-r)}_{\underline{1}}(\sigma),\varphi^{(4m-s)}_{\underline{2}}(\sigma')\right\} = \left[C^{(4m-r\,r)}_{\underline{12}},\frac{1}{2}q_{r+s}J^{(4m-r-s)}_{1\underline{2}}(\sigma')+(\nabla_1\Pi)^{(4m-r-s)}_{2}(\sigma')\right]\delta(\sigma-\sigma'), \tag{32}$$

here $r=1,2,\dots,2m-1, s=1,2,\dots,2m-1$. From (25)–(27), one can conclude that $\varphi_{(0)}$ is a first class constraint. The algebra among $\varphi_{(r)}$, φ_{4m-r}, $r=1,2,\dots,2m-1$, are more complicated and form a non trivial algebra. They are reducible because of the constraints (14) and (15). One can decompose $\varphi^{(r)}$ and $\varphi^{(4m-r)}, r=1,2,\dots,2m-1$ into the first and second class constraints using some relevant projection. The second class constraints can be used to define the Dirac brackets.

The definition of the transition matrix is [21–25]

$$T(\sigma_1,\sigma_2,z)=P\mathrm{e}^{\int_{\sigma_2}^{\sigma_1}\mathrm{d}\sigma\ J_1(\sigma,z)}, \tag{33}$$

here P means path ordering, $J_1(\sigma,z)$ is the spatial component of the current (5). As shown in Ref. [21–25] the Poisson brackets between the transition matrix $T_{\underline{1}}(\sigma_1,\sigma_2,z_1)$ and $T_{\underline{2}}(\sigma'_1,\sigma'_2,z_2)$ are equal to

$$\begin{aligned}&\{T_{\underline{1}}(\sigma_1,\sigma_2,z_1),T_{\underline{2}}(\sigma'_1,\sigma'_2,z_2)\}\\ =&\int_{\sigma_2}^{\sigma_1}\mathrm{d}\sigma\int_{\sigma'_2}^{\sigma'_1}\mathrm{d}\sigma'\,(T_{\underline{1}}(\sigma_1,\sigma,z_1)\otimes T_{\underline{2}}(\sigma'_1,\sigma',z_2))\\ &\times\{J_{\underline{1}1}(\sigma,z_1),J_{\underline{2}1}(\sigma',z_2)\}(T_{\underline{1}}(\sigma,\sigma_2,z_1)\\ &\otimes T_{\underline{2}}(\sigma',\sigma'_2,z_2)).\end{aligned} \tag{34}$$

The above relation shows that one needs to evaluate $\{J_{\underline{1}}(\sigma,z_1),J_{\underline{2}}(\sigma',z_2)\}$ in order to compute the Poisson bracket between the transition matrices. Here we calculate the Poisson bracket between the currents which can be thought of as a first step in the complete calculation of the algebra of the transition matrices. Using the equations (2), (5), (8)–(9) and (10)–(13), after some tedious calculation, we have

$$\begin{aligned}&\{J_{1\underline{1}}(\sigma,z_1),J_{1\underline{2}}(\sigma',z_2)\}\\ =&\Big(\alpha[C^{(2m\,2m)}_{\underline{12}},J_{1\underline{1}}(\sigma,z_1)]+\beta[C^{(2m\,2m)}_{\underline{12}},J_{1\underline{2}}(\sigma,z_2)]\\ &+\gamma[C^{(00)}_{\underline{12}},J_{1\underline{1}}(\sigma,z_1)+J_{1\underline{2}}(\sigma,z_2)]\Big)\delta(\sigma-\sigma')\\ &+\sum_{r=1}^{2m-1}\xi_2(r)\left\{J_{1\underline{1}}(\sigma,z_1),\varphi^{(r)}_{\underline{2}}(\sigma')\right\}\\ &+\sum_{r=1}^{2m-1}\chi_2(r)\left\{J_{1\underline{1}}(\sigma,z_1),\varphi^{(4m-r)}_{\underline{2}}(\sigma')\right\}\\ &+\sum_{r=1}^{2m-1}\xi_1(r)\left\{\varphi^{(r)}_{\underline{1}}(\sigma'),J_{1\underline{2}}(\sigma,z_1)\right\}\\ &+\sum_{r=1}^{2m-1}\chi_1(r)\left\{\varphi^{(4m-r)}_{\underline{1}}(\sigma',z_1),J_{1\underline{2}}(\sigma,z_1)\right\}\\ &+\Lambda\partial_\sigma\delta(\sigma-\sigma'),\end{aligned}$$

where

$$\begin{aligned}\alpha &= \frac{\beta_2^2}{\alpha_2(2m)\beta_1-\alpha_1(2m)\beta_2},\\ \beta &= \frac{\beta_1^2}{\alpha_2(2m)\beta_1-\alpha_1(2m)\beta_2},\\ \gamma &= \frac{\beta_1\beta_2}{\alpha_2(2m)\beta_1-\alpha_1(2m)\beta_2},\\ \xi_2(r) &= \gamma\alpha_2(r),\quad \xi_1(r)=-\gamma\alpha_1(r)\end{aligned}$$

$$\chi_2(r) = \gamma\alpha_2(4m-r), \quad \chi_1(r) = -\gamma\alpha_1(4m-r),$$

$$\begin{aligned} \Lambda = & -(\alpha_1(2m)\beta_2+\alpha_2(2m)\beta_1)C_{\underline{12}}^{(2m2m)} \\ & -\sum_{r=1}^{2m-1}(\alpha_1(4m-r)\xi_2(r) \\ & +\chi_1(r)\alpha_2(r))C_{\underline{12}}^{(4m-r\ r)} \\ & -\sum_{r=1}^{2m-1}(\alpha_1(r)\chi_2(r) \\ & +\xi_1(r)\alpha_2(4m-r))C_{\underline{12}}^{(r\ 4m-r)}, \end{aligned}$$

with

$$\beta_1 = \beta(z_1), \beta_2 = \beta(z_2), \alpha_1(i) = e(i,z_1),$$

$$\alpha_2(i) = e(i,z_2), i = 1,2,\cdots,\ 4m-1.$$

In the above equation, there are additional terms depending on $\xi_i(r)$, $\chi_i(r)$ and non-ultralocal terms involving Λ. When one separates the constraints into first class constraints and second class constraints and calculates the Dirac brackets of the currents, these terms may be absent and the algebra may have a closed structure. The relevant work is interesting and under investigation.

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