Elementary Theory for Optimum Extraction of Space-Charge-Dominated Ion Beams from Plasma Boundaries*

G. D. Alton[1)] H. Bilheux

(Oak Ridge National Laboratory, Oak Ridge, TN 37831-6372, USA)

Abstract The problem of extracting space-charge-limited ion beams from spherical emission boundaries is analyzed for simple, two electrode, parallel-plate and spherical sector electrode systems by application of Langmuir-Blodgett theory with account taken for the divergent lens effect caused by the aperture in the extraction electrode. Results derived from simulation studies for the three electrode system, designed for use with the Oak Ridge National Laboratory ECR ion source, complement predictions made from elementary analytical theory with or without magnetic field in the extraction region of the source. Under minimum half-angular divergence (minimum emittance) conditions, the plasma emission boundary has an optimum curvature and the perveance, P, (i.e, current density, j_+ and extraction gap, d), has an optimum value for a given charge-state. From these studies, we find that the optimum perveance for any electrode system can be determined from the Child-Langmuir relation for the parallel-plate electrode system multiplied by a factor, F with value $0.49 \leqslant F \leqslant 1$.

Key words space-charge dominated beam extraction theory, ion beam extraction simulation, optimum perveance, optimum angular divergence

1 Introduction

Langmuir-Blodgett-Compton formulation for electron flow in curved electrode systems[1—3] after correction for mass and charge, can be manipulated into analytical approximations useful in understanding, on an elementary level, the ion optics of space-charge-dominated ion beams extracted from cylindrical and spherical-sector plasma boundaries and accelerated through simple electrode structures (see, for example, Refs. [4–6]). In such analyses, the plasma emission boundary is treated as a curved fluid surface, the radius of curvature, of which changes due to changes in plasma density or extraction field. The extraction process is further complicated by the presence of an aperture in the extraction electrode, necessary for ion extraction. The aperture effect can be accounted for by application of the appropriate Davisson-Calbick relation[7] to account for optical effects, including space-charge influences on the divergences of beams[4, 5, 8].

2 Elementary extraction optics theory (without magnetic field)

2.1 Langmuir-blodgett formalism

Solutions to Poisson's equation for space-charge-limited flow from spherical emission boundaries can be derived from the Langmuir-Blodgett relation[1, 2]. The appropriate equation for space-charge-limited flow of electrical current, I, of a beam of particles of charge-state q and mass M, in a spherical sector elec-

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1) E-mail: altongd@ornl.gov

trode system, maintained at a potential difference, $\Delta\phi_{\rm ex}$, is given by:

$$I_{\rm ex} \cong (8\pi\varepsilon_0/9)(2q/M)^{1/2}\Delta\phi_{\rm ex}^{3/2}(1-\cos\theta)/(-\alpha)^2 \ . \quad (1)$$

where ε_0 is the permittivity of free space and α is a dimensionless parameter. α can be shown to be a solution to a non-linear differential equation expressed in series solution by:

$$\alpha = -\mu + 0.3\mu^2 - 0.075\mu^3 + 0.00143\mu^4 - \cdots \quad (2)$$

where $\mu = \ln(r_{\rm s}/r_{\rm ex})$. The series is valid for $r_{\rm s}/r_{\rm ex} > 1$. The perveance, P, emission from a spherical sector plasma boundary system is defined as:

$$\begin{aligned} P &= I/\Delta\phi_{\rm ex}^{3/2} \cong [\{4\varepsilon_0/9\}(2e/M)^{1/2}\pi a^2/d^2] \times \\ &[2(1-\cos\theta)d^2/a^2(-\alpha)^2] = \\ &2P_{\rm pp}[(1-\cos\theta)d^2/a^2(-\alpha)^2]. \end{aligned} \quad (3)$$

where $P_{\rm pp}$ is the perveance for extraction of space-charge-limited ion beams in a parallel-plate (planar) geometry electrode system, as first derived by Child[9], and independently by Langmuir[10] given by

$$P_{\rm pp} = \{4\varepsilon_0/9\}[2qe/M]^{1/2}\pi a^2/d^2 \ . \quad (4)$$

In Eq. (2), ε_0 is the permittivity of free space, e is the electronic charge; q is the charge-state and M the mass of the extracted beam. After expansion of Eq. (3), to order $d/r_{\rm s}$ for $d \ll r_{\rm s}$, the following approximation is obtained:

$$P = I/\Delta\phi_{\rm ex}^{3/2} \cong P_{\rm pp}(1 - 1.6d/r_{\rm s}). \quad (5)$$

Tabulations of the function α can be found in several references, including Ref. [8]. Further details of the solution of Eq. (2), resulting in expressions appropriate for optimizing extraction of space-charge dominated beams from parallel-plate, spherical-sector and the remotely positional three electrode system used for extracting beams from the ORNL ECR ion source will be the subject of a forthcoming article.

3 Ion extraction from spherical-sector plasma boundaries

The Langmuir-Blodgett relation[1, 2], represented by Eq. (3), can be manipulated to approximate extraction of space-charge-dominated beams from spherical-sector plasma emitting diode structures with appropriate corrections for the lens action on the beam during passage from one electric field region to another.

3.1 Electrode systems

Figures 1—3 respectively, schematically illustrate two-electrode, parallel-plate (Fig. 1), two electrode, spherical sector (Fig. 2) and the ORNL three electrode system (Fig. 3) for extraction of convergent, space-charge limited positive ion beams from concave spherical-sector plasma emission boundaries (spherical radii: $r_{\rm s}$, and circular emission apertures: a) to extraction electrodes of aperture radii; b) Extraction is effected between the source at potential, and the extraction electrode, at potential, $\phi_{\rm ex}$, separated by a distance $z = d = r_{\rm s} - r_{\rm ex}$. For the three electrode system, $\phi_{\rm g}$, is the potential of the ground electrode. In each of the respective Figures, θ is the half-angular convergence of the beam at the emission boundary; ψ is the half-angular convergence imparted to the beam by the electric field in the extraction aperture; ω is the final angular divergence of the beam after passing through the electrode system.

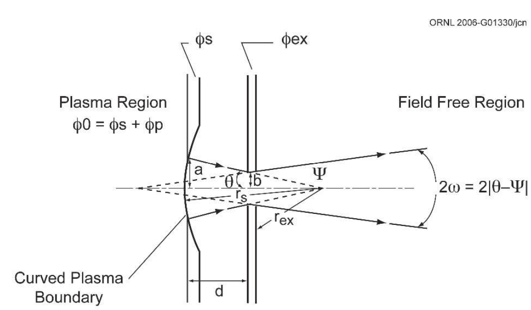

Fig. 1. Simplified parallel-plate electrode system, two-electrode for ion extraction from a concave spherical-sector plasma boundary.

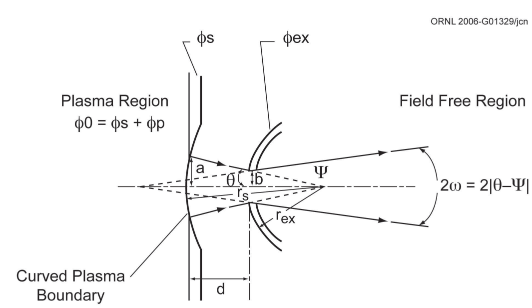

Fig. 2. Simplified spherical-sector electrode two-electrode system for ion extraction from a concave spherical-sector plasma boundary.

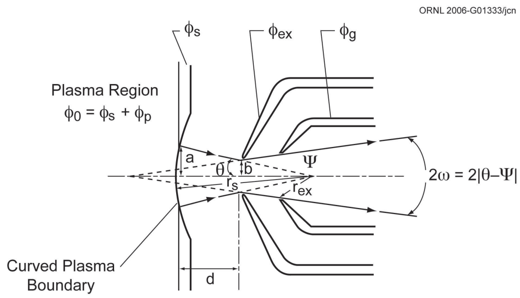

Fig. 3. Schematic drawing of the three-electrode extraction system used for extracting beams from the ORNL ECR ion source.

3.2 Minimum half-angular divergence

The final half-angular divergences for an ion beam extracted from plasma across an extraction dap d can be shown to be:

$$\omega = \theta - \psi \cong 0.5(1 - 1.67P/P_{\rm pp})a/d \quad \text{(parallel-plate system)}, \tag{6}$$

$$\omega = \theta - \psi \cong 0.398[1 - 2.266P/P_{\rm pp} + 0.427(P/P_{\rm pp})^2]a/d \quad \text{(spherical-sector system)}, \tag{7}$$

$$\omega = \theta - \psi \cong E[1 - 1.3P/P_{\rm pp}]a/d \quad \text{(three electrode system)}, \tag{8}$$

where θ is the emission angle of the beam as it leaves the plasma boundary; ψ is the change in angle attributable to the effect of the aperture lens in the extraction electrode; $P_{\rm pp}$ is the perveance for the parallel-plate extraction system given by the Child-Langmuir relation[9, 10].

3.3 Optimum perveance

The value of the perveance at the minimum half-angular divergence ($\omega \cong 0$) is referred to the optimum perveance, $P_{\rm opt}$. The condition, $\omega \cong 0$, occurs whenever

$$P_{\rm optp} = I_{\rm optp}/\Delta\phi_{\rm ex}^{3/2} \cong 0.6P_{\rm pp} \quad \text{(parallel-plate extraction system)} \tag{9}$$

and

$$P_{\rm opts} = I_{\rm opts}/\Delta\phi_{\rm ex}^{3/2} \cong 0.49P_{\rm pp} \quad \text{(spherical-sector extraction system)}. \tag{10}$$

From the simulation studies for the three electrode system displayed in Fig. 3, we find that the extraction process can be very closely approximated by the following relation

$$P_{\rm opt} = I_{\rm opt}/\Delta\phi_{\rm ex}^{3/2} \cong 0.77P_{\rm pp} \ . \tag{11}$$

From these elementary theoretical analyses for the parallel-plate, spherical sector electrode systems and confirmed by simulation studies for the three electrode system, we conclude that any electrode system can be characterized by the relationship between spaced charge-limited current and extraction voltage given by

$$P_{\rm opt} \cong FP_{\rm pp} \ , \tag{12}$$

where F is termed the optimum perveance factor. The optimum perveance for any electrode system can be determined from the Child-Langmuir relation for the parallel-plate electrode system multiplied by a factor, F with value $0.49 \leqslant F \leqslant 1$. Once known for a particular electrode system, the system is characterized by the value of F.

Therefore, it is important to be able to optimize the extraction gap during the extraction process in order to ensure highest quality beams of the charge-state of interest. Optimum angular divergences, correlated to optimum perveances or optimum extraction gaps, have been experimentally observed, (see, e.g., Refs. [6, 11, 12]).

3.4 Optimum extraction gap

Expressions for optimum extraction gaps, $d_{\rm opt}$, can be derived for the two- and three-electrode systems, given respectively, by

$$d_{\rm optp} = 0.516\{\varepsilon_0\}^{1/2}[2eq/M]^{1/4}[A_{\rm s}/I_{\rm optp}]^{1/2}\Delta\phi_{\rm ex}^{3/4} \quad \text{(parallel-plate system)} \tag{13}$$

and

$$d_{\rm opts} = 0.467\{\varepsilon_0\}^{1/2}[2eq/M]^{1/4}[A_{\rm s}/I_{\rm opts}]^{1/2}\Delta\phi_{\rm ex}^{3/4}. \quad \text{(spherical-geometry system)} \tag{14}$$

while for the three-electrode system, described in this report, we find

$$d_{\rm optt} \cong 0.585\{\varepsilon_0\}^{1/2}[2eq/M]^{1/4}[A_{\rm s}/I_{\rm optt}]^{1/2}\Delta\phi_{\rm ex}^{3/4}. \quad \text{(three electrode system)} \tag{15}$$

where $I_{\rm optp}$, $I_{\rm opts}$ and $I_{\rm optt}$ are the optimum extracted currents for the respective extraction electrode systems.

3.5 Optimum current density

The current density for a given has an optimal value at fixed extraction voltage. Since the optimum current density for a given system is directly related to the optimum perveance for a given system,

$$j_{+\rm optp}=P_{\rm opt}\Delta\phi_{\rm ex}^{3/2}/A_{\rm s}=0.6P_{\rm pp}\Delta\phi_{\rm ex}^{3/2}/A_{\rm s}. \quad \text{(parallel-plate system)} \tag{16}$$

$$j_{+\rm opts}=P_{\rm opt}\Delta\phi_{\rm ex}^{3/2}/A_{\rm s}=0.49P_{\rm pp}\Delta\phi_{\rm ex}^{3/2}/A_{\rm s}. \quad \text{(spherical-sector system)} \tag{17}$$

From the results of simulation studies, we find that the optimum current density conforms closely to the following relation:

$$j_{+\rm opt}=P_{\rm opt}\Delta\phi_{\rm ex}^{3/2}/A_{\rm s}\cong 0.77P_{\rm pp}\Delta\phi_{\rm ex}^{3/2}/A_{\rm s}. \quad \text{(three electrode system).} \tag{18}$$

3.6 Optimum plasma boundary radius of curvature

For a given species and charge-state, changes in the curvature of the emission plasma boundary, $r_{\rm s}$, can be effected by: 1) changing the plasma density, n_{0e}, at fixed extraction gap, d, and extraction voltage, $\Delta\phi_{\rm ex}$; 2) changing the extraction voltage, $\Delta\phi_{\rm ex}$, at fixed plasma density, n_{0e}, and extraction gap, d; or 3) changing the extraction gap, d, at fixed plasma density, n_{0e}, and extraction voltage, $\Delta\phi_{\rm ex}$. Since plasma conditions are usually optimized for generation of high charge-state ion beams, for which case, the first option is usually not available. The last option is preferred because it allows the extraction field gradient to be changed without having to change the extraction voltage and plasma parameters.

By equating Eqs. (5) and (9) for the parallel-plate electrode system, the minimum half-angular divergence, ω, occurs whenever

$$r_{\rm s}\cong 4d\ , \tag{19}$$

while the corresponding optimum radius of curvature of the plasma is obtained by equating Eqs. (5) and (10), for the spherical-sector electrode system.

$$r_{\rm s}\cong 3.1d\ . \tag{20}$$

Analogously, the optimum radius of curvature for the three electrode system, can be determined by equating Eqs. (5) and (11) and found to have

$$r_{\rm s}\cong 6.96d\ .$$

By comparing the corresponding perveance factors for each system, we note that the optimum radius of curvature increases as the perveance factor, F, approaches that of the Child-Langmuir parallel-plate emission boundary factor (F=1).

4 Determination of the optimum perveance factor *F*

The perveance factor $F=0.77$ for the three electrode extraction system (Fig. 3) was determined from simulation studies. The half-angular divergences and normalized RMS emittances of beams versus perveance, current density and extraction gap, respectively, exhibit minima with values that closely agree with those derived from elementary theory for the extraction process, independent of the presence of a magnetic field; in principle, this agreement enables the use of analytical expressions for predicting optimum extraction-gap settings that minimize half-angular divergence and hence the emittances of extracted beams.

5 Conclusions

The results of theoretical analyses and simulation studies clearly demonstrate the necessity of providing means for varying the extraction gap for assuring the extraction of high quality space-charge-dominated beams. The results derived from the simulations of the extraction optics agree closely with those predicted from elementary extraction theory. The optimum perveance for any electrode system can be determined from the Child-Langmuir relation for the parallel-plate electrode system when multiplied by a factor, F with value $0.49\leqslant F\leqslant 1$. This factor can be readily determined by computational simulation of the extraction process for a given electrode system.

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