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Light four-quark states and QCD sum rule*

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Abstract The relations among four-quark states, diquarks and QCD sum rules are discussed. The situation of the existing, but incomplete studies of four-quark states with QCD sum rules is analyzed. Masses of some diquark clusters were attempted to be determined by QCD sum rules, and masses of some light tetraquark states were obtained in terms of the diquarks.

Key words four-quark state, sum rule, diquark

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1 Introduction

A four-quark state was early predicted to exist in the description of hadron scattering amplitudes[1]. As an exotic meson, its properties have been studied extensively with many methods such as: MIT bag model[2], Color junction model[3], Potential model[4], Effective Lagrangian[5], Relativistic quark model[6], QCD sum rules[7—12] and many other approaches[13, 14]. Some related reviews can be found in Ref. [15]. So far, no four-quark state has been confirmed. However, recent experiments[16] and theoretical investigations revive people's interest on this topic.

In the constituent quark model, a four-quark state consists of two quarks and two anti-quarks. Since a quark/anti-quark has many degrees of freedom: color, flavor, spin, etc., a four-quark state has a complicated internal structure. Intrinsic quarks/anti-quarks may have different correlations and make different clusters. According to the spacial extension of the clusters, four-quark states are usually supposed to have two types: $(\mathrm{qq})(\bar{\mathrm{q}}\bar{\mathrm{q}})$ and $(\mathrm{q}\bar{\mathrm{q}})(\mathrm{q}\bar{\mathrm{q}})$. The dynamics among quarks in this two different kinds of four-quark states may be different, but this two kinds of four-quark states will mix. They will mix with related normal $\mathrm{q}\bar{\mathrm{q}}$ mesons also. Unfortunately, no observable has been established to distinguish the intrinsic correlations or structures in these mesons.

2 QCD sum rules and four-quark state

QCD sum rule is an effective non-perturbative method of relating fundamental parameters of the QCD Lagrangian and vacuum to parameters of hadrons[17]. To proceed with a reasonable sum rule analysis, the employment of a suitable interpolating current (local operator) is very important. In the literature, different interpolating currents have been employed for the study of four-quark states.

In Ref. [7], the following $(\mathrm{q}\bar{\mathrm{q}})(\mathrm{q}\bar{\mathrm{q}})$ currents

$$\begin{aligned} j_1(x) &= (\bar{q}\mathit{\Gamma}\mathit{\Lambda}^{\mathrm{m}}q)(\bar{q}\mathit{\Gamma}\mathit{\Lambda}^{n}q), \\ j_2(x) &= f^{\mathrm{ab_1c_1}}f^{\mathrm{ab_2c_2}}(\bar{q}^{\mathrm{b_1}}\mathit{\Gamma}\mathit{\Lambda}^{\mathrm{m}}q^{\mathrm{c_1}})(\bar{q}^{\mathrm{b_2}}\mathit{\Gamma}\mathit{\Lambda}^{\mathrm{n}}q^{\mathrm{c_2}}), \\ j_3(x) &= (\bar{q}\mathit{\Gamma}h_{\mathrm{v}})(\bar{q}\mathit{\Gamma}\mathit{\Lambda}^{\mathrm{m}}q), \\ j_4(x) &= f^{\mathrm{ab_1c_1}}f^{\mathrm{ab_2c_2}}(\bar{q}^{\mathrm{b_1}}\mathit{\Gamma}h_{\mathrm{v}}^{\mathrm{c_1}})(\bar{q}^{\mathrm{b_2}}\mathit{\Gamma}\mathit{\Lambda}^{\mathrm{m}}q^{\mathrm{c_2}}) \end{aligned} \qquad (1)$$

are used.

In Ref. [8], both $(\mathrm{qq})(\bar{\mathrm{q}}\bar{\mathrm{q}})$ and $(\mathrm{q}\bar{\mathrm{q}})(\mathrm{q}\bar{\mathrm{q}})$ currents

$$\begin{aligned} j_{(\mathrm{q}\bar{\mathrm{q}})^2} &= (\bar{q}\gamma_5\tau^{\mathrm{a}}q)(\bar{q}\gamma_5\tau^{\mathrm{a}}q), \\ j_{(\mathrm{dq})^2} &= \epsilon^{\mathrm{abc}}\epsilon^{\mathrm{ade}}(q^{\mathrm{b}}C\gamma_5\tau_2q^{\mathrm{c}})(\bar{q}^{\mathrm{d}}C\gamma_5\tau_2\bar{q}^{\mathrm{e}}) \end{aligned} \qquad (2)$$

are analyzed.

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In Ref. [9], (cq)($\bar{\mathrm{q}}\bar{\mathrm{q}}$) currents

$$
\begin{aligned}
j_0 &= \epsilon_{\mathrm{abc}}\epsilon_{\mathrm{dec}}(q_{\mathrm{a}}^{\mathrm{T}}C\gamma_5c_{\mathrm{b}})(\bar{u}_{\mathrm{d}}\gamma_5C\bar{d}_{\mathrm{e}}^{\mathrm{T}}),\\
j_{\mathrm{s}} &= \frac{\epsilon_{\mathrm{abc}}\epsilon_{\mathrm{dec}}}{\sqrt{2}}\left[(u_{\mathrm{a}}^{\mathrm{T}}C\gamma_5c_{\mathrm{b}})(\bar{u}_{\mathrm{d}}\gamma_5C\bar{s}_{\mathrm{e}}^{\mathrm{T}})+\mathrm{u}\leftrightarrow\mathrm{d}\right], \quad (3)\\
j_{\mathrm{ss}} &= \epsilon_{\mathrm{abc}}\epsilon_{\mathrm{dec}}(s_{\mathrm{a}}^{\mathrm{T}}C\gamma_5c_{\mathrm{b}})(\bar{q}_{\mathrm{d}}\gamma_5C\bar{s}_{\mathrm{e}}^{\mathrm{T}})
\end{aligned}
$$

are used.

In Ref. [10], (cu)($\bar{\mathrm{s}}\bar{\mathrm{u}}$) currents

$$
J_{\mathrm{k}} = \frac{1}{\sqrt{2}}\left[\epsilon_{\mathrm{ade}}\epsilon_{\mathrm{afg}}c_{\mathrm{d}}^{\mathrm{T}}\mathit{\Gamma}_{\mathrm{k}}u_{\mathrm{e}}\bar{s}_{\mathrm{f}}\tilde{\mathit{\Gamma}}_{\mathrm{k}}\bar{u}_{\mathrm{g}}^{\mathrm{T}}+(\mathrm{u}\rightarrow\mathrm{d})\right] \quad (4)
$$

are used.

In Ref. [11], (ud)($\bar{\mathrm{s}}\bar{\mathrm{s}}$) currents

$$
\begin{aligned}
S_{\mathrm{abcd}} &= (\bar{s}_{\mathrm{a}}\gamma_5C\bar{s}_{\mathrm{b}}^{\mathrm{T}})(u_{\mathrm{c}}^{\mathrm{T}}C\gamma_5d_{\mathrm{d}}),\\
V_{\mathrm{abcd}} &= (\bar{s}_{\mathrm{a}}\gamma_{\mu}\gamma_5C\bar{s}_{\mathrm{b}}^{\mathrm{T}})(u_{\mathrm{c}}^{\mathrm{T}}C\gamma^{\mu}\gamma_5d_{\mathrm{d}}),\\
T_{\mathrm{abcd}} &= (\bar{s}_{\mathrm{a}}\sigma_{\mu\nu}C\bar{s}_{\mathrm{b}}^{\mathrm{T}})(u_{\mathrm{c}}^{\mathrm{T}}C\sigma^{\mu\nu}d_{\mathrm{d}}), \quad (5)\\
A_{\mathrm{abcd}} &= (\bar{s}_{\mathrm{a}}\gamma_{\mu}C\bar{s}_{\mathrm{b}}^{\mathrm{T}})(u_{\mathrm{c}}^{\mathrm{T}}C\gamma^{\mu}d_{\mathrm{d}}),\\
P_{\mathrm{abcd}} &= (\bar{s}_{\mathrm{a}}C\bar{s}_{\mathrm{b}}^{\mathrm{T}})(u_{\mathrm{c}}^{\mathrm{T}}Cd_{\mathrm{d}})
\end{aligned}
$$

are used. Similar currents were employed in some other studies of four-quark states which we have not mentioned here.

Obviously, both (qq)($\bar{\mathrm{q}}\bar{\mathrm{q}}$) and (q$\bar{\mathrm{q}}$)(q$\bar{\mathrm{q}}$) currents were used. Based on the structures of these currents, many conclusions on (qq)($\bar{\mathrm{q}}\bar{\mathrm{q}}$) and (q$\bar{\mathrm{q}}$)(q$\bar{\mathrm{q}}$) four-quark states have been drawn.

Have we really touched the structures of four-quark states in terms of these currents? The answer is NO. As pointed out in Ref. [12], in the framework of QCD sum rules, the internal constituent quark structures of multi-quark states can hardly be detected through the couplings of the interpolating currents to hadrons. In other words, the intrinsic quark correlations can hardly be detected with local operators.

This point is easy to be understood in another way. On one hand, currents (qq)($\bar{\mathrm{q}}\bar{\mathrm{q}}$) and (q$\bar{\mathrm{q}}$)(q$\bar{\mathrm{q}}$) will mix with each other under renormalization. On the other hand, under a Fierz transformation (rearrangement), the current (qq)($\bar{\mathrm{q}}\bar{\mathrm{q}}$) can be transformed into a current (q$\bar{\mathrm{q}}$)(q$\bar{\mathrm{q}}$) as follows

$$
\begin{aligned}
j_{(\mathrm{dq})^2} = \frac{1}{4}\Big\{&(\bar{q}\gamma_5\bar{\tau}^{\mathrm{a}}q)(\bar{q}\gamma_5\bar{\tau}^{\mathrm{a}}q)+(\bar{q}\bar{\tau}^{\mathrm{a}}q)(\bar{q}\bar{\tau}^{\mathrm{a}}q)+\\
&(\bar{q}\gamma_{\mu}\bar{\tau}^{\mathrm{a}}q)(\bar{q}\gamma_{\mu}\bar{\tau}^{\mathrm{a}}q)+(\bar{q}\gamma_5\gamma_{\mu}\bar{\tau}^{\mathrm{a}}q)(\bar{q}\gamma_5\gamma_{\mu}\bar{\tau}^{\mathrm{a}}q)-\\
&\frac{1}{2}(\bar{q}\sigma_{\mu\nu}\bar{\tau}^{\mathrm{a}}q)(\bar{q}\sigma_{\mu\nu}\bar{\tau}^{\mathrm{a}}q)\Big\}. \quad (6)
\end{aligned}
$$

Therefore, there is no definite difference among these currents. Accordingly, conclusions on four-quark states in terms of these four-quark currents are not reasonable. In fact, there is no direct correspondence between the current (operator) picture with the constituent quark picture.

Furthermore, in the existing literature, all calculations are terminated at the leading order of α_{s} because of the difficulties in the operator product expansion, which may result in large deviations.

Recent publications drawing different conclusions on the lowest scalar in the framework of QCD sum rules appears[18, 19].

3 Diquark and light tetraquark state

Diquark clusters in hadrons are in fact a kind of strong correlation between pairs of quarks, which was first mentioned by Gell-Mann[20]. So far, people have not found fact confirming the existence of diquarks in hadrons. However, the concept of diquarks has been applied successfully to many strong-interaction phenomena[21—28].

Diquarks appear often in the interpolating currents within the QCD sum rule approach. However, as we pointed out in Ref. [12], the diquark is a superficial concept in that literature. A diquark is meaningful in the constituent quark picture, but it is not meaningful in the operator picture in the framework of QCD sum rules.

As is well known, a diquark is not an isolated cluster in a multi-quark state, but it may be approximately regarded as a bound state composed of two quarks and may be used as a degree of freedom. In history[22, 28], the diquark picture was applied to weak hadron decays within the QCD sum rule approach.

In Ref. [12], an updated analysis was performed in terms of the diquark current with flavor (sq)

$$
j_{\mathrm{i}}(x)=\epsilon_{\mathrm{ijk}}s_{\mathrm{j}}^{\mathrm{T}}(x)COq_{\mathrm{k}}(x). \quad (7)
$$

Accordingly, the most "suitable" m_{qq} and m_{sq} were determined as:

$$
m_{\mathrm{qq}}\sim 400\ \mathrm{MeV},\quad s_0=1.2\ \mathrm{GeV}^2,
$$

$$
m_{\mathrm{sq}}\sim 460\ \mathrm{MeV},\quad s_0=1.2\ \mathrm{GeV}^2.
$$

The dependence of the diquark masses on flavor is explicit. The mass scale of the diquark is the same as that of the constituent quarks. Our results are consistent with the fit of Maiani et al[14].

Once the masses determined here are regarded as the constituent diquark masses[12], the masses of 0^{++} tetraquark follow as in Ref. [14]

$$
\begin{aligned}
&0^{++}\ [\mathrm{qq}][\bar{\mathrm{q}}\bar{\mathrm{q}}]: \sim 490\ \mathrm{MeV},\\
&0^{++}\ [\mathrm{sq}][\bar{\mathrm{q}}\bar{\mathrm{q}}]: \sim 610\ \mathrm{MeV}, \quad (8)\\
&0^{++}\ [\mathrm{sq}][\bar{\mathrm{s}}\bar{\mathrm{q}}]: \sim 730\ \mathrm{MeV}.
\end{aligned}
$$

Similarly, the masses of the $L=1$ excited tetraquark state follow

$$\begin{aligned}1^{--}\ [\mathrm{qq}][\bar{\mathrm{q}}\bar{\mathrm{q}}]:&\ \sim 490+B'_{\mathrm{q}}\ \mathrm{MeV},\\ 1^{--}\ [\mathrm{sq}][\bar{\mathrm{q}}\bar{\mathrm{q}}]:&\ \sim 610+B'_{\mathrm{q}}\ \mathrm{MeV},\\ 1^{--}\ [\mathrm{sq}][\bar{\mathrm{s}}\bar{\mathrm{q}}]:&\ \sim 730+B'_{\mathrm{s}}\ \mathrm{MeV},\end{aligned} \tag{9}$$

with

$$B'_{\mathrm{q}}=\left[\frac{\alpha_{\mathrm{s}}(m^2_{\mathrm{qq}})}{\alpha_{\mathrm{s}}(m^2_{\mathrm{q}})}\right]^2\frac{m_{\mathrm{qq}}}{m_{\mathrm{q}}}B_{\mathrm{q}},\quad B_{\mathrm{q}}=0.495\ \mathrm{GeV},$$

where B'_{q} is very sensitive to Λ.

Through this exploration, it seems reasonable to identify $\mathrm{f}_0(600)$ (or σ), $\mathrm{f}_0(980)$, $\mathrm{a}_0(980)$ and the unconfirmed $\kappa(800)$ as the 0^{++} light tetraquark states.

4 Conclusions and discussions

The relations among four-quark states, diquarks and QCD sum rules are discussed. It is known that both $(\mathrm{qq})(\bar{\mathrm{q}}\bar{\mathrm{q}})$ and $(\mathrm{q}\bar{\mathrm{q}})(\mathrm{q}\bar{\mathrm{q}})$ interpolating currents were employed within the QCD sum rule approach. However, the properties of $(\mathrm{qq})(\bar{\mathrm{q}}\bar{\mathrm{q}})$ and $(\mathrm{q}\bar{\mathrm{q}})(\mathrm{q}\bar{\mathrm{q}})$ four-quark states can not be detected through these currents. In principle, there is no difference between these currents. On one hand, currents $(\mathrm{qq})(\bar{\mathrm{q}}\bar{\mathrm{q}})$ and $(\mathrm{q}\bar{\mathrm{q}})(\mathrm{q}\bar{\mathrm{q}})$ will mix under renormalization. On the other hand, the current $(\mathrm{qq})(\bar{\mathrm{q}}\bar{\mathrm{q}})$ can be tranformed into the current $(\mathrm{q}\bar{\mathrm{q}})(\mathrm{q}\bar{\mathrm{q}})$ under a Fierz transformation.

Diquarks are useful in the constituent quark model, but it is a superficial concept in the construction of interpolating currents. In fact, there is no correspondence between the constituent quark picture and the current (operator) picture.

The QCD sum rule was combined with the constituent quark model to study the tetraquark state. In this model the masses of diquarks were determined within the QCD sum rule and were regarded as the masses of the constituent diquarks. The obtained masses of the diquarks depend explicitly on flavor. Subsequently, similar as in the constituent quark model, the masses of the 0^{++} and excited 1^{--} tetraquark states were obtained in terms of those diquarks. It is pointed out that the identification of $\mathrm{f}_0(600)$ (or σ), $\mathrm{f}_0(980)$, $\mathrm{a}_0(980)$ and the unconfirmed $\kappa(800)$ as the 0^{++} light tetraquark states seems reasonable.

Of course, the diquark has not yet been confirmed. Whether it can be studied with QCD sum rules requires more exploration. Furthermore, the dynamics in hadron is still not clear, the uncertainty of our model is not clear either.

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