# Study of pure annihilation decays $B_{d,s} \to D^0\overline{D}^0$ *

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**Abstract** Within the heavy quark limit and the hierarchy approximation $\lambda_{QCD} \ll m_D \ll m_B$, we analyze the $B \to D^0\overline{D}^0$ and $B_s \to D^0\overline{D}^0$ decays, which occur purely via annihilation type diagrams. As a rough estimate, we calculate their branching ratios and $CP$ asymmetries in the perturbative QCD (PQCD) approach. The branching ratio of $B \to D^0\overline{D}^0$ is about $3.8\times10^{-5}$ that is just below the latest experimental upper limit. The branching ratio of $B_s \to D^0\overline{D}^0$ is about $6.8\times10^{-4}$, which could be measured in LHC-b. From the calculation, it is found that this branching ratio is not sensitive to the weak phase angle $\gamma$. In these two decay modes, there exist $CP$ asymmetries because of the interference between weak and strong interaction. However, these asymmetries are too small to be measured easily.

**Key words** perturbative QCD approach, B meson, $CP$ violation

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## 1 Introduction

In the Standard Model (SM), $CP$-violation ($CP$V) arises from a complex phase in the Cabibbo-Kobayashi- Maskawa (CKM) quark mixing matrix, and the angles of the unitary triangle are defined as[1]:

$$\beta = \arg\left[-\frac{V_{cb}^*V_{cd}}{V_{tb}^*V_{td}}\right], \quad \alpha = \arg\left[-\frac{V_{tb}^*V_{td}}{V_{ub}^*V_{ud}}\right],$$
$$\gamma = \arg\left[-\frac{V_{ub}^*V_{ud}}{V_{cb}^*V_{cd}}\right]. \tag{1}$$

In order to test the SM and search for new physics, many measurements of $CP$-violation observables can be used to constrain above mentioned angles. It is well known that we measure $\beta$ precisely using the golden decay mode $B \to J/\psi K_s$; the angle $\alpha$ can be determined with the decay $B \to \pi\pi$ and $\gamma$ could be measured precisely in the Large Hadron Collider (LHC) with the decay mode $B_s \to D_sK$.

Besides the above mentioned channels, many other channels are used to cross check the measurements. Among these decays, $B \to DD$ decay is considered to test the $\beta$ measurement. For the $B \to DD$ decay, the analysis based on $SU(3)$ symmetry[2], iso-spin symmetry[3], factorization approach[4, 5] and other approaches[6] have been done in the past several years. However, the calculation of the decay $B^0 \to D^0\overline{D}^0$ has difficulties. It is a pure-annihilation diagram decay, also named W-exchange diagram decay, which is power suppressed in factorization language. The quark diagrams of this decay are shown in Fig. 1. Theoretically, the QCD factorization approach (QCDF)[7] and the soft collinear effective theory (SCET)[8] cannot deal effectively with decays with two heavy charmed mesons. In Refs. [9, 10], perturbative QCD (PQCD) has been exploited to B meson decays with one charmed meson in the final state and the results agree well with the experimental data. Specially, the pure annihilation-type B decays with charmed mesons were studied in Ref. [10].

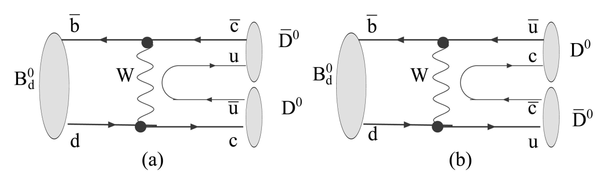


Fig. 1. The quark level Feynman diagrams for the $B_d \to D^0\bar{D}^0$ process.

In the standard model picture, the W boson exchange causes $\bar{b}d \to \bar{c}c$, and the $\bar{u}u$ quarks are produced from a gluon. This gluon attaches to any one of the quarks participating in the W boson exchange. In the decay $B \to D^0\overline{D}^0$, the momentum of the final state

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D meson is $\frac{1}{2}m_{\rm B}(1-2r^2)$, with $r=m_{\rm D}/m_{\rm B}$. If we consider the heavy quark limit and the hierarchy approximation $\lambda_{\rm QCD}\ll m_{\rm D}\ll m_{\rm B}$, the D meson momentum is nearly $m_{\rm B}/2$. According to the distribution amplitude used in Ref. [9], the light quark in the D meson carries nearly 40% of the D meson momentum. So, this light quark is still a collinear quark with 1 GeV energy, like that in $\rm B\to DM$[9, 10], $\rm B\to K(\pi)\pi$[11, 12] decays. The gluon could approximatively be viewed as a hard gluon, so that we can treat the process perturbatively, where the four-quark operator exchanges a hard gluon with an $\rm u\bar{u}$ quark pair. Of course, we are able to calculate the diagrams if the charm quark and the up quark are exchanged. As a rough estimation, we give the branching ratio and $CP$-violation of $\rm B_{d,s}\to D^0\overline{D}^0$.

In the next section we will develop the analytic formulae for the decay amplitudes. In Section 3, we give the numerical results and summarize this article in Section 4.

## 2 Analytic formulae

For simplicity, we set the B meson at rest in our calculation. In light-cone coordinates, the momentum of B, $\rm D^0$ and $\rm \overline{D}^0$ are:

$$P_{\rm B} = \frac{M_{\rm B}}{\sqrt{2}}(1,1,\mathbf{0});\quad P_2=\frac{M_{\rm B}}{\sqrt{2}}(1-r^2,r^2,\mathbf{0});$$
$$P_3 = \frac{M_{\rm B}}{\sqrt{2}}(r^2,1-r^2,\mathbf{0}). \tag{2}$$

We define the light (anti-)quark momenta in the B, $\rm D^0$ and $\rm \overline{D}^0$ mesons, $k_1$, $k_2$, and $k_3$ as:

$$k_1 = (x_1P_1^+,0,\boldsymbol{k}_{1T}),\quad k_2=(x_2P_2^+,0,\boldsymbol{k}_{2T}),$$
$$k_3 = (0,x_3P_3^-,\boldsymbol{k}_{3T}). \tag{3}$$

In PQCD, we factorize the decay amplitude into soft ($\Phi$), hard ($H$), and harder ($C$) dynamics, characterized by different scales[11, 12],

$$\mathcal{A}\sim \int {\rm d}x_1{\rm d}x_2{\rm d}x_3b_1{\rm d}b_1b_2{\rm d}b_2b_3{\rm d}b_3\times {\rm Tr}\Big[C(t)\Phi_{\rm B}(x_1,b_1)\Phi_{\rm D}(x_2,b_2)\Phi_{\rm D}(x_3,b_3)\times H(x_i,b_i,t)S_t(x_i)\,{\rm e}^{-S(t)}\Big]. \tag{4}$$

In the above equation, $b_i$ is the conjugate space coordinate of the transverse momentum $\boldsymbol{k}_{iT}$, and $t$ is the largest energy scale. $C$ is the Wilson coefficient, and $\Phi$ is the wave function. The last term, ${\rm e}^{-S(t)}$, contains two kinds of contributions. One is due to the resummation of the large double logarithms from the renormalization of the ultra-violet divergence $\ln tb$. The other contribution comes from the resummation of the double logarithm $\ln^2 b$ from the overlap of collinear and soft gluon corrections, which is called Sudakov form factor. The hard part $H$ can be calculated perturbatively, and it is channel dependent. More detailed explanations of the above formula and reviews on PQCD can be found in many references, such as[11—13].

As a heavy meson, the B meson wave function is not well defined, neither is that of the D meson. In the heavy quark limit, we take them as:

$$\Phi_{\rm B}(x,b)=\frac{\rm i}{\sqrt{6}}\left[\not{P}+M_{\rm B}\right]\gamma_5\phi_{\rm B}(x,b), \tag{5}$$

$$\Phi_{\rm D}(x,b)=\frac{\rm i}{\sqrt{6}}\gamma_5\left[\not{P}+M_{\rm D}\right]\phi_{\rm D}(x,b). \tag{6}$$

The Lorentz structure of the two mesons are different because the B meson is the initial state and D meson is the final state.

The effective Hamiltonian for $\rm \bar{b}\to\bar{q}(q=d,s)$ is given by[14]:

$$\mathcal{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}}\Big\{V_{\rm cq}V_{\rm cb}^*\left[C_1(\mu)O_1^{\rm c}(\mu)+C_2(\mu)O_2^{\rm c}(\mu)\right]+ V_{\rm uq}V_{\rm ub}^*\left[C_1(\mu)O_1^{\rm u}(\mu)+C_2(\mu)O_2^{\rm u}(\mu)\right]- V_{\rm tb}^*V_{\rm tq}\sum_{i=3}^{10}C_i(\mu)O_i(\mu)\Big\}, \tag{7}$$

where $C_i(\mu)(i=1,\cdots,10)$ are the Wilson coefficients at the renormalization scale $\mu$ and the four quark operators $O_i(i=1,\cdots,10)$ are

$$\begin{aligned}
O_1^{\rm c} &= (\bar{b}_ic_j)_{V-A}(\bar{c}_jq_i)_{V-A},\\
O_2^{\rm c} &= (\bar{b}_ic_i)_{V-A}(\bar{c}_jq_j)_{V-A},\\
O_1^{\rm u} &= (\bar{b}_iu_j)_{V-A}(\bar{u}_jq_i)_{V-A},\\
O_2^{\rm u} &= (\bar{b}_iu_i)_{V-A}(\bar{u}_jq_j)_{V-A},\\
O_3 &= (\bar{b}_iq_i)_{V-A}\sum\nolimits_q(\bar{q}_jq_j)_{V-A},\\
O_4 &= (\bar{b}_iq_j)_{V-A}\sum\nolimits_q(\bar{q}_jq_i)_{V-A},\\
O_5 &= (\bar{b}_iq_i)_{V-A}\sum\nolimits_q(\bar{q}_jq_j)_{V+A},\\
O_6 &= (\bar{b}_iq_j)_{V-A}\sum\nolimits_q(\bar{q}_jq_i)_{V+A},\\
O_7 &= \frac{3}{2}(\bar{b}_iq_i)_{V-A}\sum\nolimits_q e_q(\bar{q}_jq_j)_{V+A},\\
O_8 &= \frac{3}{2}(\bar{b}_iq_j)_{V-A}\sum\nolimits_q e_q(\bar{q}_jq_i)_{V+A},\\
O_9 &= \frac{3}{2}(\bar{b}_iq_i)_{V-A}\sum\nolimits_q e_q(\bar{q}_jq_j)_{V-A},\\
O_{10} &= \frac{3}{2}(\bar{b}_iq_j)_{V-A}\sum\nolimits_q e_q(\bar{q}_jq_i)_{V-A}.
\end{aligned} \tag{8}$$

Here $i$ and $j$ are the $SU(3)$ color indices; in $O_{3,\cdots,10}$ the sum over q runs over the quark fields that are

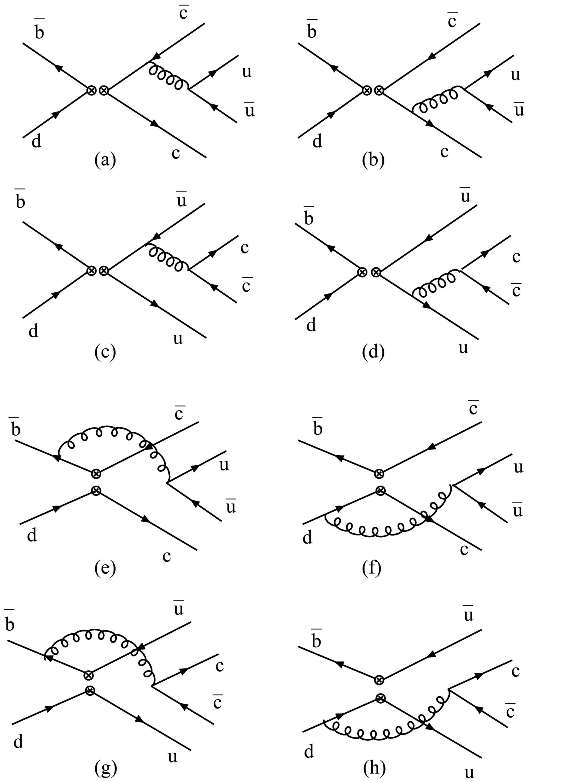


Fig. 2. The leading order Feynman diagrams for $B_d \to D^0\overline{D}^0$ process in PQCD approach.

active at the scale $\mu = O(m_b)$, i.e., $q \in \{u, d, s, c, b\}$. For the Wilson coefficients, we will also use the leading logarithm summation for the QCD corrections, although the next-to-leading order calculation already exists[14]. This is the consistent way to cancel the explicit $\mu$ dependence in the theoretical formulae.

According to the effective Hamiltonian in Eqs. (7), (8), the lowest order diagrams of $B \to D^0\overline{D}^0$ are drawn in Fig. 2. We first calculate the usual factorizable diagrams (a), (b), (c) and (d). For the $(V-A)(V-A)$ operators, their contributions of (a) and (c) are always canceled by diagrams (b) and (d), respectively (because of current conservation). For the $(V-A)(V+A)$ operators, these diagrams do not contribute, either, ie., factorizable diagrams have no contribution. For the non-factorizable diagrams (e), (f), (g) and (h), we find that the hard part of the $(V-A)(V-A)$ operators are the same as those of the $(V-A)(V+A)$ operators. We combine the contribution of diagrams (e) and (f) into $M_a$, as follows:

$$\begin{aligned} M_a[C_i] \;=\; & \frac{64\pi C_F M_B^2}{\sqrt{2N_C}}\int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \times \\ & \phi_B(x_1,b_1)\phi_D(x_2,b_2)\phi_D(x_3,b_2)\times \\ & \Big\{\big[x_1+x_2+(2x_3-x_2)r^2\big]C_i(t_a^1)E(t_a^1)\times \\ & h_a^{(1)}(x_1,x_2,x_3,b_1,b_2)+ \\ & \big[-x_3+(2x_1-2x_2+x_3)r^2\big]\times \\ & C_i(t_a^2)E(t_a^2)h_a^{(2)}(x_1,x_2,x_3,b_1,b_2)\Big\}, \end{aligned} \tag{9}$$

where $C_F = 4/3$ is the group factor of the $SU(3)_c$ gauge group. The function $E_m$ is defined as

$$E(t) = \alpha_s(t)\, e^{-S_B(t)-S_D(t)-S_D(t)}, \tag{10}$$

and $S_B$, $S_D$ result from the Sudakov factor and the single logarithms due to the renormalization of the ultra-violet divergence. The functions $h_a$ are the Fourier transformations of the virtual quark and gluon propagators. They are defined by

$$\begin{aligned} h_a^{(j)}(x_1,x_2,x_3,b_1,b_2) = & \Big\{\frac{\pi i}{2}H_0^{(1)}(M_B\sqrt{x_2x_3(1-2r^2)}\,b_1)\times \\ & J_0(M_B\sqrt{x_2x_3(1-2r^2)}\,b_2)\theta(b_1-b_2)+(b_1\leftrightarrow b_2)\Big\}\times \\ & \begin{pmatrix} K_0(M_B F_{a(j)} b_1), & \text{for} \quad F^2_{a(j)}>0 \\ \frac{\pi i}{2}H_0^{(1)}(M_B\sqrt{|F^2_{a(j)}|}\,b_1), & \text{for} \quad F^2_{a(j)}<0 \end{pmatrix}, \end{aligned} \tag{11}$$

with:

$$\begin{aligned} F^2_{a(1)} \;=\; & -x_1-x_2-x_3+x_1x_3+x_2x_3+ \\ & (x_2+x_3-x_1x_3-2x_2x_3)r^2; \end{aligned} \tag{12}$$

$$F^2_{a(2)} \;=\; x_2x_3-x_1x_3+(x_1x_3-2x_2x_3)r^2. \tag{13}$$

In the above equation, $H_0^{(1)}(z) = J_0(z)+i\,Y_0(z)$ is a Hankel function of the first kind. In order to reduce the large logarithmic radiative corrections, the hard scale $t$ in the amplitudes is selected as the largest energy scale in the hard part:

$$t_a^j = \max(M_B\sqrt{|F^2_{a(j)}|},\, M_B\sqrt{(1-2r^2)x_2x_3}, 1/b_1, 1/b_2). \tag{14}$$

Analogically, we can get the $M_b$, which comes from the contribution of diagrams (g) and (h):

$$\begin{aligned} M_b[C_i] \;=\; & \frac{64\pi C_F M_B^2}{\sqrt{2N_C}}\int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2\, \phi_B(x_1,b_1)\phi_D(x_2,b_2)\phi_D(x_3,b_2)\times \\ & \Big\{\big[1-x_3+(2+2x_1-2x_2+x_3)r^2\big]C_i(t_b^1)E(t_b^1)h_b^{(1)}(x_1,x_2,x_3,b_1,b_2)+ \\ & \big[x_1+x_2-1+(-2-x_2+2x_3)r^2\big]C_i(t_b^2)E(t_b^2)h_b^{(2)}(x_1,x_2,x_3,b_1,b_2)\Big\}, \end{aligned} \tag{15}$$

where the following functions have been defined as:

$$h_{\rm b}^{(\rm j)}(x_1,x_2,x_3,b_1,b_2)=\left\{\frac{\pi \rm i}{2}H_0^{(1)}(M_{\rm B}\sqrt{1-x_2-x_3+x_2x_3+(x_2+x_3-2x_2x_3)r^2}\,b_1)\times J_0(M_{\rm B}\sqrt{1-x_2-x_3+x_2x_3+(x_2+x_3-2x_2x_3)r^2}\,b_2)\theta(b_1-b_2)+(b_1\leftrightarrow b_2)\right\}\times \begin{pmatrix} K_0(M_{\rm B}F_{\rm b(j)}b_1), & \text{for} \quad F^2_{\rm b(j)}>0 \\ \frac{\pi \rm i}{2}H_0^{(1)}(M_{\rm B}\sqrt{|F^2_{\rm b(j)}|}\,b_1), & \text{for} \quad F^2_{\rm b(j)}<0 \end{pmatrix}; \tag{16}$$

$$F^2_{\rm b(1)} = -1-x_1x_3+x_2x_3+(x_1x_3-2x_2x_3)r^2, \tag{17}$$

$$F^2_{\rm b(2)} = 1-x_1-x_2-x_3+x_1x_3+x_2x_3+(x_2+x_3-x_1x_3-2x_2x_3)r^2, \tag{18}$$

$$t_{\rm b}^{\rm j} = \max(M_{\rm B}\sqrt{|F^2_{\rm b(j)}|},\ \ M_{\rm B}\sqrt{1-x_2-x_3+x_2x_3+(x_2+x_3-2x_2x_3)r^2}, 1/b_1, 1/b_2). \tag{19}$$

We obtain then the decay amplitude of the decay $\rm B_d \to D^0\overline{D}^0$ as:

$$\begin{aligned}\mathcal{A}_1 &= V^*_{\rm cb}V_{\rm cd}M_{\rm a}[C_2]-V^*_{\rm tb}V_{\rm td}M_{\rm a}[C_5+C_7]+ \\ &V^*_{\rm ub}V_{\rm ud}M_{\rm b}[C_2]-V^*_{\rm tb}V_{\rm td}M_{\rm b}[C_5+C_7]= \\ &V^*_{\rm cb}V_{\rm cd}T_1-V^*_{\rm tb}V_{\rm td}P_1= \\ &V^*_{\rm tb}V_{\rm td}P_1(1+z_1{\rm e}^{{\rm i}(\beta+\delta_1)}),\end{aligned} \tag{20}$$

where $\beta$ is the weak phase angle defined in Eq. (1), and $\delta_1$ is the strong phase, which plays an important role in studying $CP$-violation. In the above calculation we used the following notation:

$$\begin{aligned}T_1 &= M_{\rm a}[C_2]-M_{\rm b}[C_2], \\ P_1 &= M_{\rm a}[C_5+C_7]+M_{\rm b}[C_5+C_7]+M_{\rm b}[C_2],\end{aligned} \tag{21}$$

and

$$z_1=\left|\frac{V^*_{\rm cb}V_{\rm cd}}{V^*_{\rm tb}V_{\rm td}}\right|\left|\frac{T_1}{P_1}\right|, \tag{22}$$

which describes the ratio between the tree diagram and the penguin diagram. The corresponding charge conjugate decay is described by the amplitude

$$\overline{\mathcal{A}_1}=V_{\rm tb}V^*_{\rm td}P_1\big(1+z_1{\rm e}^{{\rm i}(-\beta+\delta_1)}\big). \tag{23}$$

Therefore, the averaged decay width $\Gamma$ for $\rm B^0\to D^0\overline{D}^0$ decay is then given by

$$\Gamma(\rm B^0\to D^0\overline{D}^0) = \frac{G_{\rm F}^2M_{\rm B}^3}{128\pi}(1-2r^2)\big|V^*_{\rm tb}V_{\rm td}P_1\big|^2\times \big|1+z_1^2+2z_1\cos\beta\cos\delta_1\big|. \tag{24}$$

From this equation, we know that the averaged branching ratio is a function of CKM angle $\beta$, if $z_1\neq 0$. Derived from Eq. (20) and Eq. (23), the direct $CP$-violation can be formulated as:

$$A_{CP}^{\rm dir}(\rm B\to D^0\overline{D}^0) = \frac{\big|A_{\rm B_d\to D^0\overline{D}^0}\big|^2-\big|A_{\rm \overline{B}_d\to \overline{D}^0D^0}\big|^2}{\big|A_{\rm B_d\to D^0\overline{D}^0}\big|^2+\big|A_{\rm \overline{B}_d\to \overline{D}^0D^0}\big|^2} = \frac{-2z_1\sin\beta\sin\delta_1}{1+z_1^2+2z_1\cos\beta\cos\delta_1}. \tag{25}$$

For $\rm B^0_s\to D^0\overline{D}^0$ and its conjugate decay, we write the decay amplitudes and rearrange them as:

$$\begin{aligned}\mathcal{A}_2 &= V^*_{\rm cb}V_{\rm cs}M_{\rm a}[C_2]-V^*_{\rm tb}V_{\rm ts}M_{\rm a}[C_5+C_7]+ \\ &V^*_{\rm ub}V_{\rm us}M_{\rm b}[C_2]-V^*_{\rm tb}V_{\rm ts}M_{\rm b}[C_5+C_7]= \\ &V^*_{\rm ub}V_{\rm us}M_{\rm b}[C_2]-V^*_{\rm tb}V_{\rm ts}\Big\{M_{\rm a}[C_5+C_7]+ \\ &M_{\rm b}[C_5+C_7]-\frac{V^*_{\rm cb}V_{\rm cs}}{V^*_{\rm tb}V_{\rm ts}}M_{\rm a}[C_2]\Big\}= \\ &V^*_{\rm ub}V_{\rm us}T_2-V^*_{\rm tb}V_{\rm ts}P_2= \\ &V^*_{\rm ub}V_{\rm us}T_2\Big[1+z_2{\rm e}^{{\rm i}(-\gamma+\delta_2)}\Big],\end{aligned} \tag{26}$$

$$\overline{\mathcal{A}_2} = V_{\rm ub}V^*_{\rm us}T_2\Big[1+z_2{\rm e}^{{\rm i}(\gamma+\delta_2)}\Big], \tag{27}$$

where $T_2$, $P_2$ and $z_2$ are defined as:

$$\begin{aligned}T_2 &= M_{\rm b}[C_2], \\ P_2 &= M_{\rm a}[C_5+C_7]+M_{\rm b}[C_5+C_7]-\frac{V^*_{\rm cb}V_{\rm cd}}{V^*_{\rm tb}V_{\rm ts}}M_{\rm a}[C_2], \\ z_2 &= \left|\frac{V^*_{\rm tb}V_{\rm ts}}{V^*_{\rm ub}V_{\rm us}}\right|\left|\frac{T_2}{P_2}\right|.\end{aligned} \tag{28}$$

So, the averaged decay width and direct $CP$ violation can be formulated as:

$$\Gamma(\rm B_s\to D^0\overline{D}^0) = \frac{G_{\rm F}^2M_{\rm B}^3}{128\pi}(1-2r^2)\big|V_{\rm ub}V^*_{\rm us}T_2\big|^2\times (1+z_2^2+2z_2\cos\delta_2\cos\gamma), \tag{29}$$

$$A_{CP}^{\rm dir}(B_s\to D^0\overline{D}^0)=\frac{|A_{B_s\to D^0\overline{D}^0}|^2-|A_{\overline{B}_s\to\overline{D}^0D^0}|^2}{|A_{B_s\to D^0\overline{D}^0}|^2+|A_{\overline{B}_s\to\overline{D}^0D^0}|^2}=\frac{2z_2\sin\gamma\sin\delta_2}{1+z_2^2+2z_2\cos\gamma\cos\delta_2}. \tag{30}$$

In our calculation, we set $m_c\approx m_D$, just because $m_D-m_c\approx\Lambda_{QCD}$ and $\frac{\Lambda_{QCD}}{m_B}\to 0$ in the heavy quark limit.

## 3 Numerical results

For the B meson, the distribution amplitude is well determined by the charmless B decays[11, 12], which is chosen as

$$\phi_B(x,b)=N_Bx^2(1-x)^2\exp\left[-\frac{M_B^2\,x^2}{2\omega_b^2}-\frac{1}{2}(\omega_b b)^2\right], \tag{31}$$

with parameters $\omega_b=0.4$ GeV, and $N_B=91.745$ GeV which is the normalization constant using $f_B=190$ MeV. For the $B_s$ meson, we use the same distribution amplitude according to $SU(3)$ symmetry, where $\omega_b=0.4$ GeV, $N_{B_s}=119.4$ GeV and $f_{B_s}=230$ MeV.

Since the c quark is much heavier than the u quark, the c quark in the D meson picks up more momentum, and therefor the distribution amplitude should be asymmetric with respect to $x=1/2$. The asymmetry is parameterized by $a_D$. Similar to the $b$-dependence of the wave function of the B meson, for controlling the size of charmed mesons, we also introduce the intrinsic $b$-dependence similar to those of charmed mesons. Hence, we use the wave function of the D meson as[15]

$$\phi_D(x,b)=\frac{3}{\sqrt{2N_c}}f_Dx(1-x)\Big[1+a_D(1-2x)\Big]\times\exp\left[-\frac{1}{2}(\omega_D b)^2\right]. \tag{32}$$

We use $a_D=0.7$ and $\omega_D=0.4$ in the above function. Other parameters, such as the meson masses, decay constants, the CKM matrix elements and the lifetime of the B meson are listed below[1, 16]:

$$\begin{gathered}M_B=5.28\text{ GeV},\ M_{B_s}=5.36\text{ GeV},\ M_D=1.87\text{ GeV},\\ f_D=210\text{ MeV},|V_{ud}|=0.974,\ |V_{ub}|=4.3\times10^{-3},\\ |V_{cd}|=0.23,\ |V_{cb}|=41.6\times10^{-3},\ |V_{td}|=7.4\times10^{-3},\\ |V_{tb}|=1.0,\ |V_{us}|=0.226,\ |V_{cs}|=0.957,\\ |V_{ts}|=41.6\times10^{-3},\ \tau_{B_d^0}=1.54\times10^{-12}\text{ s},\\ \tau_{B_s^0}=1.46\times10^{-12}\text{ s}.\end{gathered} \tag{33}$$

With these parameters fixed, we calculated the decay amplitudes of the $B^0\to D^0\overline{D}^0$ and $B_s\to D^0\overline{D}^0$ decays in Table 1. From the table, we notice that the main contribution comes from the tree diagram (e) and (f). And our predictions for the branching ratio of each mode corresponding to $\beta=23°$ and $\gamma=63°$ are listed below,

$$\begin{aligned}BR(B_d\to D^0\overline{D}^0)&=2.3\times10^{-5};\\ BR(B_s\to D^0\overline{D}^0)&=6.8\times10^{-4}.\end{aligned} \tag{34}$$

Fig. 3 shows the branching ratio of $B_s\to D^0\overline{D}^0$ as a function of the $\gamma$. It can be seen that the branching ratio is not sensitive to the CKM angle $\gamma$. From the experimental side, only upper limits for decay $B_d\to D^0\overline{D}^0$ are given at a 90% confidence level:

$$\begin{aligned}BR(B_d\to D^0\overline{D}^0)&<6.0\times10^{-5};\quad \text{BarBar}^{[17]}\\ BR(B_d\to D^0\overline{D}^0)&<4.2\times10^{-5}.\quad \text{Belle}^{[18]}\end{aligned} \tag{35}$$

Obviously, our result is consistent with the data. For the $B_d\to D^0\overline{D}^0$ decay mode, $z_1$ is about 6.5, and the strong phase $\delta_1$ is 34°, so $A_{CP}^{\rm dir}$ is about −6% with the definition in Eq. (25). As far as the decay mode $B_s\to D^0\overline{D}^0$ is concerned, $z_2$ is about 205 and $\delta_2=155°$, and the relation between direct $CP$ violation and $\gamma$ is shown in Fig. 4. From the figure we see that the $CP$ asymmetry is about 0.4%, which is rather small. It is necessary to state that the $z_1$ and $z_2$ are not the true ratio between the tree contribution and the penguin, because mathematical technique are used in Eqs. (20) and (27).

Table 1. Amplitudes ($10^{-3}$ GeV) of $B_d\to D^0\overline{D}^0$ and $B_s\to D^0\overline{D}^0$.

| | $B_d\to D^0\overline{D}^0$ | $B_s\to D^0\overline{D}^0$ |
|---|---|---|
| $T(e)+T(f)$ | $68+17i$ | $66+27i$ |
| $P(e)+P(f)$ | $0.80+0.23i$ | $0.77+3.68i$ |
| $T(g)+T(h)$ | $9.81-2.99i$ | $14.0-0.6i$ |
| $P(g)+P(h)$ | $0.08-0.02i$ | $-0.01+0.01i$ |

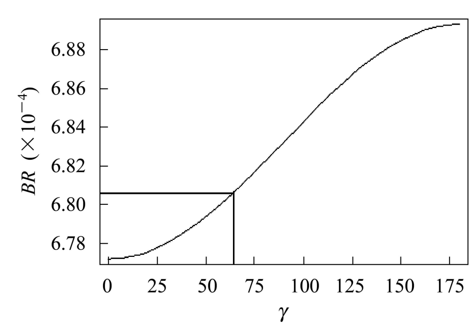


Fig. 3. The change of the branching ratio of $B_s\to D^0\overline{D}^0$ changes with the CKM angle $\gamma$.

In addition to the perturbative annihilation contributions, there is also a hadronic picture for the $B_d\to D^0\overline{D}^0$ decay, named soft final states interaction[19]. The B meson decays into $D^+$ and $D^-$, the secondary particles then exchange a ρ meson, and then scatter into $D^0\overline{D}^0$ through final state interaction. For the $B_s$ decay, the $B_s$ meson decays into $D_s^+$ and $D^+$ then scatters into $D^0\overline{D}^0$ by exchanging a Kaon. But this picture cannot be calculated accurately because of the lack of the knowledge of many effective vertices. We have ignored this contribution here, though it may be important[19].

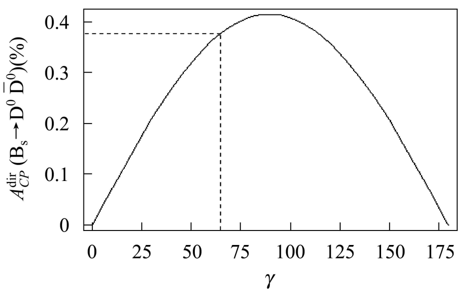


Fig. 4. The direct $CP$-violation of $B_s \to D^0\overline{D}^0$ as a function of the CKM angle $\gamma$.

There are many uncertainties in our calculation such as higher order corrections, the parameters listed in Eq. (33) and the distribution amplitudes of heavy mesons. We will not discuss uncertainties arising from higher order corrections as we only roughly estimate the branching ratios and $CP$ asymmetries. Nevertheless the higher order corrections have been considered for some special channels[20, 21] and showed a 15%—20% uncertainty. The parameters in Eq. (33), fixed by experiments, are proportional to the amplitudes, so we will not analyze this kind of uncertainties either. In our calculation, we find that the results are sensitive to the distribution amplitudes, especially to that of the D meson. Since the heavy D wave function is less constrained, we set $a_D \in (0.6-0.8)$ GeV and $\omega_D \in (0.35-0.45)$ GeV to exploit the uncertainties. Table 2 shows the sensitivity of the branching ratios to a change of $\omega_b$, $\omega_D$ and $a_D$. It is found that the uncertainty of the predictions on PQCD predictions is mainly due to $\omega_D$, which describes the behavior in the end-point region of the D meson, however it is very hard to be determined. Considering the experimental upper limit, our results favor large $\omega_b$, large $\omega_D$ and small $a_D$.

Table 2. The sensitivity of the decay branching ratios and $CP$ asymmetries to change of $\omega_b$, $\omega_D$ and $a_D$.

| | $BR(B_d \to D^0\overline{D}^0)$ $(\times10^{-5})$ | $BR(B_s \to D^0\overline{D}^0)$ $(\times10^{-4})$ | $A_{CP}^{\rm dir}(B_d \to D^0\overline{D}^0)$ (%) | $A_{CP}^{\rm dir}(B_s \to D^0\overline{D}^0)$ (%) |
|---|---|---|---|---|
| $\omega_b(B\backslash B_s)$ | | | | |
| $0.35\backslash0.45$ | 4.3 | 7.8 | −7.2 | 0.4 |
| $0.40\backslash0.50$ | 3.8 | 6.8 | −5.3 | 0.4 |
| $0.45\backslash0.55$ | 3.2 | 5.9 | −5.8 | 0.4 |
| $\omega_D$ | | | | |
| 0.35 | 5.0 | 9.7 | −4.2 | 0.3 |
| 0.40 | 3.8 | 6.8 | −5.3 | 0.4 |
| 0.45 | 2.2 | 4.2 | −7.8 | 0.5 |
| $a_D$ | | | | |
| 0.6 | 3.2 | 5.9 | −6.9 | 0.4 |
| 0.7 | 3.8 | 6.8 | −5.3 | 0.4 |
| 0.8 | 4.3 | 7.8 | −6.1 | 0.4 |

At last, we give the prediction of the branching ratios with err bars included as follows:

$$BR\ (B_d \to D^0\overline{D}^0) = \left(3.8^{+0.5+1.2+0.5}_{-0.6-1.6-0.6}\right)\times10^{-5}\times \left(\frac{f_B\cdot f_D\cdot f_D}{190\ \text{MeV}\cdot210\ \text{MeV}\cdot210\ \text{MeV}}\right)^2;$$

$$BR\ (B_s \to D^0\overline{D}^0) = \left(6.8^{+1.0+2.9+1.0}_{-0.9-2.6-0.9}\right)\times10^{-4}\times \left(\frac{f_{B_s}\cdot f_D\cdot f_D}{230\ \text{MeV}\cdot210\ \text{MeV}\cdot210\ \text{MeV}}\right)^2. \quad (36)$$

We believe that the $B_d \to D^0\overline{D}^0$ will be measured soon because this ratio is just below the present upper limit, and $B_d \to D^0\overline{D}^0$ will be measured in LHC-b next year as a channel to cross check the $\gamma$ measurements.

## 4 Summary

Within the heavy quark limit and the hierarchy approximation $\lambda_{QCD} \ll m_D \ll m_B$, we analyzed the $B \to D^0\overline{D}^0$ and $B_s \to D^0\overline{D}^0$ decays, which occur purely via annihilation type diagrams. As a rough estimation, we calculate the branching ratios and $CP$ asymmetries in the PQCD approach. The branching ratios are still sizable. The branching ratio of $B \to D^0\overline{D}^0$ is about $3.8\times10^{-5}$, which is just below the experimental upper limited result[17, 18], and we think that it will be measured in the near future. For $B_s \to D^0\overline{D}^0$, the branching ratio is about $6.8\times10^{-4}$, which could be measured in LHC-b. From the calculation, it is found that this branching ratio is not sensitive to the angle $\gamma$. In these two decays, there exist $CP$ asymmetries because of the interference between weak and strong interactions, though they are very small.

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