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Constraints on the Tritium Beta Decay and the Neutrinoless Double Beta Decay in the Minimal Seesaw Model*

GUO Wan-Lei[1)] XING Zhi-Zhong[2)]

(Institute of High Energy Physics, CAS, Beijing 100049, China)

Abstract We calculate the effective mass terms of the tritium beta decay ($\langle m\rangle_{\rm e}$) and the neutrinoless double beta decay ($\langle m\rangle_{\rm ee}$) in the minimal seesaw model with two heavy right-handed Majorana neutrinos. By using current neutrino oscillation data, we obtain the ranges of $\langle m\rangle_{\rm e}$ and $\langle m\rangle_{\rm ee}$ for two possible patterns of the neutrino mass spectrum: (1) $0.00424{\rm eV} \leqslant \langle m\rangle_{\rm e} \leqslant 0.0116{\rm eV}$ and $0.00031{\rm eV} \leqslant \langle m\rangle_{\rm ee} \leqslant 0.0052{\rm eV}$ for the normal neutrino mass hierarchy; (2) $0.0398{\rm eV} \leqslant \langle m\rangle_{\rm e} \leqslant 0.0571{\rm eV}$ and $0.0090{\rm eV} \leqslant \langle m\rangle_{\rm ee} \leqslant 0.0571{\rm eV}$ for the inverted neutrino mass hierarchy. The sensitivity of $\langle m\rangle_{\rm ee}$ on the smallest neutrino mixing angle and the Majorana CP-violating phase is also discussed.

Key words neutrino, seesaw mechanism, tritium beta decay, neutrinoless double beta decay

1 Introduction

Current solar[1], atmospheric[2], reactor[3] and accelerator[4] neutrino experiments have provided us with very convincing evidence for the existence of neutrino oscillations, a quantum phenomenon which can naturally occur if neutrinos are massive and lepton flavors are mixed. In order to interpret the small neutrino mass-squared differences and the large lepton flavor mixing angles observed in solar and atmospheric neutrino oscillation experiments, a lot of neutrino models have been proposed at either low-energy scales or high-energy scales[5]. Among them, the so-called minimal seesaw model[6] is particularly simple, suggestive and predictive. Its phenomenological consequences on the cosmological matter-antimatter asymmetry and neutrino oscillations have been discussed by many authors[7—10]. The main purpose of this letter is to explore possible consequences of the minimal seesaw model on the tritium beta decay ($^3_1{\rm H} \rightarrow {}^3_2{\rm He}+{\rm e}^-+\overline{\nu}_{\rm e}$) and the neutrinoless double beta decay (${}^{A}_{Z}{\rm X} \rightarrow {}_{Z+2}^{\;\;A}{\rm X}+2{\rm e}^-$), whose effective mass terms are defined as

$$\langle m\rangle_{\rm e} \equiv \sqrt{m_1^2|V_{\rm e1}|^2+m_2^2|V_{\rm e2}|^2+m_3^2|V_{\rm e3}|^2} \qquad (1)$$

and

$$\langle m\rangle_{\rm ee} \equiv |m_1V_{\rm e1}^2+m_2V_{\rm e2}^2+m_3V_{\rm e3}^2| \,, \qquad (2)$$

respectively[11]. In Eqs. (1) and (2), m_i (for $i=1,2,3$) denote the neutrino masses, and $V_{{\rm e}i}$ (for $i=1,2,3$) stand for the elements of the 3×3 lepton flavor mixing matrix. Such a phenomenological analysis, which has been lacking, will be useful to test the minimal seesaw model in the future experiments of the tritium beta decay and the neutrinoless double beta decay.

2 The minimal seesaw model

In the minimal seesaw model, only two heavy right-handed Majorana neutrinos (N_1 and N_2) are introduced and the Lagrangian of electroweak interac-

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* Supported by National Natural Science Foundation of China (10425522)
1) E-mail: guowl@ihep.ac.cn
2) E-mail: xingzz@ihep.ac.cn

tions is kept to be invariant under the $SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge transformation[6]. The Lagrangian relevant for lepton masses can be written as

$$-\mathscr{L} = \bar{l}_{\rm L} Y_{\rm l} e_{\rm R} H^{\rm c} + \bar{l}_{\rm L} Y_{\nu} N_{\rm R} H + \frac{1}{2}\overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}\,, \quad (3)$$

where $l_{\rm L}$ denotes the left-handed lepton doublet; $e_{\rm R}$ and $N_{\rm R} = (N_1, N_2)^{\rm T}$ stand for the right-handed charged lepton and Majorana neutrino singlets, respectively. After spontaneous gauge symmetry breaking, the Higgs-boson H acquires the vacuum expectation value $v \approx 174$GeV. Then one obtains the charged lepton mass matrix $M_{\rm l} = vY_{\rm l}$ and the Dirac neutrino mass matrix $M_{\rm D} = vY_{\nu}$. The heavy right-handed Majorana neutrino mass matrix $M_{\rm R}$ is a 2×2 symmetric matrix. Without loss of generality, we work in the basis where both $M_{\rm l}$ and $M_{\rm R}$ are diagonal, real and positive; i.e., $M_{\rm l} = {\rm Diag}\{m_{\rm e}, m_{\mu}, m_{\tau}\}$ and $M_{\rm R} = {\rm Diag}\{M_1, M_2\}$. Note that $M_{\rm D}$ is a complex 3×2 matrix. The effective (light) neutrino mass matrix M_{ν} can be expressed in terms of $M_{\rm D}$ and $M_{\rm R}$ through the famous seesaw relation[12]:

$$M_{\nu} \approx M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^{\rm T}\,. \quad (4)$$

As $M_{\rm R}$ is at most of rank 2, $|{\rm Det}(M_{\nu})| = m_1 m_2 m_3 = 0$ must hold1). Taking account of $m_2 > m_1$ extracted from the solar neutrino oscillation data[1], we are left with two possibilities: either $m_1 = 0$ (normal hierarchy) or $m_3 = 0$ (inverted hierarchy). This peculiar feature of the minimal seesaw model implies that the 3×3 lepton flavor mixing matrix consists of only two nontrivial CP-violating phases:

$$V = \begin{pmatrix} c_x c_z & s_x c_z & s_z \\ -c_x s_y s_z - s_x c_y {\rm e}^{-{\rm i}\delta} & -s_x s_y s_z + c_x c_y {\rm e}^{-{\rm i}\delta} & s_y c_z \\ -c_x c_y s_z + s_x s_y {\rm e}^{-{\rm i}\delta} & -s_x c_y s_z - c_x s_y {\rm e}^{-{\rm i}\delta} & c_y c_z \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & {\rm e}^{{\rm i}\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (5)$$

with $s_x \equiv \sin\theta_x$, $c_x \equiv \cos\theta_x$ and so on. Three mixing angles of V are directly related to the solar, atmospheric and CHOOZ[13] neutrino oscillations; i.e., $\theta_x \approx \theta_{\rm sun}$, $\theta_y \approx \theta_{\rm atm}$ and $\theta_z \approx \theta_{\rm chz}$ hold as a very good approximation. A global analysis of current experimental data[14] yields

$$30^\circ \leqslant \theta_x \leqslant 38^\circ\,,$$
$$36^\circ \leqslant \theta_y \leqslant 54^\circ\,,$$
$$0^\circ \leqslant \theta_z < 10^\circ\,, \quad (6)$$

at the 99% confidence level. The mass-squared differences of solar and atmospheric neutrino oscillations are defined as $\Delta m^2_{\rm sun} \equiv m_2^2 - m_1^2$ and $\Delta m^2_{\rm atm} \equiv |m_3^2 - m_2^2|$, respectively. At the 99% confidence level, we have[14]

$$7.2\times 10^{-5}{\rm eV}^2 \leqslant \Delta m^2_{\rm sun} \leqslant 8.9\times 10^{-5}{\rm eV}^2\,,$$
$$1.7\times 10^{-3}{\rm eV}^2 \leqslant \Delta m^2_{\rm atm} \leqslant 3.3\times 10^{-3}{\rm eV}^2\,. \quad (7)$$

Whether $m_2 < m_3$ or $m_2 > m_3$ remains an open question.

3 Numerical predictions and further discussions

If $m_1 = 0$ holds in the minimal seesaw model, we can easily obtain

$$m_2 = \sqrt{\Delta m^2_{\rm sun}}\,,$$
$$m_3 = \sqrt{\Delta m^2_{\rm sun} + \Delta m^2_{\rm atm}}\,. \quad (8)$$

On the other hand, $m_3 = 0$ may directly lead to

$$m_1 = \sqrt{\Delta m^2_{\rm atm} - \Delta m^2_{\rm sun}}\,,$$
$$m_2 = \sqrt{\Delta m^2_{\rm atm}}\,. \quad (9)$$

Taking account of Eq. (7), we are then able to constrain the ranges of m_2 and m_3 by using Eq. (8) or the ranges of m_1 and m_2 by using Eq. (9). Our numerical results are shown in Fig. 1(a) and Fig. 1(b), respectively.

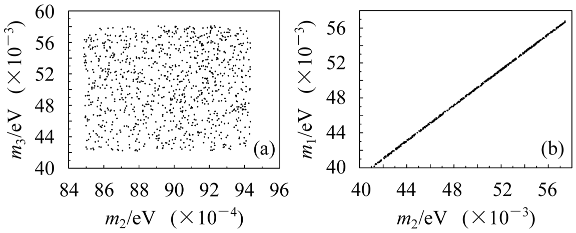

Fig. 1. The allowed region of (a) m_2 and m_3 for $m_1 = 0$ or (b) m_1 and m_2 for $m_3 = 0$ in the minimal seesaw model.

1) As already shown in Ref. [9], $|{\rm Det}(M_{\nu})| = 0$ is stable against radiative corrections from the seesaw scale to the electroweak scale (or vice versa). Hence one of the neutrino mass eigenvalues (m_1 or m_3) is always vanishing in the minimal seesaw model.

Namely,

$$0.00849\text{eV} \leqslant m_2 \leqslant 0.00943\text{eV}\,,$$
$$0.0421\text{eV} \leqslant m_3 \leqslant 0.0582\text{eV} \qquad (10)$$

for the normal neutrino mass hierarchy ($m_1 = 0$); and

$$0.0401\text{eV} \leqslant m_1 \leqslant 0.0568\text{eV}\,,$$
$$0.0412\text{eV} \leqslant m_2 \leqslant 0.0574\text{eV} \qquad (11)$$

for the inverted neutrino mass hierarchy ($m_3 = 0$).

Now we proceed to calculate the effective mass terms $\langle m\rangle_{\text{e}}$ and $\langle m\rangle_{\text{ee}}$ in the minimal seesaw model. Combining Eqs. (1), (5), (8) and (9), we obtain

$$\langle m\rangle_{\text{e}} = \begin{cases} \sqrt{\Delta m^2_{\text{sun}} s_x^2 c_z^2 + (\Delta m^2_{\text{sun}} + \Delta m^2_{\text{atm}})\, s_z^2}, \\ (m_1 = 0)\,, \\ \sqrt{(\Delta m^2_{\text{atm}} - \Delta m^2_{\text{sun}} c_x^2)\, c_z^2}\,,\ (m_3 = 0)\,. \end{cases} \qquad (12)$$

On the other hand, the expression of $\langle m\rangle_{\text{ee}}$ can be obtained by combining Eqs. (2), (5), (8) and (9):

$$\langle m\rangle_{\text{ee}} = \begin{cases} \sqrt{\Delta m^2_{\text{sun}} s_x^4 c_z^4 + (\Delta m^2_{\text{sun}} + \Delta m^2_{\text{atm}})\, s_z^4 + T_1 \cos 2\sigma}, & (m_1 = 0)\,, \\ \sqrt{\Delta m^2_{\text{atm}} s_x^4 c_z^4 + (\Delta m^2_{\text{atm}} - \Delta m^2_{\text{sun}})\, c_x^4 c_z^4 + T_3 \cos 2\sigma}, & (m_3 = 0)\,, \end{cases} \qquad (13)$$

where

$$T_1 = 2\sqrt{\Delta m^2_{\text{sun}}\,(\Delta m^2_{\text{sun}} + \Delta m^2_{\text{atm}})}\ s_x^2 c_z^2 s_z^2\,,$$
$$T_3 = 2\sqrt{\Delta m^2_{\text{atm}}\,(\Delta m^2_{\text{atm}} - \Delta m^2_{\text{sun}})}\ c_x^2 s_x^2 c_z^4\,. \qquad (14)$$

Just as expected, $\langle m\rangle_{\text{ee}}$ depends on the Majorana CP-violating phase σ. This phase parameter does not affect CP violation in neutrino-neutrino and antineutrino-antineutrino oscillations, but it may play a significant role in the leptogenesis[15] due to the lepton-number-violating and CP-violating decays of two heavy right-handed Majorana neutrinos.

With the help of current experimental data listed in Eqs. (6) and (7), we can obtain the numerical predictions for $\langle m\rangle_{\text{e}}$ and $\langle m\rangle_{\text{ee}}$ by using Eqs. (12) and (13). The results are shown in Fig. 2 for two different patterns of the neutrino mass spectrum. It is then straightforward to arrive at

$$0.00424\text{eV} \leqslant \langle m\rangle_{\text{e}} \leqslant 0.0116\text{eV}\,,$$
$$0.00031\text{eV} \leqslant \langle m\rangle_{\text{ee}} \leqslant 0.0052\text{eV} \qquad (15)$$

for $m_1 = 0$; and

$$0.0398\text{eV} \leqslant \langle m\rangle_{\text{e}} \leqslant 0.0571\text{eV}\,,$$
$$0.0090\text{eV} \leqslant \langle m\rangle_{\text{ee}} \leqslant 0.0571\text{eV} \qquad (16)$$

for $m_3 = 0$. Two comments are in order.

(a) Whether $\langle m\rangle_{\text{e}}$ and $\langle m\rangle_{\text{ee}}$ can be measured remains an open question. The present experimental upper bounds are $\langle m\rangle_{\text{e}} < 3\text{eV}$[16] and $\langle m\rangle_{\text{ee}} < 0.35\text{eV}$ at the 90% confidence level[17]. They are much larger than our predictions for the upper bounds of $\langle m\rangle_{\text{e}}$ and $\langle m\rangle_{\text{ee}}$ in the minimal seesaw model. The proposed KATRIN experiment is possible to reach the sensitivity $\langle m\rangle_{\text{e}} \sim 0.3\text{eV}$[18]. If a signal of $\langle m\rangle_{\text{e}} \sim 0.1\text{eV}$ is seen, the minimal seesaw model will definitely be ruled out. On the other hand, a number of the next-generation experiments for the neutrinoless double beta decay[19] is possible to probe $\langle m\rangle_{\text{ee}}$ at the level of 10meV to 50meV. Such experiments are expected to test our prediction for $\langle m\rangle_{\text{ee}}$ given in Eq. (16); i.e., in the case of $m_3 = 0$.

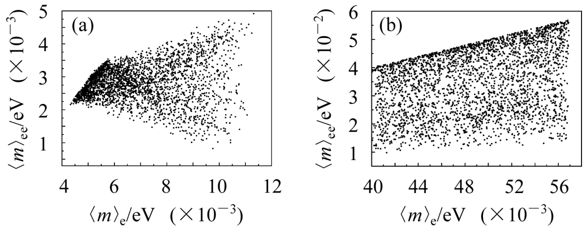

Fig. 2. The allowed region of $\langle m\rangle_{\text{e}}$ and $\langle m\rangle_{\text{ee}}$ in the minimal seesaw model: (a) $m_1 = 0$ and (b) $m_3 = 0$.

(b) Now that the magnitude of $\langle m\rangle_{\text{ee}}$ in the case of $m_3 = 0$ is experimentally accessible in the future, its sensitivity to the unknown parameters θ_z and σ is worthy of some discussions. Eq. (13) shows that $\langle m\rangle_{\text{ee}}$ depends only on c_z for $m_3 = 0$. Hence we conclude that the magnitude of $\langle m\rangle_{\text{ee}}$ is insensitive to the change of θ_z in its allowed range (i.e., $0^\circ \leqslant \theta_z < 10^\circ$ [14]). The dependence of $\langle m\rangle_{\text{ee}}$ on the Majorana CP-violating phase σ is illustrated in Fig. 3. We observe that $\langle m\rangle_{\text{ee}}$ is significantly sensitive to σ. Thus a measurement of $\langle m\rangle_{\text{ee}}$ will allow us to determine (or constrain) this important phase parameter in the minimal seesaw model.

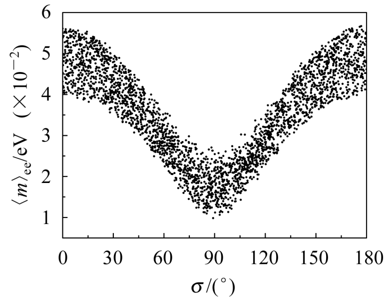

Fig. 3. The dependence of $\langle m\rangle_{ee}$ on the Majorana CP-violating phase σ for $m_3 = 0$ in the minimal seesaw model.

4 Summary

In summary, we have analyzed the effective mass terms of the tritium beta decay ($\langle m\rangle_{e}$) and the neutrinoless double beta decay ($\langle m\rangle_{ee}$) in the minimal seesaw model. By using current neutrino oscillation data, we have obtained the ranges of $\langle m\rangle_{e}$ and $\langle m\rangle_{ee}$ for two possible patterns of the neutrino mass spectrum: (1) 0.00424eV $\leqslant \langle m\rangle_{e} \leqslant$ 0.0116eV and 0.00031eV $\leqslant \langle m\rangle_{ee} \leqslant$ 0.0052eV for the normal neutrino mass hierarchy ($m_1 = 0$); (2) 0.0398eV $\leqslant \langle m\rangle_{e} \leqslant$ 0.0571eV and 0.0090eV $\leqslant \langle m\rangle_{ee} \leqslant$ 0.0571eV for the inverted neutrino mass hierarchy ($m_3 = 0$). In the latter case, the magnitude of $\langle m\rangle_{ee}$ is accessible to the sensitivity of the future neutrinoless double beta decay experiments. A measurement of $\langle m\rangle_{ee}$ can therefore shed light on the single Majorana CP-violating phase in the minimal seesaw model.

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最小seesaw模型对氚β衰变和无中微子双β衰变的限制*

郭万磊[1]　邢志忠[2]

(中国科学院高能物理研究所　北京　100049)

摘要　在最小seesaw模型下计算了氚β衰变的有效质量$\langle m\rangle_{\rm e}$以及无中微子双β衰变的有效质量$\langle m\rangle_{\rm ee}$. 利用最新的中微子振荡数据, 在正质量等级情况下得到了$0.00424{\rm eV}\leqslant\langle m\rangle_{\rm e}\leqslant0.0116{\rm eV}$和$0.00031{\rm eV}\leqslant\langle m\rangle_{\rm ee}\leqslant0.0052{\rm eV}$; 如果中微子的质量谱是倒质量等级情况, 能够得到$0.0398{\rm eV}\leqslant\langle m\rangle_{\rm e}\leqslant0.0571{\rm eV}$和$0.0090{\rm eV}\leqslant\langle m\rangle_{\rm ee}\leqslant0.0571{\rm eV}$. 最后还讨论了最小的中微子混合角和Majorana CP破坏位相对$\langle m\rangle_{\rm ee}$的影响.

关键词　中微子　跷跷板机制　氚β衰变　无中微子双β衰变

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1) E-mail: guowl@ihep.ac.cn

2) E-mail: xingzz@ihep.ac.cn