第 31 卷 第 9 期 高 能 物 理 与 核 物 理 Vol. 31, No. 9
2007 年 9 月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS Sep., 2007

CP Asymmetry Prediction for Neutral Charmed Meson Decays into CP Eigenstates by Using Amplitude Ratios*

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Abstract CP asymmetries for neutral charmed meson decays into CP eigenstates are calculated by using amplitude ratios. The formulas and numerical results are presented. The impact on experiments is briefly discussed.

Key words neutral charmed meson, CP asymmetry, CP eigen states

1 Introduction

Up to now, we still do not have any experimental evidence for CP violation in the charm sector. Theoretically, the prediction for charm mixing in the standard model is very small. This leads to small CP violating effects in charm decays. However, searching for large mixing and CP violation in charm decays is still very interesting not only for testing standard model but also for finding new physics, for a recent review, see Ref. [1]. Because CP eigenstates are very special, if $\mathrm{D^0}$-$\overline{\mathrm{D^0}}$ decay into the same CP eigenstates, then the CP violating asymmetry could be enhanced by interference. Another advantage of CP eigenstates is that the amplitude ratio $A(\overline{\mathrm{D^0}}\to \mathrm{f})/A(\mathrm{D^0}\to \mathrm{f})$ can be estimated without computing the amplitudes directly. This makes the computation of the CP asymmetries easier. In this paper, we shall concentrate on the case of CP eigenstates into which charm decays.

2 Time-dependent CP asymmetry

Define

$$\begin{aligned}
&CP\ |D^0\rangle = |\overline{D^0}\rangle,\\
&|D_\mathrm{S}\rangle = p\ |D^0\rangle + q\ |\overline{D^0}\rangle,\\
&|D_\mathrm{L}\rangle = p\ |D^0\rangle - q\ |\overline{D^0}\rangle,\\
&|p|^2+|q|^2=1\,.
\end{aligned}\tag{1}$$

The corresponding eigenvalues of $|D_\mathrm{S}\rangle$, $|D_\mathrm{L}\rangle$ are

$$\lambda_\mathrm{S} = m_\mathrm{S} - \mathrm{i}\frac{\gamma_\mathrm{S}}{2}\,,\qquad \lambda_\mathrm{L} = m_\mathrm{L} - \mathrm{i}\frac{\gamma_\mathrm{L}}{2}\,.$$

Assuming CPT invariance, the time-evolved states are

$$\begin{aligned}
|D^0_\mathrm{p}(t)\rangle &= g_+(t)|D^0\rangle + \frac{q}{p}\ g_-(t)|\overline{D^0}\rangle\,,\\
|\overline{D^0_\mathrm{p}}(t)\rangle &= \frac{p}{q}\ g_-(t)|D^0\rangle + g_+(t)|\overline{D^0}\rangle\,,
\end{aligned}\tag{2}$$

where

$$\left\{\begin{aligned}
g_\pm &= \frac{1}{2}\left(\mathrm{e}^{-\mathrm{i}\lambda_\mathrm{S}t}\pm\mathrm{e}^{-\mathrm{i}\lambda_\mathrm{L}t}\right) = \frac{1}{2}\mathrm{e}^{-\mathrm{i}mt-\frac{\gamma}{2}t}\times\\
&\qquad \left\{\mathrm{e}^{\mathrm{i}\frac{\Delta m}{2}t-\frac{\Delta\gamma}{4}t}\pm\mathrm{e}^{-\mathrm{i}\frac{\Delta m}{2}+\frac{\Delta\gamma}{4}t}\right\}\,,\\
\Delta m &= m_\mathrm{L}-m_\mathrm{S}\,,\quad m=(m_\mathrm{L}+m_\mathrm{S})/2\,,\\
\Delta\gamma &= \gamma_\mathrm{S}-\gamma_\mathrm{L}\,,\quad \gamma=(\gamma_\mathrm{L}+\gamma_\mathrm{S})/2\,.
\end{aligned}\right.\tag{3}$$

Define the mixing parameter

$$x=\frac{\Delta m}{\gamma}\,,\qquad y=\frac{\Delta\gamma}{2\gamma}\,,\tag{4}$$

then the decay amplitudes for final state f are

$$\begin{aligned}
A(D^0_\mathrm{p}(t)\to\mathrm{f}) &= \langle f|H_\mathrm{eff}|D^0_\mathrm{p}(t)\rangle =\\
&\qquad A(\mathrm{f})\left\{g_+(t)+\lambda_\mathrm{f}g_-(t)\right\}\,,
\end{aligned}\tag{5}$$

Received 30 March 2007
* Supported by NSFC (90103011, 10375073, 90403024)
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where

$$\begin{aligned} A(\mathrm{f}) &= \langle f|H_{\mathrm{eff}}|D^0\rangle\,, \\ \overline{A}(\mathrm{f}) &= \langle f|H_{\mathrm{eff}}|\overline{D^0}\rangle\,, \\ \lambda_{\mathrm{f}} &= \frac{q}{p}\,\frac{\overline{A}(\mathrm{f})}{A(\mathrm{f})}\,. \end{aligned} \tag{6}$$

Similarly, we put

$$\begin{aligned} A(\overline{\mathrm{f}}) &= \langle \overline{f}|H_{\mathrm{eff}}|D^0\rangle\,, \\ \overline{A}(\overline{\mathrm{f}}) &= \langle \overline{f}|H_{\mathrm{eff}}|\overline{D^0}\rangle\,, \\ \overline{\lambda}_{\overline{\mathrm{f}}} &= \frac{p}{q}\,\frac{A(\overline{\mathrm{f}})}{\overline{A}(\overline{\mathrm{f}})}\,. \end{aligned} \tag{7}$$

where $\overline{\mathrm{f}}$ is the CP conjugate state of final state f, and

$$|\overline{f}\rangle = CP\;|f\rangle = \eta_{\mathrm{CP}}|f\rangle\,,$$

with $\eta_{\mathrm{CP}}=\pm1$ is the CP eigenvalue (or CP parity). For the amplitude of $\overline{\mathrm{D}^0_{\mathrm{p}}}(t)\to\overline{\mathrm{f}}$, we have from Eq. (2)

$$A(\overline{\mathrm{D}^0_{\mathrm{p}}}(t)\to\overline{\mathrm{f}}) = \langle\overline{f}|H_{\mathrm{eff}}|\overline{\mathrm{D}^0_{\mathrm{p}}}(t)\rangle = \overline{A}(\overline{\mathrm{f}})\left\{g_+(t)+\overline{\lambda}_{\overline{\mathrm{f}}}g_-(t)\right\}\,. \tag{8}$$

Now, it is easy to calculate the time-dependent width $\varGamma(\mathrm{D}^0_{\mathrm{p}}(t)\to\mathrm{f})$ and $\varGamma(\overline{\mathrm{D}^0_{\mathrm{p}}}(t)\to\overline{\mathrm{f}})$. Using Eqs. (3), (5) and (8), we have

$$\varGamma(\mathrm{D}^0_{\mathrm{p}}(t)\to\mathrm{f}) = |A(\mathrm{f})|^2\left\{|g_+(t)|^2+ 2\mathrm{Re}\left[\lambda_{\mathrm{f}}g_+^*(t)g_-(t)\right]+|\lambda_{\mathrm{f}}|^2|g_-(t)|^2\right\}\,, \tag{9}$$

$$\varGamma(\overline{\mathrm{D}^0_{\mathrm{p}}}(t)\to\mathrm{f}) = \left|\overline{A}(\overline{\mathrm{f}})\right|^2\left\{|g_+(t)|^2+ 2\mathrm{Re}\left[\overline{\lambda}_{\overline{\mathrm{f}}}g_+^*(t)g_-(t)\right]+|\overline{\lambda}_{\overline{\mathrm{f}}}|^2|g_-(t)|^2\right\}\,. \tag{10}$$

In order to compute λ_{f} and $\overline{\lambda}_{\overline{\mathrm{f}}}$, we need first to compute the amplitude ratios $\overline{A}(\mathrm{f})/A(\mathrm{f})$ and $A(\overline{\mathrm{f}})/\overline{A}(\overline{\mathrm{f}})$. As an example, we consider $\mathrm{D}^0,\overline{\mathrm{D}^0}\to\mathrm{K}^+\mathrm{K}^-$. Draw the decay diagrams (Fig. 1), we see that if we neglect the penguin diagram contribution, the D^0 and $\overline{\mathrm{D}^0}$ decay diagrams involve only one CKM factor $V_{\mathrm{us}}V^*_{\mathrm{cs}}$ and $V^*_{\mathrm{us}}V_{\mathrm{cs}}$, respectively. The only difference of D^0 and $\overline{\mathrm{D}^0}$ decay diagrams is that the initial and final particles change into their CP counterparts. So

$$\frac{\overline{A}(\mathrm{f})}{A(\mathrm{f})} = \frac{\overline{A}(\overline{\mathrm{D}^0}\to\mathrm{K}^+\mathrm{K}^-)}{A(\mathrm{D}^0\to\mathrm{K}^+\mathrm{K}^-)} = \eta_{\mathrm{CP}}(\mathrm{K}^+\mathrm{K}^-)\frac{V^*_{\mathrm{us}}V_{\mathrm{cs}}}{V_{\mathrm{us}}V^*_{\mathrm{cs}}} = \eta_{\mathrm{CP}}(\mathrm{K}^+\mathrm{K}^-)=+1\,. \tag{11}$$

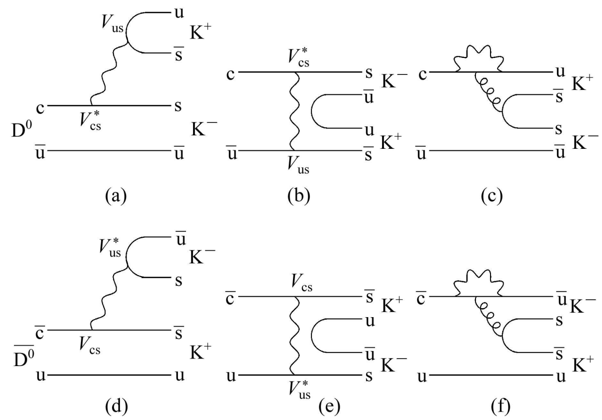

Fig. 1. Decay diagrams for $\mathrm{D}^0,\overline{\mathrm{D}^0}\to\mathrm{K}^+\mathrm{K}^-$.

In Eq. (11), V_{cs} and V_{us} are both real in Wolfenstein parametrization for CKM matrix and $\eta_{\mathrm{CP}}(\mathrm{f})$ is the CP parity of the final state f. Usually $\eta_{\mathrm{CP}}=\pm1$ for different f. Actually, we can prove that (see the appendix in Ref. [2]), if the decays of D^0 and $\overline{\mathrm{D}^0}$ only involve one CKM factor respectively, then the ratio

$$\frac{\overline{A}(\mathrm{f})}{A(\mathrm{f})} = \eta_{\mathrm{CP}}(\mathrm{f})\frac{\mathrm{e}^{-\mathrm{i}\varphi_{\mathrm{wk}}}}{\mathrm{e}^{\mathrm{i}\varphi_{\mathrm{wk}}}} = \eta_{\mathrm{CP}}(\mathrm{f})\,. \tag{12}$$

The last equality holds only for charm decay because all the CKM matrix elements involved are real, if we neglect the penguin contribution.

Define

$$\rho_{\mathrm{f}} = \frac{\overline{A}(\mathrm{f})}{A(\mathrm{f})}\,,\qquad \overline{\rho}_{\overline{\mathrm{f}}} = \frac{A(\overline{\mathrm{f}})}{\overline{A}(\overline{\mathrm{f}})}\,. \tag{13}$$

From Eqs. (6) and (7), we have

$$\begin{aligned} \lambda_{\mathrm{f}} &= \frac{q}{p}\rho_{\mathrm{f}} = \eta_{\mathrm{CP}}(\mathrm{f})\left|\frac{q}{p}\right|\mathrm{e}^{-\mathrm{i}\varphi}\,, \\ \overline{\lambda}_{\overline{\mathrm{f}}} &= \frac{p}{q}\overline{\rho}_{\overline{\mathrm{f}}} = \eta_{\mathrm{CP}}(\mathrm{f})\left|\frac{p}{q}\right|\mathrm{e}^{\mathrm{i}\varphi}\,. \end{aligned} \tag{14}$$

After a straightforward calculation we arrive at

$$\begin{aligned} \varGamma(\mathrm{D}^0_{\mathrm{p}}(t)\to\mathrm{f}) = \frac{1}{4}\mathrm{e}^{-\gamma t}|A(\mathrm{f})|^2\Bigg\{&\left(1+\left|\frac{q}{p}\right|^2\right)\times (\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t})+2\left(1-\left|\frac{q}{p}\right|^2\right)\cos\Delta mt+ \\ &2\eta_{\mathrm{CP}}(\mathrm{f})\left|\frac{q}{p}\right|\left[(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t})\cos\varphi+ 2\sin\varphi\sin\Delta mt\right]\Bigg\}\,, \end{aligned} \tag{15}$$

$$\Gamma(\overline{\mathrm{D}^0_{\mathrm{p}}}(t)\to\mathrm{f})=\frac{1}{4}\mathrm{e}^{-\gamma t}|\overline{A}(\overline{\mathrm{f}})|^2\Bigg\{\left(1+\left|\frac{p}{q}\right|^2\right)\times (\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t})+2\left(1-\left|\frac{p}{q}\right|^2\right)\cos\Delta mt+2\eta_{\mathrm{CP}}(\mathrm{f})\times \left|\frac{p}{q}\right|\left[(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t})\cos\varphi-2\sin\varphi\sin\Delta mt\right]\Bigg\}\,. \tag{16}$$

Assume

$$|A(\mathrm{f})|=|\overline{A}(\overline{\mathrm{f}})|\,. \tag{17}$$

This is guaranteed by our approximation of neglecting the penguin, because in that case only one CKM factor appears [3].

The time-dependent CP asymmetry is

$$C_{\mathrm{f}}(t)=\frac{\Gamma(\mathrm{D}^0_{\mathrm{p}}(t)\to\mathrm{f})-\Gamma(\overline{\mathrm{D}^0_{\mathrm{p}}}(t)\to\overline{\mathrm{f}})}{\Gamma(\mathrm{D}^0_{\mathrm{p}}(t)\to\mathrm{f})+\Gamma(\overline{\mathrm{D}^0_{\mathrm{p}}}(t)\to\overline{\mathrm{f}})}\equiv\frac{N_{\mathrm{f}}(t)}{D_{\mathrm{f}}(t)}\,, \tag{18}$$

$$\begin{aligned}N_{\mathrm{f}}(t)\;=\;&\left(\left|\frac{q}{p}\right|^2-\left|\frac{p}{q}\right|^2\right)(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t})-\\&2\left(\left|\frac{q}{p}\right|^2-\left|\frac{p}{q}\right|^2\right)\cos\Delta mt+\\&2\eta_{\mathrm{CP}}(\mathrm{f})\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right)(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}-\mathrm{e}^{\frac{1}{2}\Delta\gamma t})\cos\varphi+\\&4\eta_{\mathrm{CP}}(\mathrm{f})\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right)\sin\varphi\sin\Delta mt\,,\end{aligned} \tag{19}$$

$$\begin{aligned}D_{\mathrm{f}}(t)\;=\;&\left(2+\left|\frac{q}{p}\right|^2+\left|\frac{p}{q}\right|^2\right)(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}+\mathrm{e}^{\frac{1}{2}\Delta\gamma t})+\\&2\left(2-\left|\frac{q}{p}\right|^2-\left|\frac{p}{q}\right|^2\right)\cos\Delta mt+\\&2\eta_{\mathrm{CP}}(\mathrm{f})\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right)(\mathrm{e}^{-\frac{1}{2}\Delta\gamma t}-\mathrm{e}^{\frac{1}{2}\Delta\gamma t})\cos\varphi+\\&4\eta_{\mathrm{CP}}(\mathrm{f})\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right)\sin\varphi\sin\Delta mt\,.\end{aligned} \tag{20}$$

In Eqs. (19) and (20), there are several parameters we need to know: the phase φ, $x=\Delta m/\gamma$, $y=\Delta\gamma/2\gamma$, $|q|/|p|$, etc. But we do know that $|x|\sim|y|\lesssim 10^{-2}$, and $|q|/|p|$ is very close to unity. Some people assume[4] that $|q|/|p|-|p|/|q|\lesssim\pm1\%$. As for the phase φ, we just keep it as a free parameter. For $\mathrm{e}^{\pm\frac{1}{2}\Delta\gamma t}$, using $\Delta\gamma/(2\gamma)=y$, we have $\mathrm{e}^{\pm\frac{1}{2}\Delta\gamma t}=\mathrm{e}^{\pm y\gamma t}=\mathrm{e}^{\pm yt/\tau_{\mathrm{D}^0}}$, because $y\lesssim 10^{-2}$, $\mathrm{e}^{\pm yt/\tau_{\mathrm{D}^0}}$ is around unity. Thus we have

$$\begin{aligned}N_{\mathrm{f}}(t)\;\simeq\;&-4\eta_{\mathrm{CP}}(\mathrm{f})\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right)(y\gamma t)\cos\varphi+\\&8\eta_{\mathrm{CP}}(\mathrm{f})\sin\varphi\sin\Delta mt\,,\end{aligned} \tag{21}$$

$$D_{\mathrm{f}}(t)\;\simeq\;8\,, \tag{22}$$

$$\begin{aligned}&C_{\mathrm{f}}(t)\approx\\&\eta_{\mathrm{CP}}(\mathrm{f})\left\{\frac{y\gamma t}{2}\left(\left|\frac{p}{q}\right|-\left|\frac{q}{p}\right|\right)\cos\varphi+\sin\varphi\sin\Delta mt\right\}=\\&\eta_{\mathrm{CP}}(\mathrm{f})\left\{\frac{1}{2}y\gamma t\left(\left|\frac{p}{q}\right|-\left|\frac{q}{p}\right|\right)\cos\varphi+\sin\varphi\sin(x\gamma t)\right\}.\end{aligned} \tag{23}$$

In Ref. [3], the first term in Eq. (26) is omitted and the CP parity factor $\eta_{\mathrm{CP}}(\mathrm{f})$ is missing.

3　Time-integrated CP asymmetry

In order to have more statistics, we integrate the time-dependent observables with time. We first list some useful quantities:

$$G_+=\int_0^\infty \mathrm{d}t\;|g_+(t)|^2=\frac{2+x^2-y^2}{2\gamma(1+x^2)(1-y^2)}\approx\frac{1}{\gamma}\,, \tag{24}$$

$$G_-=\int_0^\infty \mathrm{d}t\;|g_-(t)|^2=\frac{x^2+y^2}{2\gamma(1+x^2)(1-y^2)}\approx\frac{x^2+y^2}{2\gamma}\,, \tag{25}$$

$$\begin{aligned}G_{+-}\;=\;&\int_0^\infty \mathrm{d}t\;g_+^*(t)g_-(t)=\\&\frac{-y(1+x^2)+\mathrm{i}x(1-y^2)}{2\gamma(1+x^2)(1-y^2)}\approx\frac{-y+\mathrm{i}x}{2\gamma}\,,\end{aligned} \tag{26}$$

for x^2, $y^2\ll 1$. It is straightforward to get the integrated decay width. From Eqs. (9) and (10) we have

$$\begin{aligned}&\Gamma(\mathrm{D}^0_{\mathrm{p}}\to\mathrm{f})=\int_0^\infty \mathrm{d}t\;\Gamma(\mathrm{D}^0_{\mathrm{p}}(t)\to\mathrm{f})=\\&\qquad|A(\mathrm{f})|^2\left\{G_++2\mathrm{Re}(\lambda_{\mathrm{f}}G_{+-})+|\lambda_{\mathrm{f}}|^2G_-\right\}\,,\end{aligned} \tag{27}$$

$$\begin{aligned}&\Gamma(\overline{\mathrm{D}^0_{\mathrm{p}}}\to\overline{\mathrm{f}})=\int_0^\infty \mathrm{d}t\;\Gamma(\overline{\mathrm{D}^0_{\mathrm{p}}}(t)\to\overline{\mathrm{f}})=\\&\qquad|\overline{A}(\overline{\mathrm{f}})|^2\left\{G_++2\mathrm{Re}(\overline{\lambda}_{\overline{\mathrm{f}}}G_{+-})+|\overline{\lambda}_{\overline{\mathrm{f}}}|^2G_-\right\}\,.\end{aligned} \tag{28}$$

Again we assume $|A(\mathrm{f})|=|\overline{A}(\overline{\mathrm{f}})|$, then the time-integrated CP asymmetry is

$$C_{\mathrm{f}}=\frac{\Gamma(\mathrm{D}^0_{\mathrm{p}}\to\mathrm{f})-\Gamma(\overline{\mathrm{D}^0_{\mathrm{p}}}\to\overline{\mathrm{f}})}{\Gamma(\mathrm{D}^0_{\mathrm{p}}\to\mathrm{f})+\Gamma(\overline{\mathrm{D}^0_{\mathrm{p}}}\to\overline{\mathrm{f}})}\equiv\frac{N_{\mathrm{f}}}{D_{\mathrm{f}}}\,, \tag{29}$$

$$N_{\rm f} = -2\eta_{\rm CP}({\rm f})\left[y\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right)\cos\varphi - x\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right)\sin\varphi\right] + (x^2+y^2)\left(\left|\frac{q}{p}\right|^2-\left|\frac{p}{q}\right|^2\right), \tag{30}$$

$$D_{\rm f} = 4-2\eta_{\rm CP}({\rm f})\left[y\left(\left|\frac{q}{p}\right|+\left|\frac{p}{q}\right|\right)\cos\varphi + x\left(\left|\frac{p}{q}\right|-\left|\frac{q}{p}\right|\right)\sin\varphi\right] + (x^2+y^2)\left(\left|\frac{q}{p}\right|^2+\left|\frac{p}{q}\right|^2\right)\approx 4. \tag{31}$$

Finally, neglecting the (x^2+y^2) term in Eq. (30), one can obtain

$$C_{\rm f}\simeq\eta_{\rm CP}({\rm f})\left\{-\frac{y}{2}\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right)\cos\varphi+x\sin\varphi\right\}. \tag{32}$$

Up to now, we have only discussed incoherent D^0-$\overline{D^0}$ decays. Sometimes D^0-$\overline{D^0}$ pairs are produced coherently, such as in e^+e^- colliding machines (BES and CLEO-c).

The time-evolved coherent state of D^0-$\overline{D^0}$ pair can be written as[3]

$$|{\rm i}\rangle = |D^0(k_1,t_1)\overline{D^0}(k_2,t_2)\rangle+\eta|\overline{D^0}(k_1,t_1)D^0(k_2,t_2)\rangle, \tag{33}$$

where η is the charge conjugation parity or orbital angular momentum parity of the D^0-$\overline{D^0}$ pair.

Because $D^0\to l^+X$ and $\overline{D^0}\to l^-X$ only, we can use the semileptonic decay to tagg one of the two time-evolved states $D^0_{\rm p}(t)$ and $\overline{D^0_{\rm p}}(t)$. We define the leptonic-tagging CP asymmetry $C_{\rm fl}$ as

$$C_{\rm fl}=\frac{N(l^-,{\rm f})-N(l^+,\bar{\rm f})}{N(l^-,{\rm f})+N(l^+,\bar{\rm f})}, \tag{34}$$

where

$$N(l^-,{\rm f})=\int_0^\infty {\rm d}t_1{\rm d}t_2\ |\langle l^-,f|H_{\rm eff}|{\rm i}\rangle|^2 \tag{35}$$

is proportional to the number of events in which $D^0_{\rm p}(k,t)\to l^-X$ as tagging in one side and the other side is the decay $\overline{D^0_{\rm p}}(k,t)\to{\rm f}$ or vice versa. Similarly,

$$N(l^+,\bar{\rm f})=\int_0^\infty {\rm d}t_1{\rm d}t_2\ |\langle l^+,\bar{f}|H_{\rm eff}|{\rm i}\rangle|^2. \tag{36}$$

Assuming $|A({\rm f})|=|\overline{A}(\bar{\rm f})|$ and $|A(l^+)|=|\overline{A}(l^-)|$, after a tedious calculation, we have

$$N(l^-,{\rm f})=|\overline{A}(l^-)A({\rm f})|^2\{G_+^2+G_-^2+2|\lambda_{\rm f}|^2\times[G_+G_-+\eta|G_{+-}|^2]+2(1+\eta)G_-{\rm Re}(\lambda_{\rm f}G^*_{+-})+2(1+\eta)G_+{\rm Re}(\lambda_{\rm f}G_{+-})+2\eta{\rm Re}(G^2_{+-})\}, \tag{37}$$

$$N(l^+,\bar{\rm f})=|A(l^+)\overline{A}(\bar{\rm f})|^2\{G_+^2+G_-^2+2|\overline{\lambda}_{\bar{\rm f}}|^2\times[G_+G_-+\eta|G_{+-}|^2]+2(1+\eta)G_-{\rm Re}(\overline{\lambda}_{\bar{\rm f}}G^*_{+-})+2(1+\eta)G_+{\rm Re}(\overline{\lambda}_{\bar{\rm f}}G_{+-})+2\eta{\rm Re}(G^2_{+-})\}, \tag{38}$$

After taking the same approximation,finally we have

$$C_{\rm fl}=\frac{N_{\rm fl}}{N_{\rm fl}}=(1+\eta)\eta_{\rm CP}({\rm f})\times\left\{-\frac{y}{2}\left(\left|\frac{q}{p}\right|-\left|\frac{p}{q}\right|\right)\cos\varphi+x\sin\varphi\right\}. \tag{39}$$

Comparing Eq. (39) with Eq. (32), we find that $C_{\rm fl}$ is just twice as large as $C_{\rm f}$ when the charge conjugation parity or the orbital angular momentum l is even. This is surprising. From Eq. (32), the order of magnitude of $C_{\rm f}$ is $\lesssim 10^{-3}$, because $x\sim y\lesssim 10^{-2}$. Now we present $C_{\rm f}$ (theory), $C_{\rm f}$ (exp.), branching fractions for D^0, $\overline{D^0}$ decay into CP eigenstates and the number of D-$\overline{\rm D}$ pairs needed for testing CP asymmetry for 1σ signal lower bound in Table 1, where for the

Table 1. The number of D$\overline{\rm D}$ pairs needed for testing CP asymmetry.

$D^0\to$ f	$C_{\rm f}$ (theory)	$C_{\rm f}$ (exp.)	BR	$N_{D\overline{D}}$ (1σ lower bound)
K^+K^-		0.014 ± 0.010	$(3.84\pm0.10)\times10^{-3}$	2.60×10^7
K_sK_s		-0.23 ± 0.19	$(3.7\pm0.7)\times10^{-4}$	2.70×10^8
$K^{*+}K^{*-}$			1.0×10^{-2}(BSW)	1×10^7
$\pi^+\pi^-$	$\lesssim10^{-3}$	0.013 ± 0.012	$(1.364\pm0.032)\times10^{-3}$	7.4×10^7
$\pi^0\pi^0$		0.00 ± 0.05	$(7.9\pm0.8)\times10^{-4}$	1.26×10^8
$\rho^0\pi^0$			$(3.2\pm0.4)\times10^{-3}$	3.13×10^7
$\rho^+\rho^-$			1.3×10^{-2}(BSW)	7.69×10^6
$\rho^0\rho^0$			1.2×10^{-3} (BSW)	8.33×10^7
$\phi\pi^0$			$(7.4\pm0.5)\times10^{-4}$	1.35×10^8
$\phi\eta$			$(1.4\pm0.4)\times10^{-4}$	7.14×10^8
$K^{*0}K^{*0}$			$(7\pm5)\times10^{-5}$	1.43×10^9

branching ratios we take most of them from the 2006 particle data booklet[5]. For $K^{*+}K^{*-}$, $\rho^+\rho^-$ and $\rho^0\rho^0$ final states, we use the BSW theoretical estimation[6]. We use the formula for $N_{D\overline{D}}$

$$N_{D\overline{D}} = \frac{1}{BC_f^2} \qquad \text{for } 1\sigma \text{ signature};$$

$$N_{D\overline{D}} = \frac{9}{BC_f^2} \qquad \text{for } 3\sigma \text{ signature}.$$

4　Summary and conclusion

We have computed the time-dependent and time-integrated CP asymmetry for neutral charmed meson decays into CP eigenstates. We present CP asymmetry not only for incoherent D^0-$\overline{D^0}$, but also coherent D^0-$\overline{D^0}$ pairs. We find that the time-integrated CP asymmetries are very small (order of $\lesssim 10^{-3}$) for testing the CP asymmetries. We also give the lower bound for the number of $D\overline{D}$ pairs needed for testing the CP asymmetries. At present, the integrated luminosities for e^+e^- colliders are:

BES Ⅱ:　27 pb^{-1}
BESⅢ:　20 fb^{-1}　for 4 years data taking.
CLEO − c: 281 pb^{-1}

The corresponding $D\overline{D}$ pairs are

BES Ⅱ:　10^5
BESⅢ:　10^7 .
CLEO-c: 10^6

In Table 1, for $D \to VV$ decays, only when both vector mesons are longitutinally polarized the VV final states are CP eigen states. For the corresponding branching ratios in Table 1, we assume that both longitutinally polarized final states doninate. From Table 1 we see that the only hope is relying on BESIII and B-factories. At B-factories, because the large data sample of charmed meson, both time-dependent asymmetry and time-integrated asymmetry can be measured. While for BESIII, only time-integrated CP asymmetry can be tested. Of course, if there is new physics, some surprise may happen. We can also see from Table 1 that all the measured CP asymmetries presently are consistent with zero.

I thank Hai-bo Li, Cai-dian Lü, Mao-zhi Yang and Zhi-zhong Xing for discussions.

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用振幅比值方法预言中性粲介子衰变到CP本征态的CP不对称性*

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摘要　用振幅比之方法计算了中性粲介子衰变到CP本征态的CP不对称性. 计算了时间相关和时间积分的CP不对称性. 结果表明, 时间积分的CP不对称参数约为千分之一的量级. 还讨论了在BESⅢ和B工厂上实验检验的可能性.

关键词　中性粲介子　CP不对称性　CP本征态

2007 – 03 – 30 收稿
∗国家自然科学基金(90103011, 10375073, 90403024)资助

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