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Cooling of Akmal-Pandharipande-Ravenhall neutron stars with a rotochemical heating source*

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Abstract Employing phenomenological density-dependent critical temperatures of strong singlet-state proton pairing and of moderate triplet-state neutron pairing, we investigate the effects of rotochemical heating on the thermal evolution of superfluid neutron stars whose cores consist of npe matter with the Akmal-Pandharipande-Ravenhall equation of state. Since the star is not quite in the weak interaction equilibrium state during spin-down, the departure from the chemical equilibrium leads to the rotochemical heating in a rotating NS which will increase the stellar's temperature. Our calculations show that the rotochemical heating delays the cooling of superfluid neutron stars considerably and makes the previous classification of NS cooling ambiguous. What's more, our model is currently consistent with all the observational data, and in particular some middle-aged and cold NSs (PRS J0205+6449 in 3C 58, PRS J1357-6429, RX J007.0+7303 in CTA 1, Vela) can be better explained when taking into account rotochemical heating.

Key words neutron stars, thermal evolution, rotation, rotochemical heating, superfluidity

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1 Introduction

The fundamental properties of supranuclear matter in the cores of neutron stars, such as the equation of state, the compositions and superfluidity, are still poorly known. The theoretical study of the thermal evolution of the neutron star (NS) and a comparison between its predictions and observations of thermal emission from it are of significant interest in probing properties of matter at supernuclear densities [1, 2].

It is widely accepted that there are three main cooling regulators in the cores of neutron stars.

First, the neutrino emission leads to the loss of the internal energy in the cores, but the emission efficiency is quite different for various processes which can be classified by slow and fast processes. In the general situation, some slow processes, Modified Urca (Murca) process, NN-bremsstrahlung, etc., are always allowed. At higher density, the neutrino emission can be strongly enhanced by the onset of Direct Urca (Durca) process [3] in nucleon matter or similar processes in exotic phases of matter (such as pion condensation, kaon condensation and deconfinement quark matter), which possibly results in extremely fast cooling of the neutron star.

Second, it is generally believed that dense matter can be superfluid at such high density and low temperature [4]. Baryon pairing (or quark pairing) strongly suppresses not only the neutrino emission but also the heat capacity of nuclear matter. Meanwhile, Cooper pair formation and breaking also affect cooling of the neutron star.

So far as the above two aspects are concerned, we could find different cooling scenarios, such as "standard cooling", "enhanced cooling" and "minimal cooling" [5]. Their differences are determined by the equation of state, chemical composition and superfluid property.

Third, the heating mechanisms should not be ignored in the NS cooling simulation. Rotochemical heating is one of the most important heating mechanisms. Reisenegger [6] first studied rotochemical heat-

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ing and applied the mechanism to determine the millisecond pulsar's properties. He argued that this heating mechanism significantly enhances the thermal emission from millisecond pulsars at old ages. Until now, it is not the end of this problem especially when some superfluid components appear. We will pay attention to the rotochemical heating effects in superfluid neutron stars.

What's more, the cooling scenario of the neutron star is sensitively dependent on the equation of state of neutron star matter [5]. We here focus on the Akmal-Pandharipande-Ravenhall (APR) model in consideration of the following facts: the first is that the APR model is currently assumed to be the most elaborate EOS; and the second is that the APR star with superfluid nucleon cores presents an extraordinary cooling scenario, as found by Ref. [7].

This paper is arranged as follows. In Section 2, we review the nucleon superfluidity and its effect on APR stars. Section 3 discusses the rotochemical heating from non-equilibrium weak reactions, and presents the descriptions of direct and Modified Urca processes in superfluid nucleon matter. The thermal evolution equation of the neutron star, including the rotochemical heating, is given in Section 4. In Section 5, we detail the inputs necessary for numerical calculations and describe our results. Finally, we summarize our work.

2 Nucleon superfluidity

Many microscopic studies of dense nucleon matter predict that below a certain critical temperature, nucleons will be superfluid, and the critical temperatures, $T_{\rm cp}$ and $T_{\rm cn}$, depend sensitively on the model of nucleon-nucleon interaction and many-body theory employed [8]. It is widely accepted that there are three types of nucleon superfluidities in neutron stars: singlet-state ${}^1S_0$ pairing of neutrons ($T_{\rm c}=T_{\rm cns}$) in the inner crust and outermost core; ${}^1S_0$ proton pairing ($T_{\rm c}=T_{\rm cp}$) in the core; and triplet-state (${}^3\mathrm{P}_2$) neutron pairing ($T_{\rm c}=T_{\rm cnt}$) in the core.

Almost all contemporary theories predict some common features of nucleon superfluidity: $T_{\rm cn}(\rho)$ for the singlet–state neutron SF has a maximum at subnuclear densities in the crust and vanishes at $\rho\sim 2{\times}10^{14}$ g·cm^{-3}, while $T_{\rm cn}(\rho)$ for the triplet–state neutron SF grows up at subnuclear density, reaches maximum at $\rho=(2-3)\,\rho_0$ ($\rho_0=2.8\times10^{14}$ g·cm^{-3} is the saturation density of nuclear matter), decreases with ρ and vanishes at $\rho\sim(3-5)\times10^{15}$ g·cm^{-3}. $T_{\rm cp}$ also has a maximum at several ρ_0 and vanishes at higher ρ [8]. To describe the density dependence of $T_{\rm cn}$ and $T_{\rm cp}$, several authors propose phenomenological models [9].

We will include the latter two types of superfluidity in the NS core and the detailed density dependence of these two critical temperatures is described in Fig. 1 of Ref. [7].

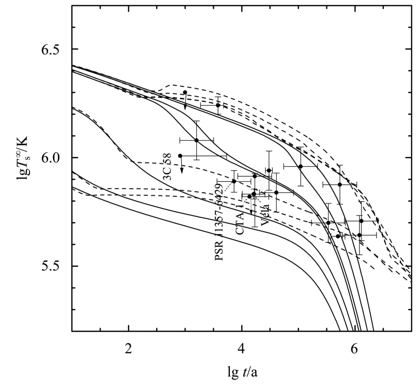

Fig. 1. Thermal evolution curves of superfluid NSs for different star mass: $1.2M_\odot$, $1.4M_\odot$, $1.6M_\odot$, $1.68M_\odot$, $1.7M_\odot$, $1.8M_\odot$ and $1.9M_\odot$ from top to bottom with and without rotochemical heating (dashed and solid curves respectively). The observational data come from Table 3 in Ref. [17]. The magnetic field is $B=10^{11}$G and the initial spin period is 1 ms.

The nucleon superfluidity suppresses the heat capacity and neutrino processes involving a superfluid nucleon, and what's more, superfluidity initiates an additional neutrino emission associated with Cooper pairing of nucleons. Our codes of NS thermal evolution include all of these effects.

It's well known that for non-superfluid NSs with the core composed of standard nuclear matter, we have two distinct cooling regimes: slow and fast cooling [8]. The transition from slow to fast cooling with increasing mass occurs in a very narrow mass range and is sharp. When the superfluidity is taken into account, the Urca neutrino emission and the NS heat capacity will be strongly suppressed, and neutrino emission due to Cooper pairing will occur [10]. These effects will make the cooling behavior of NSs different. Recently, using APR EOS and including nucleon superfluidity, Gusakov et al. [7] found a particular scenario of neutron star cooling that the theoretical cooling models of isolated middle-aged neutron stars can be divided into three distinct types: slow, moderate and fast cooling. They neglected the heating effects, while we will take these effects into account, which may make the cooling scenario different.

3 Rotochemical heating

As a neutron star spins down, some heating mechanisms may be present and will play important roles in the thermal evolution of NSs.

Rotochemical heating originates in deviations from weak interaction equilibrium caused by spin-down in a rotating NS. As a neutron star comprising standard nucleon matter spins down, the centrifugal force diminishes and correspondingly the interior density increases, changing the chemical equilibrium state of the nucleon matter. Non-equilibrium reactions tend to restore equilibrium. If the relaxation timescale for the weak processes are small compared with the timescale of the rotational evolution, the nucleon matter will maintain chemical equilibrium. However, these timescales are comparable and the nucleon matter deviates from chemical equilibrium, leading to energy release at the expense of the stored chemical energy [6].

For npe matter in a NS, their relative concentrations are adjusted by Direct Urca reactions,

$$\mathrm{n} \rightarrow \mathrm{p}+\mathrm{e}+\bar{\nu}, \tag{1}$$

$$\mathrm{p}+\mathrm{e} \rightarrow \mathrm{n}+\nu, \tag{2}$$

and Modified Urca reactions,

$$\mathrm{n}+\mathrm{N} \rightarrow \mathrm{p}+\mathrm{N}+\mathrm{e}+\bar{\nu}, \tag{3}$$

$$\mathrm{p}+\mathrm{N}+\mathrm{e} \rightarrow \mathrm{n}+\mathrm{N}+\nu. \tag{4}$$

The departure from the chemical equilibrium can be quantified by the chemical imbalance,

$$\eta_{\mathrm{npe}} = \delta\mu_{\mathrm{n}} - \delta\mu_{\mathrm{p}} - \delta\mu_{\mathrm{e}}, \tag{5}$$

where $\delta\mu_{\mathrm{i}} = \mu_{\mathrm{i}} - \mu_{\mathrm{i}}^{\mathrm{eq}}$ is the deviation from the equilibrium chemical potential of species i at a given pressure. In the non-equilibrium state, neutrino emissivities and net reaction rates per unit volume of Urca processes will be modified, which can be written as [11]

$$Q_{\alpha}(n,T,\eta_{\alpha}) = Q_{\alpha}^{\mathrm{eq}}(n,T) F_{*}\left(\frac{\eta_{\alpha}}{kT}\right), \tag{6}$$

$$\Delta\Gamma_{\alpha}(n,T,\eta_{\alpha}) = \frac{1}{kT} Q_{\alpha}^{\mathrm{eq}}(n,T) H_{*}\left(\frac{\eta_{\alpha}}{kT}\right), \tag{7}$$

where Q_{α}^{eq} is the neutrino emissivity in equilibrium, η_{α} is the chemical imbalance due to reaction α, T is the local temperature, n is the baryon number density, k is Boltzmanm's constant, and the expressions of F_{*} and H_{*} are given in the appendix of Reisenegger [6]. The total energy dissipation rate per unit volume is

$$Q_{\mathrm{H}} = \sum_{\alpha} \Delta\Gamma_{\alpha}\eta_{\alpha}. \tag{8}$$

Note that when considering the thermal evolution of NSs with superfluid cores, we should introduce the superfluid reduction factors to the quantity Q_{α}^{eq} in Eqs. (6) and (7). The reduction factors can be obtained from Ref. [12].

4 The equations of thermal evolution

The equation of thermal evolution can be written as

$$C_{\mathrm{V}} \frac{\mathrm{d}T^{\infty}}{\mathrm{d}t} = -L_{\nu}^{\infty} - L_{\gamma}^{\infty} + L_{\mathrm{H}}^{\infty}, \tag{9}$$

where C_{V} is the total stellar heat capacity, L_{H}^{∞} is the total power released by the heating mechanism, L_{ν}^{∞} is the total power emitted as neutrinos and L_{γ}^{∞} is the power released as thermal photon. These quantities are calculated as

$$L_{\mathrm{H}}^{\infty} = \int \mathrm{d}V Q_{\mathrm{H}} \mathrm{e}^{2\phi}, \tag{10}$$

$$L_{\nu}^{\infty} = \int \mathrm{d}V Q_{\nu} \mathrm{e}^{2\phi}, \tag{11}$$

$$L_{\gamma}^{\infty} = 4\pi R^2 T_{\mathrm{s}}^4 \mathrm{e}^{2\phi_{\mathrm{s}}} = 4\pi R_{\infty}^2 (T_{\mathrm{s}}^{\infty})^4, \tag{12}$$

respectively, where $\mathrm{d}V = 4\pi r^2 (g_{rr}^{1/2})\mathrm{d}r$ is the proper volume element, Q_{ν} is the total neutrino emissivity contributed by reactions. σ is the Stefan-Boltzman constant, R is the stellar coordinate radius, $\phi_{\mathrm{s}} = \phi(R)$, $R_{\infty} = R\mathrm{e}^{-\phi_{\mathrm{s}}}$ is the effective radius as measured from infinity and T_{s}^{∞} is the redshifted effective temperature.

When considering the rotochemical heating effect, we can write the time evolution of the temperature and the chemical imbalances for NS constituents of npe matter as [13]

$$\begin{aligned} \dot{T}^{\infty} = {} & [M_{\mathrm{D}}(\xi_{\mathrm{npe}}) L_{\mathrm{D}}^{\infty} + M_{\mathrm{M}}(\xi_{\mathrm{npe}}) L_{\mathrm{M}}^{\infty} \\ & - L_{\nu}^{'} - L_{\gamma}^{\infty}] / C_{\mathrm{V}}^{\infty}, \end{aligned} \tag{13}$$

$$\begin{aligned} \dot{\eta}_{\mathrm{npe}}^{\infty} = {} & -\frac{Z_{\mathrm{npe}}}{kT} [L_{\mathrm{D}}^{\infty} H_{\mathrm{D}}(\xi_{\mathrm{npe}}) + L_{\mathrm{M}}^{\infty} H_{\mathrm{M}}(\xi_{\mathrm{npe}})] \\ & + 2W_{\mathrm{npe}} \Omega\dot{\Omega}, \end{aligned} \tag{14}$$

where $\xi = \eta^{\infty}/k_{\mathrm{b}}T^{\infty}$, $L_{\nu}^{'} = L_{\mathrm{NN}}^{\infty} + L_{\mathrm{co}}^{\infty}$, L_{NN}^{∞}, L_{co}^{∞}, L_{D}^{∞} and L_{M}^{∞} are the neutrino luminosities of neutrino emission processes included in our thermal evolution codes (neutrino bremsstrahlung in nucleon-nucleon scattering, neutrino emission due to Cooper pairing of superfluid nucleons, and direct and Modified Urca reactions), the functions M_{α} and H_{α} ($\alpha = \mathrm{D,M}$) quantify the effect of reactions towards restoring chemical equilibrium, the subscripts D and M denote Direct and Modified Urca reactions, Z_{npe} is related to the structure of the rotating NS and the scalar W_{npe} quantify the departure from equilibrium due to the change in the centrifugal force ($\propto \Omega\dot{\Omega}$).

5 Physics inputs and results

Here, we will discuss the equation of the state of the dense interior of a neutron star, the neutrino emissivities, the heat capacity and the relationship of the internal temperature and the surface temperature, since these ingredients are the most important ones to solve two coupled equations for the time evolution of the internal temperature and the chemical imbalances.

In this paper, our thermal evolution model will be based on the standard (nucleon) composition in the NS core with the Akmal-Pandharipande-Ravenhall equation of state obtained by Ref. [14]. We employ the parameterized description of APR EOS proposed by Ref. [15],

$$\begin{aligned}\varepsilon &= E_{\rm comp}(n)+S(n)(1-2x)^2 \\ &= \varepsilon_0 u\frac{u-2-s}{1+su}+S_0u^{\gamma}(1-2x)^2,\end{aligned} \tag{15}$$

where $u=n/n_0$ ($n_0=0.16$ fm^{-3}, the nuclear saturation density), $n=n_{\rm p}+n_{\rm n}$ is the total baryon density, $x=n_{\rm p}/n$ is the proton fraction, $\varepsilon_0=15.8$ MeV, $s\approx 0.2$, and we choose the parameter $\gamma=0.668$. For our EOS, the most massive stable NS has $M_{\max}=1.954M_{\odot}$, and the critical mass above which the Direct Urca process allows is $M_{\rm D}=1.686M_{\odot}$.

Since we take superfluidity into account, our thermal evolution codes should include two other neutrino emission processes: neutrino bremsstrahlung in nucleon-nucleon scattering and neutrino emission due to Cooper pairing of nucleons. The neutrino emissivities, the heat capacity and their reduction factors due to neutron and proton superfluidity are given by Ref. [12].

We use the relation between the effective surface temperature $T_{\rm s}$ and the temperature $T_{\rm b}$ at the bottom of the heat-blanketing iron envelope for $B=0$ calculated by Ref. [16],

$$T_{\rm s6}^4 = g_{14}[(7\zeta)^{2.25}+(0.33\zeta)^{1.25}], \tag{16}$$

$$\zeta = T_{\rm b9}-10^{-3}g_{14}^{1/4}\sqrt{7T_{\rm b9}}. \tag{17}$$

Here, $T_{\rm s6}=T_{\rm s}/10^6$ K, $T_{\rm b9}=T_{\rm b}/10^9$ K, g_{14} is the proper surface gravity of the star in units of 10^{14} cm·s^{-2}.

Assuming the spin-down is induced by the magnetic dipole radiation (MDR), the evolution of the rotation angular velocity Ω is given by

$$\dot{\Omega}=-\frac{2}{3Ic^3}\mu^2\Omega^3\sin^2\theta, \tag{18}$$

where I is the stellar moment of inertia, $\mu=\dfrac{1}{2}BR^3$ is the magnetic dipole moment, and θ is the inclination angle between the magnetic and the rotational axes. We make further assumptions that $I=10^{45}$ g·cm^2 and $\theta=45°$.

We plot a family of cooling curves of superfluid NSs with and without rotochemical heating for different star masses: $1.2M_{\odot}$, $1.4M_{\odot}$, $1.6M_{\odot}$, $1.68M_{\odot}$, $1.7M_{\odot}$, $1.8M_{\odot}$ and $1.9M_{\odot}$ from top to bottom with magnetic fields $B=10^{11}$G in Fig. 1. This figure shows that there is a very sharp fall in $T_{\rm s}^{\infty}$ from slowly cooling ($1.68M_{\odot}$) due to Modified Urca process to intermediate cooling ($1.7M_{\odot}$) due to Direct Urca process suppressed by superfluidity. The cooling curves cannot go through the transition zone and the observational data lying in this region (PRS J0205+6449 in 3C 58, PRS J1357-6429, RX J007.0+7303 in CTA 1, Vela) cannot be explained. In addition, we could find from this figure that all stars with different masses are hotter when including the presence of a rotochemical heating source.

Figure 2 shows the thermal evolution curves of a $1.6M_{\odot}$ superfluid NS for no heating (solid line) or rotochemical heating with various magnetic field strengths: 10^9–10^{12} G (dash lines). The analog figures can be seen in Ref. [6], but they didn't include the effects of nucleon superfluidity. It could be easily found that the rotochemical heating considerably increases the surface temperature of the stellar whose interior contains superfluid nucleon. It's quite clear that the thermal evolution curves strongly depend on the magnetic field strengths, which is related to the properties of the MDR. The stronger the magnetic field is, the earlier the interesting effects of rotochemical heating occur. For the strongest field ($B=10^{12}$ G), the spin-down is rapid and the heating is distinct at early ages, while for the lowest field ($B=10^9$ G), the heating has little effect until very late ages.

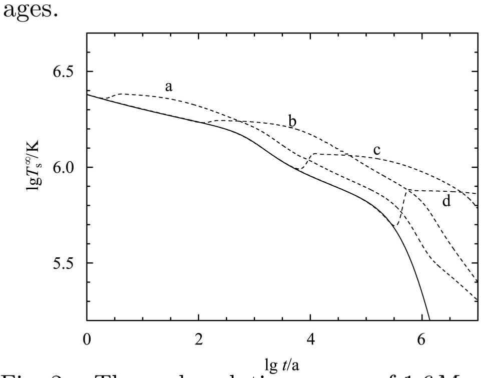

Fig. 2. Thermal evolution curves of $1.6M_{\odot}$ superfluid NSs for no heating (solid line) or rotochemical heating with magnetic field strengths (dash lines) a $B=10^{12}$ G, b $B=10^{11}$ G, c $B=10^{10}$ G and d $B=10^9$ G. The initial spin period is 1 ms.

Figure 3 is the thermal evolution curves of $1.68M_{\odot}$ and $1.7M_{\odot}$ superfluid NSs with no heating (dashed lines) and with rotochemical heating for different magnetic fields (10^9–10^{12} G). The dotted lines refer to $1.68M_{\odot}$ and the solid ones refer to $1.7M_{\odot}$. Note that the Direct Urca threshold mass is $1.686M_{\odot}$. The $1.68M_{\odot}$ star is pretty close to allow the Direct Urca processes and the $1.7M_{\odot}$ star has a small central kernel with the enchanced neutrino emission. Evidently, considering different magnetic fields, the cooling curves fill in the transition zone mentioned above, and the classification of slowly cooling and intermediate cooling become obscure. One can now easily explain those middle-aged and cold stars lying in the transition zone (PRS J0205+6449 in 3C 58, PRS J1357-6429, RX J007.0+7303 in CTA 1, Vela).

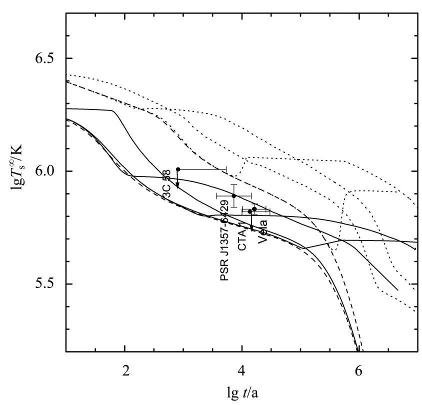

Fig. 3. Thermal evolution curves of $1.68M_{\odot}$ and $1.7M_{\odot}$ superfluid NSs with no heating (dashed lines) and rotochemical heating (dotted line, solid line for two stars respectively) for different magnetic fields $B=10^{12}$ G, $B=10^{11}$ G, $B=10^{10}$ G and $B=10^{9}$ G. The initial spin period is 1 ms.

6 Conclusion and discussion

We have studied the thermal evolution behaviors of rotating NSs with superfluid nucleon cores including the effects of the rotochemical heating, extending the study of Ref. [7], in which the cooling of superfluid neutron stars whose cores consist of nucleon matter with the Akmal-Pandharipande-Ravenhall equation of state has been investigated. The occurrence of particular nucleon superfluidity makes them obtain a representative class of massive neutron stars whose cooling is intermediate between the cooling enchanced by the neutrino emission due to Cooper pairing of neutrons in the absence of the Direct Urca process and the very fast cooling provided by the Direct Urca process non-suppressed by superfluidity. They argue that there are three distinct cooling types: slow, intermediate and fast cooling. Though their model can explain the colder stars, which are probably detected in the future, some middle-aged and cold stars (PRS J0205+6449 in 3C 58, PRS J1357-6429, RX J007.0+7303 in CTA 1, Vela) are hardly explained except by assuming that the masses of these five sources fall in a narrow range. Unfortunately, this is quite impossible. When including the rotochemical heating due to the spin-down of neutron stars, on one hand, the cooling of superfluid NSs delays; on the other hand, the heating effect has strong dependence on the magnetic field strength. Thus our model based on Ref. [7] not only makes up its deficiency, but also eliminates the difference between the slow and intermediate cooling proposed in that work.

Finally, let's mention that although the superfluid models we employ are artificial and phenomenological, they are consistent with the existing results calculated from microscopic theories despite the fact that the properties of nucleon superfluidity are very uncertain so far. With the development of superfluid theory, the relevant results in the study of compact stars will be modified, which we will pay close attention to in future work.

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