Effect of current quark masses on quark phase transitions in supernovae*

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Abstract The current quark mass model is adopted to study the phase transition of two-flavor quark matter to more stable three-flavor quark matter in the whole core of a supernova. It shows that the timescale of the process is shorter than 10^{-8} seconds, that the u- and d-quark masses can be neglected completely in this model, and that the temperature and the total neutrino energies in the core after the conversion increase nearly by 40% and 20% on the average compared with former results, respectively. The last result can further enhance the probability of success for a supernova explosion significantly.

Key words quark phase transition, current quark mass, supernova

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1 Introduction

In 1984, Witten[1] proposed a famous conjecture that matter which is composed of u-, d- and s-quarks is more stable than hadronic matter. The quark-gluon plasma, which consists of roughly comparable numbers of u-, d- and s-quarks, is called strange quark matter (SQM), which contains a small fraction of electrons in order to satisfy charge neutrality. Witten suggested that the SQM is the most stable strong interaction system and the most favorable state of matter. The possible formation scenario for SQM may be the quark transition in the collapsing process inside a massive star (e.g. supernova)[2].

Before the explosion, the supernova will be in a short-lived equilibrium state caused by the pressure of the degenerate electrons balancing the gravitation. Subsequently, the gravitational core-collapse process of the supernova will take place due to photodisintegration and electron capture[3, 4]. This process is halted when the center density exceeds the nuclear matter density, and then a shock wave is formed at the edge of the proto-neutron star (PNS) and moves fast outward. Unfortunately, a number of numerical simulations indicate that the shock can not rush out of the iron core due to photodisintegration. Although recently good progress in multi-dimensional models (taking into account some physical ingredients, such as convection, rotation, magnetic field and hydrodynamical instability) has been made, the role of the neutrino energy for the occurrence of the supernova explosion is still a matter of debate[5]. Luckily the quark phase transition in the supernova core, including nuclear matter to two-flavor quark matter[6, 7] and two-flavor quark matter to SQM[8, 9], could be the method of choice to remove the problem. The stalled shock could be revived due to a sufficient energy release from the conversion. So a careful investigation of phase transitions is essential when analyzing the formation and moving of the shock wave.

Gentile et al.[10] studied the effect of phase transitions on supernova explosions. He concluded that the shock wave would be revived by the enhanced gravitational binding energy release in the evolution of cores with a conversion from nuclear matte into quark matter. Somewhat later Dai et al.[8] further investigated the effect of the phase transition of two-flavor quark matter to three-flavor quark matter on a supernova explosion. Their results can also enhance the energy of the revived shock wave and then increase the probability for a successful occurrence of a supernova explosion. We expect that the role of quark masses should not be neglected in the conversion.

Due to the occurrence of confinement in QCD the

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precise definition of quark masses and their experimental verification are at present not fully solved problems. The s-quark masses selected previously in Refs. [8, 9] are much larger than the corresponding widely accepted current quark masses for SQM. However, it seems to be more suitable to adopt the current quark masses in a high-energy process[11]. So in this paper we adopt a current quark model under the high-energy conditions of a supernova explosion environment. The reaction rates of the quark phase transition are calculated, and then the effect of the results on the supernova explosion is analyzed. Additionally, we analyze in this model the effects of the u and d quark masses.

The investigation of the transitions occurring in the supernova core should be based on the following reasonable assumptions[8, 12]. Firstly, neutrinos produced by the conversion, u-, d- and s-quarks and electrons are all treated as a highly degenerated relativistic Fermi gas. Secondly, it is a proper approximation to treat the strange quark matter in the inner core as homogeneous matter. According to the calculations by Gentile et al.[10], the strange quark matter in the inner core does not exceed $0.3\mathrm{M}_{\odot}$. Therefore the strong interaction binding effect, caused by the quark color confinement, becomes dominant compared with the gravitational binding effect. Finally, the baryon number density and lepton abundances are assumed not to change during the conversion process.

2 Phase transition reaction rates with current quark masses

The deconfined quark matter is expected to form as a result of a first or second order phase transition during the core-collapse in a supernova. Subsequently a conversion of two-flavor quark matter to the more stable three-flavor quark matter will occur[8]. With regard to the phase transition process in the core of supernovae, neutrinos are relativistic and highly degenerate, so the reactions in which the antineutrinos take part are neglected, and also the strong interaction corrections are not considered in this paper. Finally, the relevant weak reactions to be considered are the following:

$$\mathrm{u}+\mathrm{e}^- \leftrightarrow \mathrm{d}+\mathrm{v_e}\ , \tag{1}$$

$$\mathrm{u}+\mathrm{e}^- \leftrightarrow \mathrm{s}+\mathrm{v_e}\ , \tag{2}$$

$$\mathrm{u}_{(1)}+\mathrm{d} \leftrightarrow \mathrm{u}_{(2)}+\mathrm{s}\ . \tag{3}$$

the excess d-quarks will convert into s-quarks via process (3) which contributes only to the equilibration of flavors. The semi-leptonic weak interactions (1) and (2) are the dominant contribution to the emission of neutrinos. All particles are characterized by the same temperature due to strong interactions[12]. The SQM will be in an equilibrium state due to the above weak interactions. Chemical equilibrium among the quark flavors and the leptons means:

$$\mu_\mathrm{d}+\mu_{\mathrm{v_e}}=\mu_\mathrm{s}+\mu_{\mathrm{v_e}}=\mu_\mathrm{u}+\mu_\mathrm{e}\ . \tag{4}$$

The contribution of the neutrino is not neglected here.

Heiselberg[12] gave an analytical expression for the reaction rate of Eq. (3) up to leading orders in the finite temperature $T(T\sim\mu_\mathrm{q},\ \mathrm{q}=\mathrm{u,d,s})$ and the finite chemical potential difference $\Delta\mu(\Delta\mu\ll\mu_\mathrm{q},\ \mathrm{q}=\mathrm{u,d,s})$ between s- and d-quark. Also Madsen[13] expressed the reaction rate for zero temperature and arbitrary quark chemical potential in analytical form, but they both neglected all flavor (u,d,s) quark masses. Several years later, Anand et al.[9] reinvestigated these conversions in detail for supernovae and neutron stars, taking into account the strong interaction between quarks, but the u- and d-quark masses were still not included in their work. Now three quark masses are taken into account at the same time to calculate the above three reaction rates. First, let $k_\mathrm{B}=\hbar=c=1$ (k_B, $\hbar$ and c are the Boltzmann constant, Planck constant and vacuum speed of light, respectively). The number of reactions per time and per volume for reaction (1) can be obtained using the Weinberg-Salam theory

$$\Gamma^{+}_{(\mathrm{ue}\to\mathrm{dv_e})}=\frac{24}{(2\pi)^8}G_\mathrm{F}^2\cos^2\theta_\mathrm{c}\int\Big[\prod_{i=1}^{4}\mathrm{d}\boldsymbol{p}_i^3\Big]\times \delta^{(4)}(P_1+P_2-P_3-P_4)S(P_1\cdot P_2)(P_3\cdot P_4), \tag{5}$$

where

$$S=f(E_1)f(E_2)[1-f(E_3)][1-f(E_4)]$$

and

$$f(E_i)=\left[1+\exp\left(\frac{E_i-\mu_i}{T}\right)\right]^{-1},\quad (i=1,\cdots,4)$$

are Fermi-Dirac distribution functions. $1\to4$ corresponds to particles of reaction (1) successively. μ_i, E_i, $P_i=(E_i,\boldsymbol{p}_i)$ represent the chemical potential, energy and four-momentum of particle i and T is the temperature. G_F is the Fermi constant (1.435×10^{-49} erg·cm^3) and θ_c is the Cabibbo angle ($\cos^2\theta_\mathrm{c}=0.974$). For finite quark masses we have $E_\mathrm{q}=\sqrt{p_\mathrm{q}^2+m_\mathrm{q}^2}$, for $\mathrm{q}=\mathrm{u,d,s}$. For the sake of simplicity, we introduce the following abbreviations

$$a_\mathrm{q}=\frac{E_\mathrm{q}}{p_\mathrm{q}},\quad b_\mathrm{q}=\frac{\mathrm{d}E_\mathrm{q}}{\mathrm{d}p_\mathrm{q}},\quad \mathrm{q}=\mathrm{u,d,s} \tag{6}$$

and the various number densities n_i, and abundances $Y_\mathrm{i}=n_\mathrm{i}/n_\mathrm{b}$ ($\mathrm{i}=\mathrm{u,d,s,e,v_e}$), where $n_\mathrm{b}=(n_\mathrm{u}+n_\mathrm{d}+n_\mathrm{s})/3$ is the baryon number density. Inserting n_b, Y_i and

Eq. (6) into Eq. (5), the net reaction rate per baryon of Eq. (1) reads as

$$\Gamma_1 = \frac{\Gamma_{(\mathrm{ue}\leftrightarrow \mathrm{d}\nu_\mathrm{e})}}{n_\mathrm{b}} = 6.64\times 10^5 T_{11}^3 \left(\frac{n_\mathrm{b}}{n_0}\right)^{2/3} \times \frac{(Y_\mathrm{u}Y_\mathrm{e}Y_\mathrm{d}Y_{\nu_\mathrm{e}}^2)^{1/3}}{b_\mathrm{u}b_\mathrm{d}} A_1\xi_1(\xi_1^2+4\pi^2) \ , \tag{7}$$

where

$$\begin{aligned} A_1 = \frac{1}{p_\mathrm{F}(4)}\int_0^\infty \frac{1}{x^2}\mathrm{d}x\Bigg\{ & \prod_{i=1}^{4}\sin p_\mathrm{F}(\mathrm{i})x + \\ & c_1c_2\prod_{i=1}^{4}\left(\cos p_\mathrm{F}(\mathrm{i})x - \frac{\sin p_\mathrm{F}(\mathrm{i})x}{p_\mathrm{F}(\mathrm{i})x}\right) + \\ & c_1\left(\cos p_\mathrm{F}(1)x - \frac{\sin p_\mathrm{F}(1)x}{p_\mathrm{F}(1)x}\right)\times \\ & \left(\cos p_\mathrm{F}(2)x - \frac{\sin p_\mathrm{F}(2)x}{p_\mathrm{F}(2)x}\right)\times \\ & \sin p_\mathrm{F}(3)x\sin p_\mathrm{F}(4)x + \\ & c_2\left(\cos p_\mathrm{F}(3)x - \frac{\sin p_\mathrm{F}(3)x}{p_\mathrm{F}(3)x}\right)\times \\ & \left(\cos p_\mathrm{F}(4)x - \frac{\sin p_\mathrm{F}(4)x}{p_\mathrm{F}(4)x}\right)\times \\ & \sin p_\mathrm{F}(1)x\sin p_\mathrm{F}(2)x\Bigg\}. \end{aligned} \tag{8}$$

Here T_{11} means the temperature in units of 10^{11} K, n_0 is the saturation density ($\approx 0.155\mathrm{fm}^{-3}$), $p_\mathrm{F}(\mathrm{i})$ is the Fermi momentum of particle i, $c_1 = 1/a_\mathrm{u}$, $c_2 = 1/a_\mathrm{d}$, $\xi_1 = (\mu_\mathrm{u}+\mu_\mathrm{e}-\mu_\mathrm{d}-\mu_{\nu_\mathrm{e}})/T$. ξ_1 just characterizes the deviation from chemical equilibrium as defined in Eq. (4) and A_1 represents the momentum conservation[8].

Similarly, the net reaction rate per baryon of Eq. (2) is given by

$$\Gamma_2 = \frac{\Gamma_{(\mathrm{ue}\leftrightarrow \mathrm{s}\nu_\mathrm{e})}}{n_\mathrm{b}} = 1.77\times 10^4 T_{11}^3 \left(\frac{n_\mathrm{b}}{n_0}\right)^{2/3} \times \frac{(Y_\mathrm{u}Y_\mathrm{e}Y_\mathrm{s}Y_{\nu_\mathrm{e}}^2)^{1/3}}{b_\mathrm{u}b_\mathrm{s}} A_2\xi_2(\xi_2^2+4\pi^2) \ . \tag{9}$$

A_2 can be obtained by replacing c_1 and c_2 in A_1 with $1/a_\mathrm{u}$ and $1/a_\mathrm{s}$, respectively. $\xi_2 = (\mu_\mathrm{u}+\mu_\mathrm{e}-\mu_\mathrm{s}-\mu_{\nu_\mathrm{e}})/T$.

At last the net reaction rate per baryon for reaction (3) is given by

$$\Gamma_3 = \frac{\Gamma_{(\mathrm{ud}\leftrightarrow \mathrm{us})}}{n_\mathrm{b}} = 45.9T_{11}^5\frac{(Y_\mathrm{u}^2Y_\mathrm{d})^{1/3}}{b_\mathrm{u}^2b_\mathrm{d}} \times A_3[M(\xi_3)-M(-\xi_3)]. \tag{10}$$

Likewise, A_3 can be obtained from A_1 by substituting for c_1 and c_2 the quantities $1/(a_\mathrm{u}a_\mathrm{d})$ and $1/(a_\mathrm{u}a_\mathrm{s})$, respectively. $M(\xi_1)$ is given in Ref. [8].

3 Conversion of quark matter into strange matter

The reaction rates derived from above are applied to the conversion in the inner core of supernovae. The relevant processes are governed by

$$\frac{\mathrm{d}Y_\mathrm{d}}{\mathrm{d}t} = \Gamma_1 - \Gamma_3 \ , \tag{11}$$

$$\frac{\mathrm{d}Y_\mathrm{u}}{\mathrm{d}t} = -\Gamma_1 - \Gamma_2 \ , \tag{12}$$

combined with the conditions of baryon number conservation and charge neutrality:

$$Y_\mathrm{s} = 3 - Y_\mathrm{u} - Y_\mathrm{d} \ , \tag{13}$$

$$2Y_\mathrm{u} = Y_\mathrm{d} + Y_\mathrm{s} + 3Y_\mathrm{e} \ . \tag{14}$$

Because the neutrinos are trapped, the lepton abundance Y_L is assumed to be constant during the transition. Namely, $Y_\mathrm{L} = Y_\mathrm{e} + Y_{\nu_\mathrm{e}}$ is a constant. The temperature as a function of time can be obtained from the first law of thermodynamics[8, 9],

$$\frac{\mathrm{d}T_{11}}{\mathrm{d}t} = -\left(732.9T_{11}\sum_i \frac{Y_\mathrm{i}}{p_\mathrm{F}(i)}\right)^{-1} \times \left[(\mu_\mathrm{u}+\mu_\mathrm{e}-\mu_\mathrm{s}-\mu_{\nu_\mathrm{e}})\frac{\mathrm{d}Y_\mathrm{u}}{\mathrm{d}t} + (\mu_\mathrm{d}-\mu_\mathrm{s})\frac{\mathrm{d}Y_\mathrm{d}}{\mathrm{d}t}\right]. \tag{15}$$

The chemical potential of particle i is given by

$$\mu_i = \sqrt{p_\mathrm{F}(\mathrm{i})^2 + m_\mathrm{i}^2} \quad \mathrm{i} = \mathrm{u,d,s,e,\nu_e} \ , \tag{16}$$

where the Fermi momentum $p_\mathrm{F}(\mathrm{i})$ of particle i can be obtained from Ref. [8]. The current quark masses, which are chosen from the HPQCD Collaboration, are m_u=1.9 MeV, m_d=4.4 MeV, m_s=87 MeV[14] (the uncertainties from statistics, simulation systematics, perturbation theory, and electromagnetic/isospin effects are neglected here). To get the final numerical results, it needs to combine the quark mass with other initial conditions. Those conditions involve the following three aspects. The initial temperatures are selected as 10 MeV, 20 MeV, and 30 MeV, respectively[8]. The lepton abundances Y_L are chosen as 0.32, 0.36 and 0.40, respectively[8, 15]. The baryon number density n_b in the inner core is fixed at $1.5n_0$[10], corresponding to a SQM mass in the inner core of $0.245\mathrm{M}_\odot$. The neutrino abundance can be obtained from the known lepton abundance Y_L[16]

$$Y_\nu = 0.38Y_\mathrm{L}^2 + 0.1Y_\mathrm{L} - 0.0145 \ . \tag{17}$$

Given the initial abundances Y_i of the particles, the initial temperature T_{11}, the baryon number density and quark masses, Eqs. (11), (12) and (15) can be integrated numerically. This way we obtain the particle abundances Y_i, the temperature T_{11} and the total neutrino energy production rate as a function of time and their equilibrium numerical values.

4 Numerical results and discussion

We present in this chapter the numerical results of our calculation of the conversion of two-flavor into three-flavor quark matter in a supernova core, taking into account the current quark masses. At first sight one can see from Fig. 1 and Fig. 2 that the conversion timescale is shorter than 10^{-8} seconds, which is 1 order of magnitude shorter than that given in Ref. [8]. This guarantees the validity of the assumption that the baryon number density and lepton abundances are invariant during the transition. The trends of variation with time of the temperature and the neutrino abundance are nearly the same as those found in the literature[8, 9]. But, as can be seen from Table 1, the values of the equilibrium quantities after the conversion are obviously bigger than those in the literature[8]. If we compare our results with the conclusions from Ref. [8], we find the following: For the considered initial temperature (before the phase transition) of 10, 20 and 30 MeV, our corresponding final temperatures increase nearly by 60%, 41% and 26%. At the same time our calculated total neutron energies in the entire core increase nearly by 41%, 39% and 37%. The neutrino abundances for the three cases increase almost by 20%, and the s-quark abundance nearly by 72%, respectively. These results can easily be explained within our the current quark mass model. The smaller s-quark mass (neglecting the u- and d-quark masses) leads to a lower s-quark chemical potential. The transition reactions go easier in case the difference between the chemical potentials before and after the transition is larger, i.e. more s-quarks are produced, more neutrinos and more energy can be released.

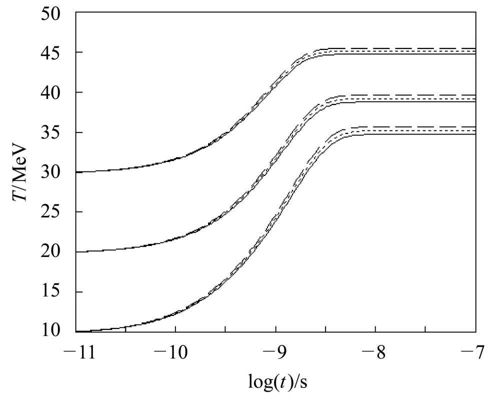

Fig. 1. The temperature vs. time for baryon number density $n_b = 1.5n_0$. The dashed, doted and solid lines correspond to the lepton abundances $Y_L = 0.32$, 0.36, 0.40, respectively. Each bundle of lines from the bottom to the top correspond to the initial temperature T=10 MeV, 20 MeV, and 30 MeV, respectively.

The effect of the u- and d-quark masses on the phase transition in the supernova core is demonstrated in Table 2, where the final parameters after the transition are given for a group of typical initial values. As far as the current quark mass model is concerned, according to Table 2, the corresponding equilibrium quantities are almost the same, irrespective whether the u- and d-quark masses are taken into account or not. Such a small effect of the u- and d-quark masses can be safely neglected if the current quark mass model is adopted to discuss the quark conversion in a supernova environment.

Table 1. The initial "i" and final "f" values of the parameters used in the conversion calculation, based on the present current quark mass model with a baryon number density $n_b = 1.5n_0$. T is given in MeV, E in units of 10^{52}erg.

$Y_{\nu_e i}$	$Y_{\nu_e f}$	T_i	T_f	Y_{ui}	Y_{uf}	Y_{di}	Y_{df}	Y_{si}	Y_{sf}	Y_{ei}	Y_{ef}	$E_{\nu_e i}$	$E_{\nu_e f}$
0.0564	0.1239	10	35.6704	1.2636	1.1961	1.7364	0.9809	0	0.8229	0.2636	0.1961	1.10	2.3960
0.0707	0.1415	10	35.1791	1.2893	1.2185	1.7107	0.9694	0	0.8121	0.2893	0.2185	1.47	2.9345
0.0863	0.1595	10	34.7021	1.3137	1.2405	1.6863	0.9581	0	0.8014	0.3137	0.2405	1.91	3.5429
0.0564	0.1239	20	39.6532	1.2636	1.1961	1.7364	0.9809	0	0.8229	0.2636	0.1961	1.29	2.6767
0.0707	0.1415	20	39.2119	1.2893	1.2185	1.7107	0.9694	0	0.8121	0.2893	0.2185	1.69	3.2365
0.0863	0.1595	20	38.7846	1.3137	1.2405	1.6863	0.9581	0	0.8014	0.3137	0.2405	2.16	3.8611
0.0564	0.1239	30	45.5234	1.2636	1.1961	1.7364	0.9809	0	0.8229	0.2636	0.1961	1.62	3.1755
0.0707	0.1415	30	45.1395	1.2893	1.2185	1.7107	0.9694	0	0.8121	0.2893	0.2185	2.07	3.7897
0.0863	0.1595	30	44.7688	1.3137	1.2405	1.6863	0.9581	0	0.8014	0.3137	0.2405	2.59	4.4693

Table 2. Final values of the parameters after the conversion using the present current quark mass model with a baryon number density $n_b = 1.5n_0$, $Y_L = 0.32$, T=10 MeV. I and II correspond to the two cases with and without considering the u- and d-quark masses.

mass model	$Y_{\nu_e f}$	T_f	Y_{uf}	Y_{df}	Y_{sf}	Y_{ef}	$E_{\nu_e f}$
I	0.1239	35.6704	1.1961	0.9809	0.8229	0.1961	2.3960
II	0.1239	35.6645	1.1961	0.9811	0.8227	0.1961	2.3954

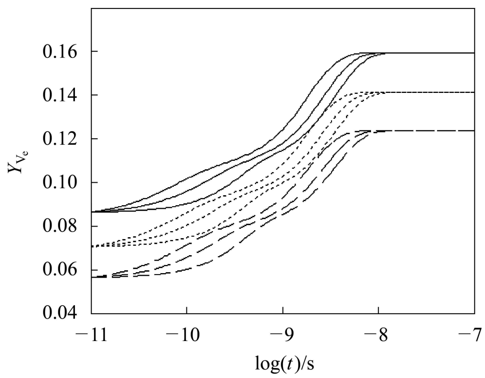

Fig. 2. The neutrino abundance vs. time for baryon number density $n_b = 1.5n_0$. The dashed, doted and solid lines correspond to the lepton abundances $Y_L = 0.32$, 0.36, and 0.40, respectively. Each bundle of lines, from the bottom to the top, corresponds to the initial temperature T=10 MeV, 20 MeV, and 30 MeV, respectively.

The decisive factor determining the average energy of the neutrinos in the delayed neutrino-heating mechanism in a supernova explosion, is the temperature of the supernova core[17, 18]. In the stage of the shock wave formation and its following outward directed progression, the deleptonization and heating of a PNS depend on the central temperature and neutrino number density[16]. The present work shows that the central temperature and neutrino density increase significantly compared with the results from other groups. This result will further affect the dynamics calculation of the supernova evolution, and greatly increase the probability for the (theoretical) occurrence of the delayed neutrino-heated explosion in supernova.

5 Conclusion

In this paper, the current quark mass model is adopted to discuss the conversion of two-flavor quark matter into three-flavor quark matter in supernova core. We find: the u, d quark mass can be neglected completely in this model; the timescale of the process is below 10^{-8} seconds; the temperature and the total neutrino energies in the core after the conversion increase greatly compared with the former results. This significantly enhances the chances for a successful supernova explosion to take place in supernova model calculations.

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