Global Quark Polarization in QGP in Non-central AA Collisions*

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Abstract There exists a large local relative orbital angular momentum between produced partons along the direction opposite to the reaction plane in the early stage of non-central heavy-ion collisions. This initial local orbital angular momentum can lead to quark polarization along the same direction due to spin-orbital coupling in QCD. We present the quark polarization by using hard thermal loop gluon propagator for quark-quark scattering in quark-gluon plasma and compare it with the result obtained from the static potential model.

Key words global polarization, impact parameter, HTL propagator

It has been pointed out[1] recently that in the early stage of non-central heavy-ion collisions at high energies, there exists a large local relative orbital angular momentum between produced partons along the direction opposite to the reaction plane, as illustrated in Fig.1. Here, the impact parameter $\boldsymbol{b}$, defined as the transverse distance of the projectile from the target nucleus, is taken as $\hat{x}$-direction. The normal $\boldsymbol{n}_b$ of the reaction plane is given by

$$\boldsymbol{n}_b \equiv \boldsymbol{p}_{\rm in} \times \boldsymbol{b}/|\boldsymbol{p}_{\rm in} \times \boldsymbol{b}|, \tag{1}$$

and is along $\hat{y}$.

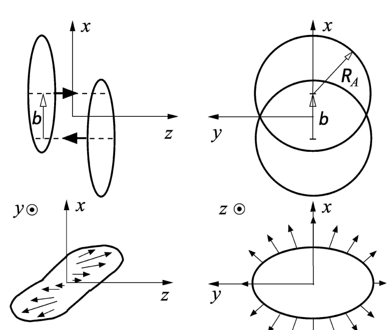

Fig. 1. Illustration of non-central heavy-ion collisions.

Using a screened static potential model in the small angle approximation, the authors of Ref. [1] have demonstrated that this orbital angular momentum can lead to polarization of the quarks and anti-quarks along the direction opposite to the reaction plane. It has also been shown that such a global quark/anti-quark polarization should have many observable consequences such as global hyperon polarization[1] and vector meson spin alignment[2]. Dedicated efforts have been made in measuring such effects at RHIC and preliminary results have been reported[3]. It is instructive to make a more realistic estimation of the quark polarization by using QCD at finite temperature, where quark-quark interaction in QGP should be described by a Hard-Thermal-Loop (HTL) resummed gluon propagator[4—6]. Such a calculation has been carried out and the results will be published soon[7]. The basic idea and main results in Refs. [1, 2, 7] have been presented by Dr. Liang last week in Shanghai[8] Here, I would like to make a more detailed discussion about the calculation using HTL gluon propagator and compare it with the results from the static potential model[1]. More details and final results can be found in Ref. [7].

For definiteness, we consider a non-identical quark-quark scattering $q_1(P_1,\lambda_1) + q_2(P_2,\lambda_2) \rightarrow$

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$q_1(P_3,\lambda_3)+q_2(P_4,\lambda_4)$, where $P_i=(E_i,\boldsymbol{p}_i)$ and λ_i in the brackets denote the four momentum and spin of the quark respectively. The cross section in momentum space is given by

$$\mathrm{d}\sigma_{\lambda_3}=\frac{c_{\rm qq}}{F}\frac{1}{4}\sum_{\lambda_1,\lambda_2,\lambda_4}\mathscr{M}\mathscr{M}^*(2\pi)^4\delta(P_1+P_2-P_3-P_4)\times \frac{\mathrm{d}^3\boldsymbol{p}_3}{(2\pi)^3 2E_3}\frac{\mathrm{d}^3\boldsymbol{p}_4}{(2\pi)^3 2E_4}\,, \tag{2}$$

where $c_{\rm qq}=2/9$ is the color factor, and $F=4\sqrt{(P_1\cdot P_2)^2-m_1^2m_2^2}$ is the flux factor. $\mathscr{M}$ is the scattering amplitude in momentum space which is only function of four momentum transfer $Q=P_3-P_1=P_2-P_4$. It can be given by

$$\mathscr{M}(Q)=\overline{u}_{\lambda_3}(P_1+Q)\gamma_\mu u_{\lambda_1}(P_1)\Delta^{\mu\nu}(Q)\times \overline{u}_{\lambda_4}(P_2-Q)\gamma_\nu u_{\lambda_2}(P_2), \tag{3}$$

where $\Delta^{\mu\nu}(Q)$ is just HTL gluon propagator. We'll always work in the center of mass frame of the q_1q_2-system where $Q=(0,\boldsymbol{q})$. In this frame HTL gluon propagator can be reduced into

$$\Delta^{\mu\nu}(Q)=\frac{g^{\mu\nu}-U^\mu U^\nu}{q^2}+\frac{U^\mu U^\nu}{q^2+{\mu_D}^2}, \tag{4}$$

where U^μ is the fluid velocity. We consider only the longitudinal momentum distribution and in this case, the center-of-mass frame of the scattering quarks coincides with the local co-moving frame of QGP and $U^\mu=(1,0,0,0)$.

We'll introduce a non-perturbative magnetic mass $\mu_{\rm m}\approx 0.255\sqrt{N_c/2}g^2T$[9] to regularize the transverse self-energy of the HTL gluon propagator. Since we are interested in the polarization of q_1 after the scattering, we thus average over the spins of the initial quarks and sum over the spin of q_2 in the final state.

Integrating over the momentum $\boldsymbol{p}_4$ of q_2 and the longitudinal component p_{3z} of the momentum of q_1, we obtain the differential cross section with respect to the transverse momentum transfer $\boldsymbol{q}_T=\boldsymbol{p}_{3T}$. It follows that the cross section in impact parameter space is obtained by making two dimensional Fourier transformation of $\boldsymbol{q}_T$, which has the expression as

$$\frac{\mathrm{d}\sigma_{\lambda_3}}{\mathrm{d}^2\boldsymbol{x}_T}=\frac{c_{\rm qq}}{16F}\sum_{\lambda_1,\lambda_2,\lambda_4}\int\frac{\mathrm{d}^2\boldsymbol{q}_T}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{k}_T}{(2\pi)^2}\mathrm{e}^{\mathrm{i}(\boldsymbol{k}_T-\boldsymbol{q}_T)\cdot\boldsymbol{x}_T}\times \frac{\mathscr{M}(\boldsymbol{q}_T)}{\Lambda(\boldsymbol{q}_T)}\frac{\mathscr{M}^*(\boldsymbol{k}_T)}{\Lambda^*(\boldsymbol{k}_T)}\,, \tag{5}$$

where $\Lambda(\boldsymbol{q}_T)=\sqrt{(E_1+E_2)|p+q_z|}$ is a kinematic factor obtained when we integrate $\delta(E_1+E_2-E_3-E_4)$ over p_{3z}.

The differential cross section can in general be divided into a spin-independent and a spin-dependent part, i.e.

$$\frac{\mathrm{d}\sigma_{\lambda_3}}{\mathrm{d}^2\boldsymbol{x}_T}=\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{x}_T}+\lambda_3\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^2\boldsymbol{x}_T}\,. \tag{6}$$

The expression for $\mathrm{d}\sigma/\mathrm{d}^2\boldsymbol{x}_T$ or $\mathrm{d}\Delta\sigma/\mathrm{d}^2\boldsymbol{x}_T$ with HTL gluon propagator is much more complicated than those we obtained in Ref. [1] by using a static potential model. However, if we use "small angle approximation", i.e. consider only small angle (low transverse momentum transfer) scattering, and neglect the quarks' mass as well, the results are still very simple. In this case, we have $q_z\sim 0$ and $q_T\equiv|\boldsymbol{q}_T|\ll p$, and we obtain that

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{x}_T}=\frac{g^4c_{\rm qq}}{8}\int\frac{\mathrm{d}^2\boldsymbol{q}_T}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{k}_T}{(2\pi)^2}\mathrm{e}^{\mathrm{i}(\boldsymbol{k}_T-\boldsymbol{q}_T)\cdot\boldsymbol{x}_T}\times \left(\frac{1}{q_T^2+\mu_m^2}+\frac{1}{q_T^2+\mu_D^2}\right)\left(\frac{1}{k_T^2+\mu_m^2}+\frac{1}{k_T^2+\mu_D^2}\right). \tag{7}$$

The spin-dependent is given by

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^2\boldsymbol{x}_T}=-\frac{1}{2p^2}(\boldsymbol{p}\times\boldsymbol{n})\cdot\boldsymbol{\nabla}_T\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{x}_T}\,. \tag{8}$$

Using the integration formulas

$$\int\frac{\mathrm{d}^2\boldsymbol{q}_T}{(2\pi)^2}\frac{\mathrm{e}^{\mathrm{i}\boldsymbol{q}_T\cdot\boldsymbol{x}_T}}{q_T^2+\mu_m^2}=\int\frac{q_T\mathrm{d}q_T}{2\pi}\frac{J_0(q_Tx_T)}{q_T^2+\mu_m^2}\,, \tag{9}$$

$$\int_0^\infty q_T\mathrm{d}q_T\frac{J_0(q_Tx_T)}{q_T^2+\mu_m^2}=K_0(\mu_mx_T)\,, \tag{10}$$

where J_0 and K_0 are the Bessel and modified Bessel functions respectively, we obtain that

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{x}_T}=\frac{c_{\rm qq}\alpha_{\rm s}^2}{2}\left[K_0(\mu_mx_T)+K_0(\mu_Dx_T)\right]^2\,, \tag{11}$$

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^2\boldsymbol{x}_T}=\frac{c_{\rm qq}\alpha_{\rm s}^2}{2}\frac{(\boldsymbol{p}\times\boldsymbol{n})\cdot\hat{\boldsymbol{x}}_T}{p^2}\left[K_0(\mu_mx_T)+K_0(\mu_Dx_T)\right]\times \left[\mu_mK_1(\mu_mx_T)+\mu_DK_1(\mu_Dx_T)\right]. \tag{12}$$

where $\hat{\boldsymbol{x}}_T=\boldsymbol{x}_T/x_T$ is the unit vector in $\boldsymbol{x}_T$-direction. If we only consider the electric contribution, they are given by

$$\left[\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{x}_T}\right]_e=\frac{c_{\rm qq}\alpha_{\rm s}^2}{2}K_0(\mu_Dx_T)^2\,, \tag{13}$$

$$\left[\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^2\boldsymbol{x}_T}\right]_e=\frac{c_{\rm qq}\alpha_{\rm s}^2}{2}\frac{(\boldsymbol{p}\times\boldsymbol{n})\cdot\hat{\boldsymbol{x}}_T}{p^2}\times \mu_DK_0(\mu_Dx_T)K_1(\mu_Dx_T). \tag{14}$$

We compare these results with what we obtained in screened static potential model where we also made small angle approximation

$$\left[\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{x}_T}\right]_S = c_T\alpha_s^2 K_0^2(\mu_D x_T), \tag{15}$$

$$\left[\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^2\boldsymbol{x}_T}\right]_S = c_T\alpha_s^2\frac{(\boldsymbol{p}\times\boldsymbol{n})\cdot\hat{\boldsymbol{x}}_T}{p^2}\mu_D K_0(\mu_D x_T)K_1(\mu_D x_T). \tag{16}$$

Fix $c_T = c_{qq}/2$ and we'll see that the results from static potential model is just the electric contribution from Eqs. (11) and (12) with HTL gluon propagator.

In non-central AA collisions with given reaction plane, the direction of the averaged relative orbital angular momentum $\boldsymbol{l}$ between the two scattered quarks q_1 and q_2 is given. Since a given direction of $\boldsymbol{l}$ corresponds to a given direction of $\boldsymbol{x}_T$, this implies that there should be a preferred direction of $\boldsymbol{x}_T$ at a given direction of $\boldsymbol{b}$. The detailed distribution of $\boldsymbol{x}_T$ at given $\boldsymbol{b}$ depends on the collective longitudinal momentum distribution which is discussed in detail in Ref. [3]. For simplicity, we considered an uniform distribution of $\boldsymbol{x}_T$ in all the possible directions in the upper half oxy-plane with $x > 0$. In this case, we need to integrate $\mathrm{d}\sigma/\mathrm{d}^2\boldsymbol{x}$ and $\mathrm{d}\sigma/\mathrm{d}^2\boldsymbol{x}$ over half plane above y-axis to obtain average cross section at a given $\boldsymbol{b}$, i.e.

$$\sigma = \int_0^{+\infty}\mathrm{d}x\int_{-\infty}^{+\infty}\mathrm{d}y\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{x}_T}, \tag{17}$$

$$\Delta\sigma = \int_0^{+\infty}\mathrm{d}x\int_{-\infty}^{+\infty}\mathrm{d}y\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^2\boldsymbol{x}_T}. \tag{18}$$

Hence the polarization of the quark is given by

$$P_q = \frac{\Delta\sigma}{\sigma}. \tag{19}$$

Integrating Eqs. (13) and (14), we can have

$$[\sigma]_e = \frac{c_{qq}\alpha_s^2\pi}{4\mu_D}, \tag{20}$$

$$[\Delta\sigma]_e = -\frac{c_{qq}\alpha_s^2\pi^2}{8p\mu_D}. \tag{21}$$

Hence, we can have

$$[P_q]_e = -\frac{\pi\mu_D}{2p}. \tag{22}$$

These are just the results given in Ref. [1]. It should be noticed that we have integrated the q_T without constraint in order to give a simple analytic polarization. This is enough for us just to give a qualitative estimate in high energy limit.

Now I'll show the analytic results from HTL gluon propagator without small angle approximation but still neglecting the quarks' mass,

$$\sigma = \frac{\pi c_{qq}\alpha_s^2}{4\hat{s}}\Bigg\{4+\frac{1}{\beta_m\tilde{T}^2}+\frac{1}{\beta_D\tilde{T}^2}-\frac{2}{1+\beta_m\tilde{T}^2}- 2\left(\beta_D\tilde{T}^2+\frac{\beta_m\tilde{T}^2-2\beta_D\tilde{T}^2-1}{\beta_m\tilde{T}^2-\beta_D\tilde{T}^2}\right)\ln\left(1+\frac{1}{\beta_D\tilde{T}^2}\right)- 2\left(\beta_m\tilde{T}^2+\frac{1+\beta_m\tilde{T}^2}{\beta_m\tilde{T}^2-\beta_D\tilde{T}^2}\right)\ln\left(1+\frac{1}{\beta_m\tilde{T}^2}\right)\Bigg\}, \tag{23}$$

$$\Delta\sigma = -\frac{c_{qq}\alpha_s^2}{8\pi\hat{s}}\int_{-1}^1\mathrm{d}t\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\times \frac{t^2\sqrt{xy}}{\sqrt{1-t^2}\sqrt{1-x^2}\sqrt{1-y^2}}\times \Bigg\{\frac{(1-t^2)(4+tx+ty)-2(t^2xy+1)}{[1-tx+2\beta_m\tilde{T}^2][1-ty+2\beta_m\tilde{T}^2]}+ \frac{2t(1+xy)(5+t^2xy)}{(x+y)[1-tx+2\beta_m\tilde{T}^2][1-ty+2\beta_m\tilde{T}^2]}+ \frac{(1-t^2)(tx+ty)+2(t^2xy+1)}{[1-tx+2\beta_D\tilde{T}][1-ty+2\beta_D\tilde{T}^2]}+ \frac{2t(1+xy)(1+t^2xy)}{(x+y)[1-tx+2\beta_D\tilde{T}][1-ty+2\beta_D\tilde{T}^2]}+ \frac{2(1-t^2)(2+tx+ty)}{[1-tx+2\beta_m\tilde{T}^2][1-ty+2\beta_D\tilde{T}^2]}+ \frac{8t(1+xy)(1+ty)}{(x+y)[1-tx+2\beta_m\tilde{T}^2][1-ty+2\beta_D\tilde{T}^2]}\Bigg\}, \tag{24}$$

where $\sqrt{\hat{s}}$ is the center of mass energy of the qq-system and $\beta_D = 4\pi\alpha_s(N_c+N_f/2)/3$, $\beta_m = 0.255^2(4\pi)^2\alpha_s^2N_c/2$. From Eqs. (23) and (24), we can see that the polarization is only the function of $\tilde{T}=T/\sqrt{\hat{s}}$. We can come to this conclusion by simple dimensional analysis. Since we have neglected the quark's mass, the polarization which is dimensionless can only be the function of dimensionless variable $\tilde{T}=T/\sqrt{\hat{s}}$.

We now carry out the integration above numerically and obtain the results for the quark polarization after one scattering with a neighboring quark. The results are shown as a function of $\sqrt{\hat{s}}/T$ in Fig. 2. The results in Fig. 2 show that the polarization is quite different for different $\sqrt{\hat{s}}/T$. It is very small both in the high energy and low energy limit. However, it can be as high as 20% at moderate $\sqrt{\hat{s}}/T$. At RHIC, we can only give a very rough estimate of the ratio $\sqrt{\hat{s}}/T \sim \Delta p_z/T$ which should be between 0.1 and 2. We see that in this range the polarization can be quite significant but can also be only of a few percent. This makes the experimental tests very difficult. In particular for measurements of vector meson spin alignments as given in Ref. [2], the effect only of the order or P_q^2. To test this, hyperon polarization should be a better choice.

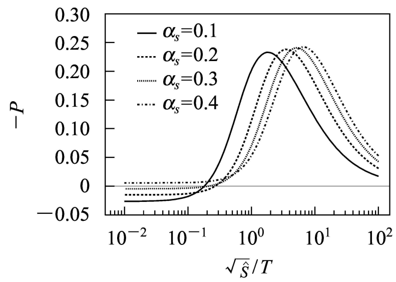

Fig. 2. Preliminary results for quark polarization $-P_q$ as a function of $\sqrt{\hat{s}}/T$ for different α_s's obtained using HTL gluon propagator.

For comparison, we show the results together with those obtained under the small angle approximation in Fig. 3. We see that, in the high energy limit where small angle approximation is expected to be valid, the result by HTL is less than that by the static potential model due to the magnetic contribution. We see also that, at RHIC energy, small angle approximation is not a good approximation[1]. We have to rely on the numerical results obtained by using HTL gluon propagator without small angle approximation.

In summary, the calculation with HTL gluon propagator shows that quarks can be polarized along the direction opposite to the reaction plane and the polarization can reach 20% at moderate $\sqrt{\hat{s}}/T$. At RHIC energy, due to the uncertainty about the temperature, the polarization can be quite significant but can also be only of a few percent. We also show that small angle approximation is not a good approximation at RHIC.The more complete calculation including the specific transverse distribution is underway.

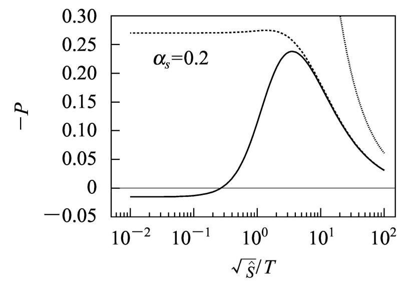

Fig. 3. Comparison of the results obtained using HTL gluon propagator (solid line) with those under "small angle approximation" (dashed line) and those using the screened static potential model under small angle approximation (dotted line).

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