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Perturbative QCD Study of $B_{(s)} \to \phi\rho$ Decays

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Abstract We study $B_{(s)} \to \phi\rho$ decays in a perturbative QCD approach based on k_T factorization. In this approach, we calculate factorizable and non-factorizable contributions, there are no annihilation contributions due to quark content. We get the branching ratios and polarization fractions for $B_{(s)} \to \phi\rho$ decays. Our predictions are consistent with the current experimental data.

Key words PQCD, B meson decay, branching ratio, polarization fraction

1 Introduction

Exclusive B meson decays, especially $B \to VV$ modes, have aroused more and more interest both theoretically and experimentally. Since the first observed charmless $B \to VV$ mode, the $B \to \phi K^*$ decay[1], many $B \to VV$ decay channels have been studied in PQCD approach, such as $B \to K^*\rho$, $K^*\omega$[2], $B \to K^*K^*$[3], $B_s \to \rho(\omega)K^*$[4], $B \to \rho(\omega)\rho(\omega)$[5], and $B^0 \to \phi\phi$[6]. It offers an excellent place to study the CP violation and search for new physics hints[7]. Because the hadronization process is non-perturbative in nature, the essential problem in handling the decay processes is the separation of different energy scales, i.e., the factorization assumption. Many approaches based on the factorization assumption have been developed, such as the naive factorization[8], the generalized factorization[9, 10], the QCD factorization[11], and the perturbative QCD approach which is based on k_T factorization[12, 13].

Recently, $B \to \phi K^*$ data reveal a large transverse polarization fraction, which has been considered as a puzzle, many theoretical efforts have been put to clarify it[14—21]. This suggests that $B \to VV$ modes must be more complicated than all the other modes and need to be studied deeply. Motivated by this, we study another $B \to VV$ mode in the perturbative approach (PQCD) within the Standard Model. For $B \to \phi\rho$ decay, only penguin operators contribute and we find that the branching ratio is at the order of 10^{-9}. For $B_s^0 \to \phi\rho^0$ decay, current-current operators and penguin operators can contribute and we find that the branching ratio is at the order of 10^{-7}. The longitudinal polarization predominates over transverse polarization and its fraction is found to go beyond 70%. Our predictions are consistent with the current experimental values. We hope that our study will help to resolve the above-mentioned puzzle a bit.

The remaining part of this paper is organized as follows. In Sec. II, we calculate analytically the related Feynman diagrams and present the various decay amplitudes for the decay modes studied. In Sec.III, we give the numerical analysis for the branching ratios and polarization fraction of the related decay modes and compare them with the measured values. The summary and some discussion are included in the final section.

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2 Theoretical framework and perturbative calculation

In PQCD approach, the decay amplitude is expressed as the convolution of the mesons' light-cone wave functions, the hard scattering kernels and the Wilson coefficients, which stand for the soft, hard and harder dynamics, respectively. The formalism can be written as:

$$\mathscr{M} \sim \int \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 b_1 \mathrm{d}b_1 b_2 \mathrm{d}b_2 b_3 \mathrm{d}b_3 \mathrm{Tr}\big[C(t) \times \Phi_{\mathrm{B}}(x_1,b_1)\Phi_{\phi}(x_2,b_2)\Phi_{\rho}(x_3,b_3) \times H(x_i,b_i,t)S_{\mathrm{t}}(x_i)\mathrm{e}^{-S(t)}\big], \quad (1)$$

where Tr denotes the trace over Dirac and color indices. $C(t)$ is Wilson coefficient of the four-quark operator which results from the radiative corrections at short distance. The wave function ϕ_{M} absorbs non-perturbative dynamics of the process, which is process independent. The hard part H is rather process-independent and can be calculated in perturbative approach. The b_i is the conjugate space coordinate of the transverse momentum, which represents the transverse interval of the meson. t is the largest energy scale in hard function H, while the jet function $S_{\mathrm{t}}(x_i)$ comes from the resummation of the double logarithms $\ln^2 x_i$, called threshold resummation[22, 23], which becomes larger near the endpoint. The Sudakov form factor $S(t)$ is from the resummation of double logarithms $\ln^2 Qb$[24].

In this paper, we use the light-cone coordinates to describe the four-dimensional momentum as $(\mathrm{p}^+,\mathrm{p}^-,\boldsymbol{P}_{\mathrm{T}})$. We work in the frame with the B meson at rest, so the meson momentum can be written as

$$\begin{aligned} P_1 &= \frac{M_{\mathrm{B}}}{\sqrt{2}}(1,1,\mathbf{0}_{\mathrm{T}}), \\ P_2 &= \frac{M_{\mathrm{B}}}{\sqrt{2}}(1-r_{\rho}^2,r_{\phi}^2,\mathbf{0}_{\mathrm{T}}), \\ P_3 &= \frac{M_{\mathrm{B}}}{\sqrt{2}}(r_{\rho}^2,1-r_{\phi}^2,\mathbf{0}_{\mathrm{T}}), \end{aligned} \quad (2)$$

in which r_{ρ}, r_{ϕ} is defined by $r_{\rho} = M_{\rho}/M_{\mathrm{B}}$ and $r_{\phi} = M_{\phi}/M_{\mathrm{B}}$. P_1, P_2, P_3 refer to B, ϕ, ρ respectively. To extract the helicity amplitudes, we parameterize the following polarization vectors. The longitudinal polarization must satisfy the orthogonality and normalization: $\varepsilon_{2\mathrm{L}}\cdot P_2 = 0$, $\varepsilon_{3\mathrm{L}}\cdot P_3 = 0$, and $\varepsilon_{2\mathrm{L}}^2 = \varepsilon_{3\mathrm{L}}^2 = -1$. Then we can give the manifest form as follows:

$$\begin{aligned} \varepsilon_{2\mathrm{L}} &= \frac{1}{\sqrt{2}r_{\phi}}(1-r_{\phi}^2,-r_{\phi}^2,\mathbf{0}_{\mathrm{T}}), \\ \varepsilon_{3\mathrm{L}} &= \frac{1}{\sqrt{2}r_{\rho}}(-r_{\rho}^2,1-r_{\rho}^2,\mathbf{0}_{\mathrm{T}}). \end{aligned} \quad (3)$$

As to the transverse polarization vectors, we can choose the simple form:

$$\begin{aligned} \varepsilon_{2\mathrm{T}} &= \frac{1}{\sqrt{2}}(0,0,\mathbf{1}_{\mathrm{T}}), \\ \varepsilon_{3\mathrm{T}} &= \frac{1}{\sqrt{3}}(0,0,\mathbf{1}_{\mathrm{T}}). \end{aligned} \quad (4)$$

The decay width for these channel is:

$$\varGamma = \frac{G_{\mathrm{F}}^2|\boldsymbol{P}_{\mathrm{c}}|}{16\pi M_{\mathrm{B}}^2}\sum_{\sigma=\mathrm{L,T}}\mathscr{M}^{\sigma\dagger}\mathscr{M}^{\sigma}, \quad (5)$$

where $|\boldsymbol{P}_{\mathrm{c}}|$ is the three-dimensional momentum of the final state meson, and $|\boldsymbol{P}_{\mathrm{c}}| = \frac{M_{\mathrm{B}}}{2}(1-r_{\rho}^2-r_{\phi}^2)$. The subscript σ denotes the helicity states of the two vector mesons with L(T) standing for the longitudinal (transverse) component. As discussed in Ref. [1], the amplitude $\mathscr{M}^{\sigma}$ is decomposed into

$$\mathscr{M}^{\sigma} = M_{\mathrm{B}}^2\mathscr{M}_{\mathrm{L}} + M_{\mathrm{B}}^2\mathscr{M}_{\mathrm{N}}\varepsilon_2^*(\sigma=\mathrm{T})\cdot\varepsilon_3^*(\sigma=\mathrm{T}) + \mathrm{i}\mathscr{M}_{\mathrm{T}}\varepsilon_{\mu\nu\rho\sigma}\varepsilon_2^{\mu*}\varepsilon_3^{\nu*}P_2^{\rho}P_3^{\sigma}. \quad (6)$$

We can define the longitudinal H_0, transverse $H_{\pm}$ helicity amplitudes

$$H_0 = M_{\mathrm{B}}^2\mathscr{M}_{\mathrm{L}}, \ H_{\pm} = M_{\mathrm{B}}^2\mathscr{M}_{\mathrm{N}} \mp M_{\phi}M_{\rho}\sqrt{r^2-1}\mathscr{M}_{\mathrm{T}}. \quad (7)$$

where $r = P_2\cdot P_3/(M_{\phi}M_{\rho})$. And we can deduce that they satisfy the relation

$$\sum_{\sigma=\mathrm{L,T}}\mathscr{M}^{\sigma\dagger}\mathscr{M}^{\sigma} = |H_0|^2+|H_+|^2+|H_-|^2. \quad (8)$$

There is another set of definition of helicity amplitudes

$$\begin{aligned} A_0 &= -\xi M_{\mathrm{B}}^2\mathscr{M}_{\mathrm{L}}, \\ A_{\parallel} &= \xi\sqrt{2}M_{\mathrm{B}}^2\mathscr{M}_{\mathrm{N}}, \\ A_{\perp} &= \xi M_{\phi}M_{\rho}\sqrt{2(r^2-1)}\mathscr{M}_{\mathrm{T}}, \end{aligned} \quad (9)$$

with the normalization factor $\xi = \sqrt{G_{\mathrm{F}}^2 P_{\mathrm{c}}/(16\pi M_{\mathrm{B}}^2\varGamma)}$. These helicity amplitudes satisfy the relation,

$$|A_0|^2+|A_{\parallel}|^2+|A_{\perp}|^2 = 1, \quad (10)$$

where the notation $A_0, A_\parallel, A_\perp$ denote the longitudinal, parallel, and perpendicular polarization amplitudes.

Our next task is to calculate the matrix elements $\mathscr{M}_{\mathrm{L}}$, $\mathscr{M}_{\mathrm{N}}$ and $\mathscr{M}_{\mathrm{T}}$ of the operators in the weak Hamiltonian with PQCD approach. We have to use the mesons' light-cone wave functions, they are universal for all decay channels. We employ the following wave functions as in other PQCD calculations[2—4].

$$\frac{1}{\sqrt{2N_{\mathrm{c}}}}(\not{P}_1+M_{\mathrm{B}})\gamma_5\Phi_{\mathrm{B}}(x,b)\,,$$

$$\frac{1}{\sqrt{2N_{\mathrm{c}}}}\left[M_\phi\,\not{\epsilon}_2(\mathrm{L})\Phi_\phi(x)+\not{\epsilon}_2(\mathrm{L})\,\not{P}_2\Phi_\phi^{\mathrm{t}}(x)+ M_\phi I\Phi_\phi^{\mathrm{s}}(x)\right],$$

$$\frac{1}{\sqrt{2N_{\mathrm{c}}}}\left[M_\phi\,\not{\epsilon}_2(\mathrm{T})\Phi_\phi^{\mathrm{v}}(x)+\not{\epsilon}_2(\mathrm{T})\,\not{P}_2\Phi_\phi^{\mathrm{T}}(x)+ \frac{M_\phi}{P_2\cdot n_-}\mathrm{i}\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon_2^\nu(\mathrm{T})P_2^\rho n_-^\sigma\Phi_\phi^{\mathrm{a}}(x)\right],$$

$$\frac{1}{\sqrt{2N_{\mathrm{c}}}}\left[M_\rho\,\not{\epsilon}_3(\mathrm{L})\Phi_\rho(x)+\not{\epsilon}_3(\mathrm{L})\,\not{P}_3\Phi_\rho^{\mathrm{t}}(x)+M_\rho I\Phi_\rho^{\mathrm{s}}(x)\right],$$

$$\frac{1}{\sqrt{2N_{\mathrm{c}}}}\left[M_\rho\,\not{\epsilon}_3(\mathrm{T})\Phi_\rho^{\mathrm{v}}(x)+\not{\epsilon}_3(\mathrm{T})\,\not{P}_3\Phi_\rho^{\mathrm{T}}(x)+ \frac{M_{\mathrm{K}^*}}{P_3\cdot n_+}\mathrm{i}\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon_3^\nu(\mathrm{T})P_3^\rho n_+^\sigma\Phi_\rho^{\mathrm{a}}(x)\right],$$

where $n_+=(1,0,\mathbf{0}_{\mathrm{T}})$ and $n_-=(0,1,\mathbf{0}_{\mathrm{T}})$ are dimensionless vectors on the light cone. x is the momentum fraction.

2.1　$\mathrm{B}\to\phi\rho$ decays

The effective Hamiltonian for the process $\mathrm{B}\to\phi\rho$ is given as[25]

$$\mathscr{H}_{\mathrm{eff}}=\frac{G_{\mathrm{F}}}{\sqrt{2}}\Big\{V_{\mathrm{u}}[C_1(\mu)O_1^{\mathrm{u}}(\mu)+C_2(\mu)O_2^{\mathrm{u}}(\mu)]- V_{\mathrm{t}}\sum_{i=3}^{10}C_i(\mu)O_i^{(\mathrm{q})}(\mu)\Big\},\tag{11}$$

where $V_{\mathrm{u}}=V_{\mathrm{ud}}^*V_{\mathrm{ub}}$, $V_{\mathrm{t}}=V_{\mathrm{td}}^*V_{\mathrm{tb}}$, $C_i(\mu)$ are the Wilson coefficients, and the operators are

$$\begin{aligned}
O_1^{\mathrm{u}} &= (\bar{d}_iu_j)_{V-A}(\bar{u}_jb_i)_{V-A},\\
O_2^{\mathrm{u}} &= (\bar{d}_iu_i)_{V-A}(\bar{u}_jb_j)_{V-A},\\
O_3 &= (\bar{d}_ib_i)_{V-A}\sum_q(\bar{q}_jq_j)_{V-A},\\
O_4 &= (\bar{d}_ib_j)_{V-A}\sum_q(\bar{q}_jq_i)_{V-A},\\
O_5 &= (\bar{d}_ib_i)_{V-A}\sum_q(\bar{q}_jq_j)_{V+A},\\
O_6 &= (\bar{d}_ib_j)_{V-A}\sum_q(\bar{q}_jq_i)_{V+A},\\
O_7 &= \frac{3}{2}(\bar{d}_ib_i)_{V-A}\sum_q e_q(\bar{q}_jq_j)_{V+A},\\
O_8 &= \frac{3}{2}(\bar{d}_ib_j)_{V-A}\sum_q e_q(\bar{q}_jq_i)_{V+A},\\
O_9 &= \frac{3}{2}(\bar{d}_ib_i)_{V-A}\sum_q e_q(\bar{q}_jq_j)_{V-A},\\
O_{10} &= \frac{3}{2}(\bar{d}_ib_j)_{V-A}\sum_q e_q(\bar{q}_jq_i)_{V-A}.
\end{aligned}\tag{12}$$

Here i and j stand for $SU(3)$ color indices. The sum over q runs over the quark fields that are active at the scale $\mu=O(m_{\mathrm{b}})$, i.e., $(\mathrm{q}\in\{\mathrm{u,d,s,c,b}\})$. From the effective Hamiltonian, we can see that the current-current operators have no contribution. For factorizable diagrams, all the penguin operators contribute, but for the non-factorizable diagrams only the operators O_4, O_6, O_8, O_{10} can contribute because of the color structure. The leading order diagrams for these decays are shown in Fig. 1. We first calculate the usual factorization Fig. 1(a) and (b).

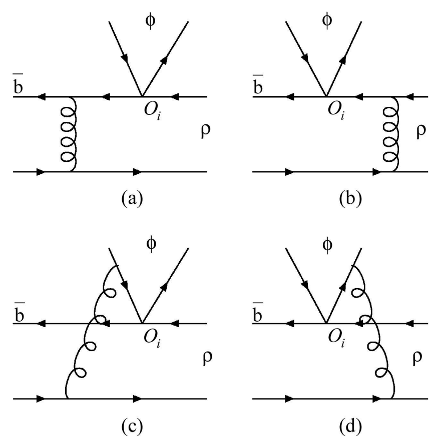

Fig. 1.　Diagrams contributing to the $\mathrm{B}\to\phi\rho$ decays.

$$\begin{aligned}
F_{\mathrm{Le}} =\ & 8\pi C_{\mathrm{F}}M_{\mathrm{B}}^2\int_0^1\mathrm{d}x_1\mathrm{d}x_3\int_0^\infty b_1\mathrm{d}b_1b_3\mathrm{d}b_3\Phi_{\mathrm{B}}(x_1,b_1)\times\\
&\{[(1+x_3)\Phi_\rho(x_3)+r_\rho(1-2x_3)(\Phi_\rho^{\mathrm{t}}(x_3)+\Phi_\rho^{\mathrm{a}}(x_3))]\times\\
&E_{\mathrm{e}}(t_{\mathrm{e}}^{(1)})h_{\mathrm{e}}(x_1,x_3,b_1,b_3)+\\
&2r_\rho\Phi_\rho^{\mathrm{s}}(x_3)E_{\mathrm{e}}(t_{\mathrm{e}}^{(2)})h_{\mathrm{e}}(x_3,x_1,b_3,b_1)\},
\end{aligned}\tag{13}$$

$$\begin{aligned}
F_{\mathrm{Ne}} =\ & 8\pi C_{\mathrm{F}}M_{\mathrm{B}}^2\int_0^1\mathrm{d}x_1\mathrm{d}x_3\int_0^\infty b_1\mathrm{d}b_1b_3\mathrm{d}b_3\Phi_{\mathrm{B}}(x_1,b_1)r_\phi\times\\
&\{[\Phi_\rho^{\mathrm{T}}(x_3)+2r_\rho\Phi_\rho^{\mathrm{v}}(x_3)+r_\rho x_3(\Phi_\rho^{\mathrm{v}}(x_3)-\Phi_\rho^{\mathrm{a}}(x_3))]\times\\
&E_{\mathrm{e}}(t_{\mathrm{e}}^{(1)})h_{\mathrm{e}}(x_1,x_3,b_1,b_3)+r_\rho[\Phi_\rho^{\mathrm{v}}(x_3)+\\
&\Phi_\rho^{\mathrm{a}}(x_3)]E_{\mathrm{e}}(t_{\mathrm{e}}^{(2)})h_{\mathrm{e}}(x_3,x_1,b_3,b_1)\},
\end{aligned}\tag{14}$$

$$F_{\mathrm{Te}}=16\pi C_{\mathrm{F}}M_{\mathrm{B}}^2\int_0^1\mathrm{d}x_1\mathrm{d}x_3\int_0^{\infty}b_1\mathrm{d}b_1b_3\mathrm{d}b_3\Phi_{\mathrm{B}}(x_1,b_1)r_{\phi}\times \{[\Phi_{\rho}^{\mathrm{T}}(x_3)+2r_{\rho}\Phi_{\rho}^{\mathrm{a}}(x_3)-r_{\rho}x_3(\Phi_{\rho}^{\mathrm{v}}(x_3)-\Phi_{\rho}^{\mathrm{a}}(x_3))]\times E_{\mathrm{e}}(t_{\mathrm{e}}^{(1)})h_{\mathrm{e}}(x_1,x_3,b_1,b_3)+r_{\rho}[\Phi_{\rho}^{\mathrm{v}}(x_3)+\Phi_{\rho}^{\mathrm{a}}(x_3)]E_{\mathrm{e}}(t_{\mathrm{e}}^{(2)})h_{\mathrm{e}}(x_3,x_1,b_3,b_1)\},\qquad(15)$$

where $C_{\mathrm{F}}=\dfrac{4}{3}$ is a color factor. The function h_{e}, including the jet function $S_{\mathrm{t}}(x_3)$(threshold resummation for non-factorizable diagrams is weaker and negligible), is the same as the h_{e} in Ref. [1]. The factors $E(t)$ contain the evolution from the W boson mass to the hard scales t in the Wilson coefficients $a(t)$, and from t to the factorization scale $1/b$ in the Sudakov factors $S(t)$:

$$E_{\mathrm{e}}(t)=\alpha_{\mathrm{s}}(t)a_{\mathrm{e}}(t)S_{\mathrm{B}}(t)S_{\rho}(t).\qquad(16)$$

The Wilson coefficients $a_{\mathrm{e}}(t)$ in Eq. (16) are given by

$$a_{\mathrm{e}}(t)=C_3+\frac{C_4}{3}+C_5+\frac{C_6}{3}-\frac{C_7}{2}-\frac{C_8}{6}-\frac{C_9}{2}-\frac{C_{10}}{6}.\qquad(17)$$

For the non-factorizable Fig. 1(c) and (d), all the three meson wave functions are involved and the amplitudes $\mathscr{M}_{\mathrm{He}}=\mathscr{M}_{\mathrm{He4}}+\mathscr{M}_{\mathrm{He6}}$ are written as

$$\mathscr{M}_{\mathrm{Le4}}=16\pi C_{\mathrm{F}}M_{\mathrm{B}}^2\sqrt{2N_{\mathrm{c}}}\int_0^1\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3\int_0^{\infty}b_1\mathrm{d}b_1b_2\mathrm{d}b_2\Phi_{\mathrm{B}}(x_1,b_1)\times\{\Phi_{\phi}(x_2)[-(x_2+x_3)\Phi_{\rho}(x_3)+r_{\rho}x_3(\Phi_{\rho}^{\mathrm{t}}(x_3)+\Phi_{\rho}^{\mathrm{s}}(x_3))]E_{\mathrm{e4}}(t_{\mathrm{d}}^{(1)})h_{\mathrm{d}}^{(1)}(x_1,x_2,x_3,b_1,b_2)+\Phi_{\phi}(x_2)[(1-x_2)\Phi_{\rho}(x_3)+r_{\rho}x_3(\Phi_{\rho}^{\mathrm{t}}(x_3)-\Phi_{\rho}^{\mathrm{s}}(x_3))]\times E_{\mathrm{e4}}(t_{\mathrm{d}}^{(2)})h_{\mathrm{d}}^{(2)}(x_1,x_2,x_3,b_1,b_2),\qquad(18)$$

$$\mathscr{M}_{\mathrm{Ne4}}=16\pi C_{\mathrm{F}}M_{\mathrm{B}}^2\sqrt{2N_{\mathrm{c}}}\int_0^1\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3\int_0^{\infty}b_1\mathrm{d}b_1b_2\mathrm{d}b_2\Phi_{\mathrm{B}}(x_1,b_1)\times r_{\phi}\{[x_2(\Phi_{\phi}^{\mathrm{v}}(x_2)+\Phi\phi^{\mathrm{a}}(x_2))\Phi_{\rho}^{\mathrm{T}}(x_3)-2r_{\rho}(x_2+x_3)(\Phi_{\phi}^{\mathrm{v}}(x_2)\Phi_{\rho}^{\mathrm{v}}(x_3)+\Phi_{\phi}^{\mathrm{a}}(x_2)\Phi_{\rho}^{\mathrm{a}}(x_3))]E_{\mathrm{e4}}(t_{\mathrm{d}}^{(1)})h_{\mathrm{d}}^{(1)}(x_1,x_2,x_3,b_1,b_2)+(1-x_2)(\Phi_{\phi}^{\mathrm{v}}(x_2)+\Phi_{\phi}^{\mathrm{a}}(x_2))\Phi_{\rho}^{\mathrm{T}}(x_3)E_{\mathrm{e4}}(t_{\mathrm{d}}^{(2)})h_{\mathrm{d}}^{(2)}(x_1,x_2,x_3,b_1,b_2)\},\qquad(19)$$

$$\mathscr{M}_{\mathrm{Te4}}=32\pi C_{\mathrm{F}}M_{\mathrm{B}}^2\sqrt{2N_{\mathrm{c}}}\int_0^1\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3\int_0^{\infty}b_1\mathrm{d}b_1b_2\mathrm{d}b_2\Phi_{\mathrm{B}}(x_1,b_1)\times r_{\phi}\{[x_2(\Phi_{\phi}^{\mathrm{v}}(x_2)+\Phi\phi^{\mathrm{a}}(x_2))\Phi_{\rho}^{\mathrm{T}}(x_3)-2r_{\rho}(x_2+x_3)(\Phi_{\phi}^{\mathrm{v}}(x_2)\Phi_{\rho}^{\mathrm{a}}(x_3)+\Phi_{\phi}^{\mathrm{a}}(x_2)\Phi_{\rho}^{\mathrm{v}}(x_3))]E_{\mathrm{e4}}(t_{\mathrm{d}}^{(1)})h_{\mathrm{d}}^{(1)}(x_1,x_2,x_3,b_1,b_2)+(1-x_2)(\Phi_{\phi}^{\mathrm{v}}(x_2)+\Phi_{\phi}^{\mathrm{a}}(x_2))\Phi_{\rho}^{\mathrm{T}}(x_3)E_{\mathrm{e4}}(t_{\mathrm{d}}^{(2)})h_{\mathrm{d}}^{(2)}(x_1,x_2,x_3,b_1,b_2)\},\qquad(20)$$

$$\mathscr{M}_{\mathrm{Le6}}=-16\pi C_{\mathrm{F}}M_{\mathrm{B}}^2\sqrt{2N_{\mathrm{c}}}\int_0^1\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3\int_0^{\infty}b_1\mathrm{d}b_1b_2\mathrm{d}b_2\Phi_{\mathrm{B}}(x_1,b_1)\times\Phi_{\phi}(x_2)\{[x_2\Phi_{\rho}(x_3)+r_{\rho}x_3(\Phi_{\rho}^{\mathrm{t}}(x_3)-\Phi_{\rho}^{\mathrm{s}}(x_3))]\times E_{\mathrm{e6}}(t_{\mathrm{d}}^{(1)})h_{\mathrm{d}}^{(1)}(x_1,x_2,x_3,b_1,b_2)+[-(1-x_2+x_3)\Phi_{\rho}(x_3)+r_{\rho}x_3(\Phi_{\rho}^{\mathrm{t}}+\Phi_{\rho}^{\mathrm{s}}(x_3))]E_{\mathrm{e6}}(t_{\mathrm{d}}^{(2)})h_{\mathrm{d}}^{(2)}(x_1,x_2,x_3,b_1,b_2),\qquad(21)$$

$$\mathscr{M}_{\mathrm{Ne6}}=-16\pi C_{\mathrm{F}}M_{\mathrm{B}}^2\sqrt{2N_{\mathrm{c}}}\int_0^1\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3\int_0^{\infty}b_1\mathrm{d}b_1b_2\mathrm{d}b_2\Phi_{\mathrm{B}}(x_1,b_1)\times r_{\phi}\{x_2(\Phi_{\phi}^{\mathrm{v}}(x_2)-\Phi\phi^{\mathrm{a}}(x_2))\Phi_{\rho}^{\mathrm{T}}(x_3)\times E_{\mathrm{e6}}(t_{\mathrm{d}}^{(1)})h_{\mathrm{d}}^{(1)}(x_1,x_2,x_3,b_1,b_2)+[(1-x_2)(\Phi_{\phi}^{\mathrm{v}}(x_2)-\Phi_{\phi}^{\mathrm{a}}(x_2))\Phi_{\rho}^{\mathrm{T}}(x_3)-2r_{\rho}(1-x_2+x_3)(\Phi_{\phi}^{\mathrm{v}}(x_2)\Phi_{\rho}^{\mathrm{v}}(x_3)-\Phi_{\phi}^{\mathrm{a}}(x_2)\Phi_{\rho}^{\mathrm{a}}(x_3))]E_{\mathrm{e6}}(t_{\mathrm{d}}^{(2)})h_{\mathrm{d}}^{(2)}(x_1,x_2,x_3,b_1,b_2)\},\qquad(22)$$

$$\mathscr{M}_{\mathrm{Te6}}=-32\pi C_{\mathrm{F}}M_{\mathrm{B}}^2\sqrt{2N_{\mathrm{c}}}\int_0^1\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3\int_0^{\infty}b_1\mathrm{d}b_1b_2\mathrm{d}b_2\Phi_{\mathrm{B}}(x_1,b_1)\times r_{\phi}\{x_2(\Phi_{\phi}^{\mathrm{v}}(x_2)-\Phi\phi^{\mathrm{a}}(x_2))\Phi_{\rho}^{\mathrm{T}}(x_3)\times E_{\mathrm{e6}}(t_{\mathrm{d}}^{(1)})h_{\mathrm{d}}^{(1)}(x_1,x_2,x_3,b_1,b_2)+[(1-x_2)(\Phi_{\phi}^{\mathrm{v}}(x_2)-\Phi_{\phi}^{\mathrm{a}}(x_2))\Phi_{\rho}^{\mathrm{T}}(x_3)-2r_{\rho}(1-x_2+x_3)(\Phi_{\phi}^{\mathrm{v}}(x_2)\Phi_{\rho}^{\mathrm{a}}(x_3)-\Phi_{\phi}^{\mathrm{a}}(x_2)\Phi_{\rho}^{\mathrm{v}}(x_3))]E_{\mathrm{e6}}(t_{\mathrm{d}}^{(2)})h_{\mathrm{d}}^{(2)}(x_1,x_2,x_3,b_1,b_2)\}.\qquad(23)$$

The evolution factors are given by

$$E_{\mathrm{ei}}(t)=\alpha_{\mathrm{s}}(t)a_{\mathrm{i}}(t)S(t)\big|_{b_3=b_1},\qquad(24)$$

with the Sudakov factor $S=S_{\mathrm{B}}S_{\phi}S_{\rho}$. The Wilson coefficients a appearing in the above formulas are

$$a_4=\frac{C_4}{3}-\frac{C_{10}}{6},\quad a_6=\frac{C_6}{3}-\frac{C_8}{6}.\qquad(25)$$

The amplitudes for $\mathrm{B}^+\to\phi\rho^+$ are written as

$$\mathscr{M}_{\mathrm{H}}=f_{\phi}V_{\mathrm{t}}^*F_{\mathrm{He}}+V_{\mathrm{t}}^*\mathscr{M}_{\mathrm{He}},\qquad(26)$$

where $F_{\rm He}$ denotes the factorizable contributions and $\mathscr{M}_{\rm He}$ the non-factorizable contributions. For the other two decay channels of $\rm B\to\phi\rho$, the amplitudes are the following: for $\rm B^-\to\phi\rho^-$,

$$\bar{\mathscr{M}}_{\rm H}=f_{\phi}V_{\rm t}F_{\rm He}+V_{\rm t}\bar{\mathscr{M}}_{\rm He}\ , \tag{27}$$

and for $\rm B^0\to\phi\rho^0$,

$$-\sqrt{2}\mathscr{M}^0_{\rm H}=V_{\rm t}^*f_{\phi}F_{\rm He}+V_{\rm t}^*\mathscr{M}_{\rm He}\ . \tag{28}$$

2.2　$\rm B_s^0\to\phi\rho^0$ decay

For the process $\rm B_s^0\to\phi\rho^0$, it is a $\rm b\to s$ transition and we use the effective Hamiltonian[25]

$$\mathscr{H}_{\rm eff}=\frac{G_{\rm F}}{\sqrt{2}}\Bigg[V_{\rm ub}V_{\rm us}^*(C_1O_1^{\rm u}+C_2O_2^{\rm u})-V_{\rm tb}V_{\rm ts}^*\left(\sum_{i=3}^{10}C_iO_i^{\rm (q)}\right)\Bigg]\quad . \tag{29}$$

We specify below the operators in $\mathscr{H}_{\rm eff}$ for $\rm b\to s$:

$$\begin{aligned}
O_1^{\rm u}&=(\bar{s}_iu_j)_{V-A}(\bar{u}_jb_i)_{V-A},\\
O_2^{\rm u}&=(\bar{s}_iu_i)_{V-A}(\bar{u}_jb_j)_{V-A},\\
O_3&=(\bar{s}_ib_i)_{V-A}\sum_q(\bar{q}_jq_j)_{V-A},\\
O_4&=(\bar{s}_ib_j)_{V-A}\sum_q(\bar{q}_jq_i)_{V-A},\\
O_5&=(\bar{s}_ib_i)_{V-A}\sum_q(\bar{q}_jq_j)_{V+A},\\
O_6&=(\bar{s}_ib_j)_{V-A}\sum_q(\bar{q}_jq_i)_{V+A},\\
O_7&=\frac{3}{2}(\bar{s}_ib_i)_{V-A}\sum_q e_{\rm q}(\bar{q}_jq_j)_{V+A},\\
O_8&=\frac{3}{2}(\bar{s}_ib_j)_{V-A}\sum_q e_{\rm q}(\bar{q}_jq_i)_{V+A},\\
O_9&=\frac{3}{2}(\bar{s}_ib_i)_{V-A}\sum_q e_{\rm q}(\bar{q}_jq_j)_{V-A},\\
O_{10}&=\frac{3}{2}(\bar{s}_ib_j)_{V-A}\sum_q e_{\rm q}(\bar{q}_jq_i)_{V-A}.
\end{aligned}\tag{30}$$

From the effective Hamiltonian we can see that the current-current operators and penguin operators can contribute. The leading order diagrams for the decay are shown in Fig. 2, the amplitude for $\rm B_s^0\to\rho^0\phi$ mode is

$$-\sqrt{2}\mathscr{M}'_{\rm H}=V_{\rm u}^*f_{\rho}F'_{\rm He}+V_{\rm u}^*\mathscr{M}'_{\rm He}-V_{\rm t}^*f_{\rho}F^{\rm P\prime}_{\rm He}-V_{\rm t}^*\mathscr{M}^{\rm P\prime}_{\rm He}, \tag{31}$$

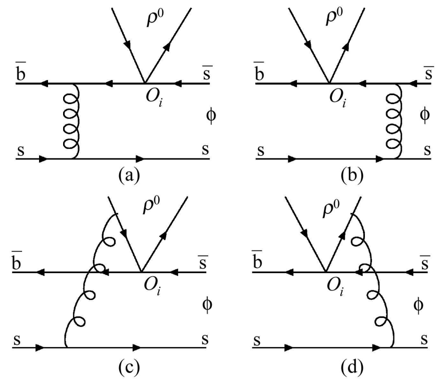

Fig. 2.　Diagrams contributing to the $\rm B_s^0\to\phi\rho^0$ decays.

where $\mathscr{M}^{\rm P\prime}_{\rm He}=\mathscr{M}^{\rm P\prime}_{\rm He4}+\mathscr{M}^{\rm P\prime}_{\rm He6}$. Here $V_{\rm u}=V_{\rm ub}V_{\rm us}^*$, $V_{\rm t}=V_{\rm tb}V_{\rm ts}^*$ and amplitude for the corresponding CP conjugate model is written as

$$-\sqrt{2}\bar{\mathscr{M}}'_{\rm H}=V_{\rm u}f_{\rho}F'_{\rm He}+V_{\rm u}\mathscr{M}'_{\rm He}-V_{\rm t}f_{\rho}F^{\rm P\prime}_{\rm He}-V_{\rm t}\mathscr{M}^{\rm P\prime}_{\rm He}. \tag{32}$$

Next we calculate the hard part in PQCD approach. For the factorizable Fig. 2(a) and (b), we have

$$\begin{aligned}
F'_{\rm Le}=&\,8\pi C_{\rm F}M_{\rm B_s}^2\int_0^1{\rm d}x_1{\rm d}x_2\int_0^{\infty}b_1{\rm d}b_1b_2{\rm d}b_2\varPhi_{\rm B_s}(x_1,b_1)\times\\
&\{[(1+x_2)\varPhi_{\phi}(x_2)+r'_{\phi}(1-2x_2)(\varPhi^{\rm t}_{\phi}(x_2)+\\
&\varPhi^{\rm a}_{\phi}(x_2))]E'_{\rm e}(t_{\rm e}^{(1)})h_{\rm e}(x_1,x_2,b_1,b_2)+\\
&2r'_{\phi}\varPhi^{\rm s}_{\phi}(x_2)E'_{\rm e}(t_{\rm e}^{(2)})h_{\rm e}(x_2,x_1,b_2,b_1)\},
\end{aligned}\tag{33}$$

$$\begin{aligned}
F'_{\rm Ne}=&\,8\pi C_{\rm F}M_{\rm B_s}^2\int_0^1{\rm d}x_1{\rm d}x_2\int_0^{\infty}b_1{\rm d}b_1b_2{\rm d}b_2\varPhi_{\rm B_s}(x_1,b_1)\times\\
&r'_{\rho}\{[\varPhi^{\rm T}_{\phi}(x_2)+2r'_{\phi}\varPhi^{\rm v}_{\rho}(x_2)+r_{\phi}x_2(\varPhi^{\rm v}_{\phi}(x_2)-\\
&\varPhi^{\rm a}_{\phi}(x_2))]\times E'_{\rm e}(t_{\rm e}^{(1)})h_{\rm e}(x_1,x_2,b_1,b_2)+\\
&r'_{\phi}[\varPhi^{\rm v}_{\phi}(x_2)+\varPhi^{\rm a}_{\phi}(x_2)]E'_{\rm e}(t_{\rm e}^{(2)})h_{\rm e}(x_2,x_1,b_2,b_1)\},
\end{aligned}\tag{34}$$

$$\begin{aligned}
F'_{\rm Te}=&\,16\pi C_{\rm F}M_{\rm B_s}^2\int_0^1{\rm d}x_1{\rm d}x_3\int_0^{\infty}b_1{\rm d}b_1b_2{\rm d}b_2\varPhi_{\rm B_s}(x_1,b_1)\times\\
&r'_{\rho}\{[\varPhi^{\rm T}_{\phi}(x_2)+2r'_{\phi}\varPhi^{\rm a}_{\phi}(x_2)r'_{\phi}x_2(\varPhi^{\rm v}_{\phi}(x_2)-\\
&\varPhi^{\rm a}_{\phi}(x_2))]\times E'_{\rm e}(t_{\rm e}^{(1)})h_{\rm e}(x_1,x_2,b_1,b_2)+\\
&r'_{\phi}[\varPhi^{\rm v}_{\rho}(x_2)+\varPhi^{\rm a}_{\phi}(x_2)]E_{\rm e}(t_{\rm e}^{(2)})h_{\rm e}(x_2,x_1,b_2,b_1)\},
\end{aligned}\tag{35}$$

with r'_{ϕ}, r'_{ρ} equal to r_{ϕ},r_{ρ} except for $M_{\rm B}$ replaced by $M_{\rm B_s}$. The expression for $F^{\rm P\prime}_{\rm He}$ are the same as $F'_{\rm He}$ but with WIlson coefficients a replaced by $a^{\rm P}$. As before, the factor $E'(t)$ are given by

$$E_{\rm e}^{\rm (P)\prime}(t)=\alpha_{\rm s}a_{\rm e}^{\rm (P)}(t)S_{\rm B}(t)S_{\phi}(t), \tag{36}$$

where the Wilson coefficients are the following

$$a = C_1 + 2/3C_2,$$
$$a^{\mathrm{P}} = 3/2C_7 + 1/2C_8 + 3/2cC_9 + 1/2C_{10}. \tag{37}$$

For non-factorizable Fig. 2(c) and (d), we find that

$$\mathscr{M}'_{\mathrm{Le4}} = 16\pi C_{\mathrm{F}} M_{\mathrm{B_s}}^2 \sqrt{2N_{\mathrm{c}}} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \times \int_0^\infty b_1 \mathrm{d}b_1 b_3 \mathrm{d}b_3 \Phi_{\mathrm{B_s}}(x_1, b_1) \times \{\Phi_\rho(x_3)[-(x_2+x_3)\Phi_\phi(x_2) + r'_\phi x_2(\Phi^{\mathrm{t}}_\phi(x_2) + \Phi^{\mathrm{s}}_\phi(x_2))] \times E^{(\mathrm{q})\prime}_{\mathrm{e4}}(t^{(1)}_{\mathrm{d}}) h^{(1)}_{\mathrm{d}}(x_1,x_2,x_3,b_1,b_3) + \Phi_\rho(x_3)[(1-x_3)\Phi_\phi(x_2) + r'_\phi x_2(\Phi^{\mathrm{t}}_\phi(x_2) - \Phi^{\mathrm{s}}_\phi(x_2))] \times E^{(\mathrm{q})\prime}_{\mathrm{e4}}(t^{(2)}_{\mathrm{d}}) h^{(2)}_{\mathrm{d}}(x_1,x_2,x_3,b_1,b_3)\}, \tag{38}$$

$$\mathscr{M}'_{\mathrm{Le6}} = -16\pi C_{\mathrm{F}} M_{\mathrm{B}}^2 \sqrt{2N_{\mathrm{c}}} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \times \int_0^\infty b_1 \mathrm{d}b_1 b_3 \mathrm{d}b_3 \Phi_{\mathrm{B_s}}(x_1, b_1) \times \Phi_\rho(x_3)\{[x_3\Phi_\phi(x_2) + r'_\phi x_2(\Phi^{\mathrm{t}}_\phi(x_2) - \Phi^{\mathrm{s}}_\phi(x_2))] \times E^{(\mathrm{q})\prime}_{\mathrm{e6}}(t^{(1)}_{\mathrm{d}}) h^{(1)}_{\mathrm{d}}(x_1,x_2,x_3,b_1,b_3) + [-(1-x_3+x_2)\Phi_\phi(x_2) + r'_\phi x_2(\Phi^{\mathrm{t}}_\phi(x_2) + \Phi^{\mathrm{s}}_\phi(x_2))] \times E^{(\mathrm{q})\prime}_{\mathrm{e6}}(t^{(2)}_{\mathrm{d}}) h^{(2)}_{\mathrm{d}}(x_1,x_2,x_3,b_1,b_3), \tag{39}$$

$$\mathscr{M}^{\mathrm{q}\prime}_{\mathrm{Ne4}} = 16\pi C_{\mathrm{F}} M_{\mathrm{B_s}}^2 \sqrt{2N_{\mathrm{c}}} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \times \int_0^\infty b_1 \mathrm{d}b_1 b_3 \mathrm{d}b_3 \Phi_{\mathrm{B_s}}(x_1, b_1) \times r'_\rho \{[x_3(\Phi^{\mathrm{v}}_\rho(x_3) + \Phi^{\mathrm{a}}_\rho(x_3))\Phi^{\mathrm{T}}_\phi(x_2) - 2r'_\phi(x_2+x_3)(\Phi^{\mathrm{v}}_\rho(x_3)\Phi^{\mathrm{v}}_\phi(x_2) + \Phi^{\mathrm{a}}_\rho(x_3)\Phi^{\mathrm{a}}_\phi(x_2))] \times E^{(\mathrm{q})\prime}_{\mathrm{e4}}(t^{(1)}_{\mathrm{d}}) h^{(1)}_{\mathrm{d}}(x_1,x_2,x_3,b_1,b_3) + (1-x_3)(\Phi^{\mathrm{v}}_\rho(x_3) + \Phi^{\mathrm{a}}_\rho(x_3))\Phi^{\mathrm{T}}_\phi(x_2) \times E^{(\mathrm{q})\prime}_{\mathrm{e4}}(t^{(2)}_{\mathrm{d}}) h^{(2)}_{\mathrm{d}}(x_1,x_2,x_3,b_1,b_3)\}, \tag{40}$$

$$\mathscr{M}'_{\mathrm{Ne6}} = -16\pi C_{\mathrm{F}} M_{\mathrm{B}}^2 \sqrt{2N_{\mathrm{c}}} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \times \int_0^\infty b_1 \mathrm{d}b_1 b_3 \mathrm{d}b_3 \Phi_{\mathrm{B_s}}(x_1, b_1) \times r'_\rho \{[x_3(\Phi^{\mathrm{v}}_\rho(x_3) - \Phi^{\mathrm{a}}_\rho(x_3))\Phi^{\mathrm{T}}_\phi(x_2)] \times E^{(\mathrm{q})\prime}_{\mathrm{e6}}(t^{(1)}_{\mathrm{d}}) h^{(1)}_{\mathrm{d}} \times (x_1,x_2,x_3,b_1,b_3) + [(1-x_3)(\Phi^{\mathrm{v}}_\rho(x_3) - \Phi^{\mathrm{a}}_\rho(x_3))\Phi^{\mathrm{T}}_\phi(x_2) - 2r'_\phi(1-x_3+x_2) \times (\Phi^{\mathrm{v}}_\rho(x_3)\Phi^{\mathrm{v}}_\phi(x_2) - \Phi^{\mathrm{a}}_\rho(x_3)\Phi^{\mathrm{a}}_\phi(x_2))] \times E^{(\mathrm{q})\prime}_{\mathrm{e6}}(t^{(2)}_{\mathrm{d}}) h^{(2)}_{\mathrm{d}}(x_1,x_2,x_3,b_1,b_3)\}, \tag{41}$$

$$\mathscr{M}'_{\mathrm{Te4}} = 32\pi C_{\mathrm{F}} M_{\mathrm{B}}^2 \sqrt{2N_{\mathrm{c}}} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \times \int_0^\infty b_1 \mathrm{d}b_1 b_3 \mathrm{d}b_3 \Phi_{\mathrm{B_s}}(x_1, b_1) \times r'_\rho \{[x_3(\Phi^{\mathrm{v}}_\rho(x_3) + \Phi^{\mathrm{a}}_\rho(x_3))\Phi^{\mathrm{T}}_\phi(x_2) - 2r'_\phi(x_2+x_3)(\Phi^{\mathrm{v}}_\rho(x_3)\Phi^{\mathrm{a}}_\phi(x_2) + \Phi^{\mathrm{a}}_\rho(x_3\Phi^{\mathrm{v}}_\phi(x_2))] \times E^{(\mathrm{q})\prime}_{\mathrm{e}}(t^{(1)}_{\mathrm{d}}) h^{(1)}_{\mathrm{d}} \times (x_1,x_2,x_3,b_1,b_3) + (1-x_3)(\Phi^{\mathrm{v}}_\rho(x_3) + \Phi^{\mathrm{a}}_\rho(x_3)) \times \Phi^{\mathrm{T}}_\phi(x_2) \times E^{(\mathrm{q})\prime}_{\mathrm{e}}(t^{(2)}_{\mathrm{d}}) h^{(2)}_{\mathrm{d}}(x_1,x_2,x_3,b_1,b_3)\}, \tag{42}$$

$$\mathscr{M}'_{\mathrm{Te6}} = -16\pi C_{\mathrm{F}} M_{\mathrm{B}}^2 \sqrt{2N_{\mathrm{c}}} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \times \int_0^\infty b_1 \mathrm{d}b_1 b_3 \mathrm{d}b_3 \Phi_{\mathrm{B_s}}(x_1, b_1) \times r'_\rho \{x_3[\Phi^{\mathrm{v}}_\rho(x_3) - \Phi^{\mathrm{a}}_\rho(x_3)]\Phi^{\mathrm{T}}_\phi(x_2) \times E^{(\mathrm{q})\prime}_{\mathrm{e}}(t^{(1)}_{\mathrm{d}}) h^{(1)}_{\mathrm{d}} \times (x_1,x_2,x_3,b_1,b_3) + [(1-x_3)(\Phi^{\mathrm{v}}_\rho(x_3) - \Phi^{\mathrm{a}}_\rho(x_3))\Phi^{\mathrm{T}}_\phi(x_2) - 2r'_\phi(1-x_3+x_2) \times (\Phi^{\mathrm{v}}_\rho(x_3)\Phi^{\mathrm{a}}_\phi(x_2) - \Phi^{\mathrm{a}}_\rho(x_3)\Phi^{\mathrm{v}}_\phi(x_2))] \times E^{(\mathrm{q})\prime}_{\mathrm{e}}(t^{(2)}_{\mathrm{d}}) h^{(2)}_{\mathrm{d}}(x_1,x_2,x_3,b_1,b_3)\}. \tag{43}$$

The evolution factors are given by

$$E^{(\mathrm{q})\prime}_{\mathrm{ei}}(t) = \alpha_{\mathrm{s}}(t) a^{(\mathrm{q})\prime}_{\mathrm{ei}}(t) S(t)\big|_{b_2=b_1}, \tag{44}$$

with the Sudakov factor $S = S_{\mathrm{B_s}} S_\phi S_\rho$ and the Wilson coefficients are given by

$$a'_1 = C_2/N_{\mathrm{c}},$$
$$a'_4 = 3/2C_{10}/N_{\mathrm{c}}, \tag{45}$$
$$a'_6 = 3/2C_8/N_{\mathrm{C}}.$$

2.3 Numerical analysis

The parameters used in our calculations are: the Fermi coupling constant G_{F}=1.16639 $\times 10^{-5}$GeV^{-2}, the meson masses M_{B}=5.28GeV, $M_{\mathrm{B_S}}$=5.37GeV, M_ρ=0.77GeV, M_ϕ=1.02GeV, the decay constant f_ρ=0.205GeV, f^{T}_ρ=0.155GeV, f_ϕ=0.237GeV, f^{T}_ϕ= 0.220GeV, the central value of the CKM matrix elements $\gamma = 60°$, $|V_{\mathrm{td}}|$=0.0075, $|V_{\mathrm{tb}}|$=0.9992, $|V_{\mathrm{ub}}|$=0.0047, $|V_{\mathrm{us}}|$=0.2196 and the meson lifetime τ_{B}=1.65ps, $\tau_{\mathrm{B_s}}$=1.461ps[26].

Using the above parameters, we get the branching ratios and helicity amplitudes of $\mathrm{B_{(s)}} \to \phi\rho$ decays

(the helicity amplitudes are in Table 1)

$$\begin{aligned} B_r(B^{\pm} \to \phi\rho^{\mp}) &= 4.1 \times 10^{-9}, \\ B_r(B^0 \to \phi\rho^0) &= 1.9 \times 10^{-9}, \\ B_r(B_s^0 \to \phi\rho^0) &= 3.09 \times 10^{-7}, \\ B_r(\bar{B}_s^0 \to \phi\rho^0) &= 3.60 \times 10^{-7}. \end{aligned} \tag{46}$$

Compared with the averaged results of QCDF[27]

$$\begin{aligned} BR(B^- \to \rho^-\phi) &= 5.5 \times 10^{-9}, \\ BR(\bar{B}^0 \to \phi\rho^0) &= 2.5 \times 10^{-9}, \end{aligned} \tag{47}$$

or those from naive factorization[28]

$$BR(\bar{B}_s^0 \to \rho^0\phi) = 2.92 \times 10^{-7}, \tag{48}$$

our predictions for $B \to \phi\rho$ are consistent with those of QCDF, the predictions for $\bar{B}_s^0 \to \rho^0\phi$ are consistent with the result of naive factorization, because the nonfactorization effects in $\bar{B}_s^0 \to \rho^0\phi$ are little.

Table 1. Branching ratios and helicity amplitudes.

channel	$\|A_0\|^2$	$\|A_\parallel\|^2$	$\|A_{\rm perp}\|^2$
$B^{\pm} \to \phi\rho^{\mp}$	0.14	0.41	0.45
$B^0(\bar{B}^0) \to \phi\rho^0$	0.14	0.41	0.45
$B_s^0 \to \phi\rho^0$	0.78	0.12	0.10
$\bar{B}_S^0 \to \phi\rho^0$	0.83	0.09	0.08

Presently, only the experimental upper limits are available at the 90% confidence level[26],

$$\begin{aligned} B_r(B^+ \to \phi\rho^+) &< 1.6 \times 10^{-5}, \\ B_r(B^0 \to \phi\rho^0) &< 1.3 \times 10^{-5}, \\ B_r(B_s^0 \to \phi\rho^0) &< 6.17 \times 10^{-4}. \end{aligned} \tag{49}$$

Obviously, our results are consistent with the data. Our predictions will be tested by the oncoming measurements.

For $B \to \phi\rho$ decays, only penguin operators contribute, so there is no direct CP violation in the $B \to \phi\rho$ decays.

For $B_s^0 \to \phi\rho^0$ decays, the CP asymmetry is time dependent

$$A_{\rm CP}(t) \simeq A_{\rm CP}^{\rm dir} \cos(\Delta m t) + a_{\epsilon+\epsilon'} \sin(\Delta m t), \tag{50}$$

where Δm is the mass difference of the two mass eigenstates of neutral B_s mesons.

The direct CP violation parameter is defined

$$A_{\rm CP}^{\rm dir} = \frac{|\mathscr{M}|^2 - |\overline{\mathscr{M}}|^2}{|\mathscr{M}|^2 + |\overline{\mathscr{M}}|^2} \ . \tag{51}$$

The direct CP violation parameter we can get in $B_s^0 \to \phi\rho^0$ is

$$A_{\rm CP}^{\rm dir}(B_s^0 \to \phi\rho^0) = -8.0\% \ . \tag{52}$$

From Table 1, we can find the longitudinal fraction in $B_s^0 \to \phi\rho^0$ decays go beyond 70%, but the longitudinal fractions in $B \to \phi\rho$ decays are very small, which is similar to $B^0 \to \rho^0\rho^0$ decay mode[29]. In $B \to \phi\rho$ decays, $O_{1,2}$ in $\mathscr{H}_{\rm eff}$ don't contribute via factorizable diagrams, the penguin operators contributing via factorizable diagrams are color suppressed, so that the nonfactorizable effects are the same order as the factorizable ones, which cause the $B \to \phi\rho$ decays not to be factorization dominated, besides the above reason, the nonfactorizable longitudinal amplitude is opposite in sign to that of the factorizable part, therefore the longitudinal amplitude gets a large cancellation between the factorizable effects and the non factorizable parts such that $|A_0|^2$ is reduced much.

According to Ref. [30], we can get the $B_{(s)} \to \rho$, ϕ vector meson transition form factors,

$$\begin{aligned} B \to \rho,\ & V(0) = 0.303, \quad A_0(0) = 0.308, \\ & A_1(0) = 0.233, \quad A_2(0) = 0.208, \\ B_s \to \phi,\ & V(0) = 0.430, \quad A_0(0) = 0.363, \\ & A_1(0) = 0.304, \quad A_2(0) = 0.276, \end{aligned}$$

which are consistent with the results with light cone sum rules[31].

3 Summary

In this paper, we calculate the branching ratios and polarization fractions of $B \to \phi\rho$ and $B_s^0 \to \phi\rho^0$ decays in perturbative QCD approach, the predicted branching ratios are compared with the experimental data and results obtained with other approaches. CP parameters in $B_s^0 \to \phi\rho^0$ are given in our paper. we compared with the experimental values, our results are consistent with the current experimental data.

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Appendix A

Wave functions

The ϕ and ρ distribution amplitudes up to twist 3 are given by

$$
\begin{aligned}
\Phi_\phi(x) &= \frac{3f_\phi}{\sqrt{2N_c}}x(1-x),\\
\Phi^t_\phi(x) &= \frac{f^T_\phi}{2\sqrt{2N_c}}\left\{3(1-2x)^2+1.68C_4^{\frac{1}{2}}(1-2x)+0.69\left[1+(1-2x)\ln\frac{x}{1-x}\right]\right\},\\
\Phi^s_\phi(x) &= \frac{f^T_\phi}{4\sqrt{2N_c}}\left[3(1-2x)(4.5-11.2x+11.2x^2)+1.38\ln\frac{x}{1-x}\right],\\
\Phi^T_\phi(x) &= \frac{3f^T_\phi}{2\sqrt{2N_c}}x(1-x)\left[1+0.2C_4^{\frac{3}{2}}(1-2x)\right],\\
\Phi^v_\phi(x) &= \frac{f^T_\phi}{2\sqrt{2N_c}}\left\{\frac{3}{4}[1+(1-2x)^2]+0.24[3(1-2x)^2-1]+0.96C_4^{\frac{1}{2}}(1-2x)\right\},\\
\Phi^a_\phi(x) &= \frac{3f^T_\phi}{4\sqrt{2N_c}}(1-2x)[1+0.93(10x^2-10x+1)].
\end{aligned}
\tag{A1}
$$

with the Gegenbauer polynomials,

$$\begin{aligned} C_2^{\frac{1}{2}}(\xi) &= \frac{1}{2}(3\xi^2-1), \\ C_4^{\frac{1}{2}}(\xi) &= \frac{1}{8}(35\xi^4-30\xi^2+3), \\ C_2^{\frac{3}{2}}(\xi) &= \frac{3}{2}(5\xi^2-1). \end{aligned} \tag{A2}$$

For ρ meson , its Lorentz structures are similar to ϕ meson and the distribution amplitudes are given by

$$\begin{aligned}
\Phi_\rho(x) &= \frac{3f_\rho}{\sqrt{2N_c}} x(1-x)\left[1+0.18C_2^{3/2}(1-2x)\right], \\
\Phi_\rho^{t}(x) &= \frac{f_\rho^{T}}{2\sqrt{2N_c}}\{3(1-2x)^2+0.3(1-2x)^2[5(1-2x)^2-3]+0.21[3-30(1-2x)^2+35(1-2x)^4]\}, \\
\Phi_\rho^{s}(x) &= \frac{3f_\rho^{T}}{2\sqrt{2N_c}}(1-2x)[1+0.76(10x^2-10x+1)], \\
\Phi_\rho^{T}(x) &= \frac{3f_\rho^{T}}{\sqrt{2N_c}} x(1-x)[1+0.2C_2^{3/2}(1-2x)], \\
\Phi_\rho^{v}(x) &= \frac{f_\rho}{2\sqrt{2N_c}}\left\{\frac{3}{4}[1+(1-2x)^2]+0.24[3(1-2x)^2-1]+0.12[3-30(1-2x)^2+35(1-2x)^4]\right\}, \\
\Phi_\rho^{a}(x) &= \frac{3f_\rho}{4\sqrt{2N_c}}(1-2x)[1+0.93(10x^2-10x+1)].
\end{aligned}$$

For the amplitudes of B and B_s, meson, we employ the following distribution amplitudes:

$$\Phi_B(x,b) = N_B x^2(1-x)^2 \exp\left[-\frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right], \tag{A3}$$

which satisfies the normalization

$$\int_0^1 \Phi_B(x)\mathrm{d}x = \frac{f_B}{2\sqrt{2N_c}} \ . \tag{A4}$$

We choose N_B=91.784GeV, ω_B=0.4GeV. Things for B_s are similar if we ignore $SU(3)$ symmetry breaking effect. As discussed in Ref. [4], we choose ω_{B_s}=0.50GeV.

用微扰 QCD 方法研究 $B_{(s)} \to \phi\rho$ 衰变

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摘要　用微扰 QCD 对 $B_{(s)} \to \phi\rho$ 衰变进行了研究, 考虑了因子化和非因子化图的贡献, 得出了 $B_{(s)} \to \phi\rho$ 衰变的分支比以及纵向极化衰变、横向极化衰变之比, 所得到的结果与现在的实验数据吻合.

关键词　微扰 QCD　B 介子衰变　分支比　极化比

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