Θ^+ with Double Y-Ansatz Confining Potential*

ZHAO Hong-Ming[1,2;1)] ZENG Zhuo-Quan[1,2] SHEN Peng-Nian[5,4,1]
DING Yi-Bing[2] LI Xue-Qian[3,4]

1 (Institute of High Energy Physics, CAS, Beijing 100049, China)
2 (College of Physical Sciences,
Graduate University of Chinese Academy of Sciences, Beijing 100049, China)
3 (Department of Physics, Nankai University, Tianjin 300071, China)
4 (Institute of Theoretical Physics, CAS, Beijing 100080, China)
5 (Center of Theoretical Nuclear Physics,
National Laboratory of Heavy Ion Accelerator of Lanzhou, Lanzhou 730000, China)

Abstract The mass of Θ^+ in the $\mathrm{qq}-\overline{\mathrm{q}}-\mathrm{qq}$ configuration is calculated by using a potential composed by a Coulomb potential originated from the One Gluon Exchange (OGE) and a double Y-mode confining potential extracted from the flux tube model. It is shown that the mass of the negative parity state of Θ^+ is 1.935GeV, which is more than 300MeV higher than the reported experimental value of 1.540GeV. While the positive parity state of Θ^+ has a mass of 2.082GeV, which is consistent with the result from the Lattice Quantum Chromodynamics (LQCD) calculation. With these masses, the reported $\Theta^+(1540)$ cannot be confirmed as a pentaquark state.

Key words pentaquark, flux-tube model, Y-mode confining potential

1 Introduction

Since LEPS Collaboration at SPring-8[1] announced the discovery of a sharp resonance with $M = 1.54 \pm 0.01$GeV and $\Gamma < 25$MeV in the $\gamma \mathrm{n} \to \mathrm{K}^+\mathrm{K}^-\mathrm{n}$ process in 2003, much attention has been attracted to this aspect. Many experimental groups, include ITEP (DIANA), JLab (CLAS), ELSA (SAPHIR) et al.[2—4] started to re-analyze their old data sets and also found a peak around 1.54GeV. These findings strongly support the existence of a resonant state called Θ^+. Because the baryon number and the strangeness of the state are all +1, and there is no Θ^{++} found in the experiment, the Θ^+ cannot be a normal baryon which is composed by three valence quarks only. Thus, the minimal constituent of such a state should be $\mathrm{uudd}\bar{\mathrm{s}}$, and this state probably is an isospin singlet state. Other quantum numbers, such as the angular momentum and parity, are still under investigation. By contrast, HERA-B and some high-energy experimental groups[5—7] reported that no evidence of $\Theta^+(1540)$ has been found. In order to confirm the existence of Θ^+, high statistical experimental data are requested. In fact, the CLAS Collaboration at JLAB recently declared[8] that their newly obtained high statistical data sets in the $\gamma + \mathrm{d} \to \mathrm{p} + \mathrm{K}^- + \mathrm{K} + \mathrm{n}$ process at g10 and in the $\gamma + \mathrm{p} \to \pi^+ + \pi^- + \mathrm{K}^+\mathrm{n}$ process at g11 no longer showed the structure around 1.540GeV reported by CLAS and SAPHIR in 2003.

Theoretical study of pentaquark can be traced back to the end of 1970's when the pentaquark was

Received 21 November 2005, Revised 20 December 2005
*Supported by National Natural Science Foundation of China (10375090, 10475042, 10475089, 10435080), CAS Knowledge Innovation Key-Project (KJCX2-SW-N02), Research Fund for the Doctoral Program of Higher Education of China and IHEP (U529)
1) E-mail: zhaohm@mail.ihep.ac.cn

predicted by using the MIT bag model[9]. In recent years, the interest in studying Θ^+ was aroused by the chiral soliton model prediction[10], in which a resonance with $S=+1$, $J^P=\frac{1}{2}^+$ and $\Gamma\leqslant 15\text{MeV}$, should exist as the lightest member of anti-decuplet multiplets. After LEPS's first observation of Θ^+, many theoretical studies on Θ^+ have been carried out, such as cluster quark model[11, 12], Quantum Chromodynamics (QCD) sum rules[13, 14], un-clustered quark model[15, 16], Lattice QCD (LQCD)[17, 18] calculations, and etc. The results from these investigations are quite different. No matter the resultant mass of Θ^+ is higher than[16, 18] or is just around[11, 13] the experimental value of 1.54GeV, the parity and the the narrow width of Θ^+ are still mysteries. The LQCD calculations also differ in result. Alexandrou et al.[17] found that the masses of Θ^+ are 1.603 ± 0.073GeV for the negative parity state and 2.36 ± 0.13GeV for the positive parity state. Takahashi et al.[18] asserted that the mass of the lowest positive-parity $4\text{q}-\bar{\text{q}}$ state is about 2.24GeV and no compact $4\text{q}-\bar{\text{q}}$ state exists below 1.75GeV in the negative-parity channel. And Mathur et al.[19] and Kieran et al.[20] showed no evidence for bound pentaquark.

Apparently, the mass of Θ^+ is closely related to the structure of pentaquark itself. Based on the flux-tube picture, Takahashi et al. suggested a double-Y-type flux tube confining structure[18], see Fig. 1(a), while Alexandrou et al. proposed a three-Steiner-point confining structure that can provide a minimal flux-tube length for a $4\text{q}-\bar{\text{q}}$ system[21], see Fig. 1(b). Song et al. conjecture that a stable structure of pentaquark should be a tetrahedron form[22]. We also compared some of those models originated from the flux-tube picture in a simplified way[23].

In the next section, the brief formalism is given. And the result and discussion are presented in section 3.

2 Brief formalism

In the framework of the flux-tube model, a simple potential form, where the linear confining potential originated from the flux tube is supplemented with a Coulomb potential induced by the one-gluon-exchange (OGE), was employed for the meson and baryon calculations[24]. That potential picture was lately verified by the LQCD calculation[25]. For a multiquark system, the flux-tube model suggests a multi-Y-shape flux-tube confining picture[17, 18]. Now, we extend our baryon spectrum study in the flux tube model[26] to the calculation of the masses of Θ^+ in both positive and negative parity states in a two-diquark cluster configuration[11]. The corresponding pentaquark configuration is $\text{qq}-\bar{\text{q}}-\text{qq}$ as shown in Fig. 1(a). In this configuration, two pairs of q-q form two $SU_\text{C}(3)$ $\bar{\mathbf{3}}$ representations, respectively, and then they are combined with the antiquark $\bar{\text{q}}$, which is also in the $SU_\text{C}(3)$ $\bar{\mathbf{3}}$ representation, in a color singlet state.

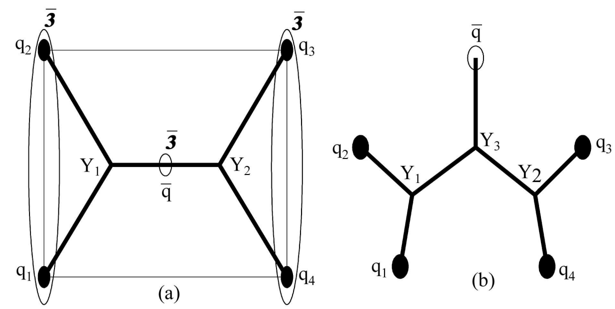

Fig. 1. Flux-tube configuration for pentaquark. (a) double-Y-type ansatz[18]; (b) three-Y-type ansatz[21].

The potential used for Θ^+ study is in the form of a double Y-shape confining potential supplemented with a Coulomb potential[27]. Then, the Hamiltonian of pentaquark can be written as

$$H=T+V^{\text{Coul}}+V_\text{Y}^{\text{conf}}, \tag{1}$$

with

$$V^{\text{Coul}}=\alpha_\text{s}\sum_{i<j}\frac{T_i^aT_j^a}{|\boldsymbol{r}_i-\boldsymbol{r}_j|}, \tag{2}$$

$T_i^a=\lambda_i^a/2$ being the color Casimir operator of the ith quark, and V_Y^{conf} being the double Y-mode confining potential originated from the flux-tube model calculation in the adiabatic approximation,

$$V_\text{Y}^{\text{conf}}=b_\text{Y}(L_{\text{min}}^{\text{Y1}}+L_{\text{min}}^{\text{Y2}})+C_{5\text{q}}^{\text{Y}}, \tag{3}$$

b_Y the string tension, $C_{5\text{q}}^{\text{Y}}$ an overall constant, $L_{\text{min}}^{\text{Y1(Y2)}}$ the minimal length of the flux-tube linking two quarks in the same q-q cluster and the antiquark. The general form of $L_{\rm min}^{\rm Y1(Y2)}$ is[28]

$$L_{\rm min}^{\rm Y1(Y2)}=\sum_i |\,\boldsymbol{r}_i-\boldsymbol{r}_0\,|, \tag{4}$$

where $\boldsymbol{r}_0$ is the coordinate of the junction point. The rule for finding the location of the junction point $\boldsymbol{r}_0$ is the following: If all the inner angles of the triangle with the two quarks in one diquark cluster and antiquark sitting at the apexes of the triangle are smaller than $2\pi/3$, the junction point is located inside the triangle and the angles spanned by two flux tubes are $2\pi/3$. If one of the inner angles of the triangle would take a value equal to or greater than $2\pi/3$, the junction point would be located at that apex. Let the lengths of the three sides of the triangle be a, b and c, respectively. $L_{\rm min}^{\rm Y1(Y2)}$ then can be expressed as

$$L_{\rm min}^{\rm Y1(Y2)}=\begin{cases}\Big[\frac{1}{2}(a^2+b^2+c^2)+\frac{\sqrt{3}}{2}\times \\ \sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}\Big]^{1/2} \\ \text{if all the inner angles are smaller than } 2\pi/3, \\ a+b+c-\max(a,b,c) \\ \text{if one of the inner angle is not smaller than } 2\pi/3.\end{cases} \tag{5}$$

The Schrödinger equation $H\Psi=E\Psi$ with Hamiltonian (1) can be solved by using variational method. The trial wave function is composed by the color, flavor, spin and spatial wave functions. These functions should be combined into a totally antisymmetric wave function to satisfy the generalized Pauli principle.

The color wave function of Θ^+ should be color singlet. The form of the color wave function with the qq$-\overline{\rm q}-$qq configuration can be expressed as

$$|(\overline{\boldsymbol{3}}_{12},\overline{\boldsymbol{3}}_{34})_{\boldsymbol{3}_{1234}},\overline{\boldsymbol{3}}_5\rangle^C= \frac{1}{\sqrt{2}}\epsilon_{jmn}C_3^mC_4^n\frac{1}{\sqrt{2}}\epsilon_{ilp}C_1^lC_2^p\frac{1}{\sqrt{2}}\epsilon_{ijk}\frac{1}{\sqrt{3}}C_5^k. \tag{6}$$

Because of the requested flavor symmetry of $\overline{\boldsymbol{10}}$ for Θ^+ and the flavor symmetry of $\bar{\boldsymbol{3}}$ for antiquark, the flavor symmetry of $\bar{\boldsymbol{6}}$ for rest of four quarks ($\rm q^4$) in Θ^+ is required. The possible flavor combinations of $\rm q^4$ are

$$|(\boldsymbol{6}_{12},\boldsymbol{6}_{34})_{\overline{\boldsymbol{6}}_{1234}},\overline{\boldsymbol{3}}_5\rangle^F=\frac{1}{2\sqrt{3}}(2{\rm uudd}+ 2{\rm dduu}-{\rm udud}-{\rm uddu}-{\rm duud}-{\rm dudu})\bar{\rm s} \tag{7}$$

and

$$|(\overline{\boldsymbol{3}}_{12},\overline{\boldsymbol{3}}_{34})_{\overline{\boldsymbol{6}}_{1234}},\overline{\boldsymbol{3}}_5\rangle^F=\frac{1}{2}({\rm ud}-{\rm du})({\rm ud}-{\rm du})\bar{\rm s}, \tag{8}$$

respectively.

Since the total angular momentum of Θ^+ is $\frac{1}{2}$, the total spin of $\rm q^4$-$\bar{\rm q}$ can be either $\frac{3}{2}$ or $\frac{1}{2}$ for the positive parity state and $\frac{1}{2}$ for the negative parity state, according to vector coupling relation $\boldsymbol{J}=\boldsymbol{L}+\boldsymbol{S}$, where $\boldsymbol{J}$, $\boldsymbol{L}$ and $\boldsymbol{S}$ denote the total angular momentum, total orbital momentum and total spin momentum of $\rm q^4$-$\bar{\rm q}$, respectively. Therefore, the spin symmetries for $\rm q^4$ in Θ^+ should be $\boldsymbol{5}$, $\boldsymbol{3}$ and $\boldsymbol{1}$. The explicit spin wave functions of Θ^+ with spin $(S,M_{\rm S})=(S,S)$ in spin multiplets $|((12),(34))_{(1234)},5\rangle^S_{S,M_{\rm S}}$ are

$$|(\boldsymbol{3}_{12},\boldsymbol{3}_{34})_{(\boldsymbol{5}_{1234})},\boldsymbol{2}_5\rangle^S_{\frac{3}{2},\frac{3}{2}}= \sqrt{\frac{4}{5}}\uparrow\uparrow\uparrow\uparrow\downarrow-\frac{1}{2}\sqrt{\frac{1}{5}}[(\uparrow\downarrow+\downarrow\uparrow)\uparrow\uparrow+\uparrow\uparrow(\uparrow\downarrow+\downarrow\uparrow)]\uparrow,$$

$$|(\boldsymbol{3}_{12},\boldsymbol{3}_{34})_{\boldsymbol{3}_{1234}},\boldsymbol{2}_5\rangle^S_{\frac{3}{2},\frac{3}{2}}= \frac{1}{2}[(\uparrow\downarrow+\downarrow\uparrow)\uparrow\uparrow+\uparrow\uparrow(\uparrow\downarrow+\downarrow\uparrow)]\uparrow,$$

$$|(\boldsymbol{3}_{12},\boldsymbol{3}_{34})_{\boldsymbol{3}_{1234}},\boldsymbol{2}_5\rangle^S_{\frac{1}{2},\frac{1}{2}}= \sqrt{\frac{1}{6}}\{[\uparrow\uparrow(\uparrow\downarrow+\downarrow\uparrow)-(\uparrow\downarrow+\downarrow\uparrow)\uparrow\uparrow]\downarrow-(\uparrow\uparrow\downarrow\downarrow-\downarrow\downarrow\uparrow\uparrow)\uparrow\},$$

$$|(\boldsymbol{3}_{12},\boldsymbol{1}_{34})_{(\boldsymbol{3}_{1234})},\boldsymbol{2}_5\rangle^S_{\frac{1}{2},\frac{1}{2}}= \frac{1}{2\sqrt{3}}[2\uparrow\uparrow(\uparrow\downarrow-\downarrow\uparrow)\downarrow+(\uparrow\downarrow+\downarrow\uparrow)(\uparrow\downarrow-\downarrow\uparrow)\uparrow],$$

$$|(\boldsymbol{1}_{12},\boldsymbol{3}_{34})_{(\boldsymbol{3}_{1234})},\boldsymbol{2}_5\rangle^S_{\frac{1}{2},\frac{1}{2}}= \frac{1}{2\sqrt{3}}[2(\uparrow\downarrow-\downarrow\uparrow)\uparrow\uparrow\downarrow+(\uparrow\downarrow-\downarrow\uparrow)(\uparrow\downarrow+\downarrow\uparrow)\uparrow],$$

$$|(\boldsymbol{3}_{12},\boldsymbol{3}_{34})_{\boldsymbol{1}_{1234}},\boldsymbol{2}_5\rangle^S_{\frac{1}{2},\frac{1}{2}}= \frac{1}{2\sqrt{3}}[2\uparrow\uparrow\downarrow\downarrow-2\downarrow\downarrow\uparrow\uparrow-\uparrow\downarrow\uparrow\downarrow-\uparrow\downarrow\downarrow\uparrow-\downarrow\uparrow\uparrow\downarrow-\downarrow\uparrow\downarrow\uparrow]\uparrow,$$

$$|(\boldsymbol{1}_{12},\boldsymbol{1}_{34})_{\boldsymbol{1}_{1234}},\boldsymbol{2}_5\rangle^S_{\frac{1}{2},\frac{1}{2}}=\frac{1}{2}(\uparrow\downarrow-\downarrow\uparrow)(\uparrow\downarrow-\downarrow\uparrow)\uparrow\ .$$

Moreover, because the parity of the antiquark is negative, for the lowest negative parity state of Θ^+, the spatial wave function should have no orbital excitation, and for the lowest positive parity state of Θ^+, the number of orbital excitation should be 1. The explicit forms of spatial wave functions depend on the coordinate system. In this work, we choose a coordinate system shown in Fig. 2. In this system, the relation of variable sets ($\boldsymbol{r}_1$, $\boldsymbol{r}_2$, $\boldsymbol{r}_3$, $\boldsymbol{r}_4$, $\boldsymbol{r}_{\bar{q}}$) and ($\boldsymbol{\rho}$, $\boldsymbol{\nu}$,

$\boldsymbol{\lambda}$, $\boldsymbol{\tau}$, $\boldsymbol{R}_{\rm cm}$) is

$$\begin{pmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \\ \boldsymbol{r}_3 \\ \boldsymbol{r}_4 \\ \boldsymbol{r}_{\overline{\rm q}} \end{pmatrix} = A \begin{pmatrix} \boldsymbol{\rho} \\ \boldsymbol{\nu} \\ \boldsymbol{\lambda} \\ \boldsymbol{\tau} \\ \boldsymbol{R}_{\rm cm} \end{pmatrix}, \tag{9}$$

where A is a unitary transformation matrix

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} & -\frac{\mu}{4m} & 1 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{2} & -\frac{\mu}{4m} & 1 \\ -0 & \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\mu}{4m} & 1 \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\mu}{4m} & 1 \\ 0 & 0 & 0 & \frac{\mu}{m_{\overline{\rm q}}} & 1 \end{pmatrix}, \tag{10}$$

and m and $m_{\overline{\rm q}}$ are the masses of the quark u(d) and anti-strange quark $\bar{\rm s}$, respectively, and $\mu = 4mm_{\overline{\rm q}}/(4m+m_{\overline{\rm q}})$ denotes the reduced mass of the $4{\rm q}-\overline{\rm q}$ system. Then, one can write the kinetic energy operator as

$$T({\rm q}^4\overline{\rm q}) = \frac{\boldsymbol{\nabla}_\tau^2}{2\mu} + \frac{\boldsymbol{\nabla}_\lambda^2}{2m} + \frac{\boldsymbol{\nabla}_\rho^2}{2m} + \frac{\boldsymbol{\nabla}_\nu^2}{2m}. \tag{11}$$

If all the quarks in Θ^+ are in S-wave, the explicit form of the spatial wave function $|\psi_{\rm LM}(\alpha)\rangle^O$ for the lowest negative parity states is

$$|\psi_{00}^S(\alpha,\xi)\rangle^O = N\left[Y_{00}(\hat{\lambda}){\rm e}^{-\alpha^2\lambda^2/2}\right]\left[Y_{00}(\hat{\rho}){\rm e}^{-\alpha^2\rho^2/2}\right]\times \left[Y_{00}(\hat{\nu}){\rm e}^{-\alpha^2\nu^2/2}\right]\left[Y_{00}(\hat{\tau}){\rm e}^{-\frac{4m_{\overline{\rm q}}}{4m+m_{\overline{\rm q}}}\alpha^2\tau^2/2}\right], \tag{12}$$

where ξ stands for the aggregate of spatial variables. When one of the quarks in Θ^+ is in P-wave, and no orbital excitation in q-q is considered, the P wave excitation can appear either between the two q-q clusters, the λ excitation mode, or between ${\rm q}^4$ and $\bar{\rm q}$, the τ excitation mode. The corresponding spatial trial wave functions can explicitly be written as

$$|\psi_{1M_{\rm L}}^\lambda(\alpha,\xi)\rangle^O = N\left[\alpha\lambda Y_{1m}(\hat{\lambda}){\rm e}^{-\alpha^2\lambda^2/2}\right]\times \left[Y_{00}(\hat{\rho}){\rm e}^{-\alpha^2\rho^2/2}\right]\left[Y_{00}(\hat{\nu}){\rm e}^{-\alpha^2\nu^2/2}\right]\times \left[Y_{00}(\hat{\tau}){\rm e}^{-\frac{4m_{\overline{\rm q}}}{4m+m_{\overline{\rm q}}}\alpha^2\tau^2/2}\right], \tag{13}$$

and

$$|\psi_{1M_{\rm L}}^\tau(\alpha,\xi)\rangle^O = N\left[Y_{00}(\hat{\lambda}){\rm e}^{-\alpha^2\lambda^2/2}\right]\times \left[Y_{00}(\hat{\rho}){\rm e}^{-\alpha^2\rho^2/2}\right]\left[Y_{00}(\hat{\nu}){\rm e}^{-\alpha^2\nu^2/2}\right]\times \left[\sqrt{\frac{4m_{\overline{\rm q}}}{4m+m_{\overline{\rm q}}}}\alpha\tau Y_{1m}(\hat{\tau}){\rm e}^{-\frac{4m_{\overline{\rm q}}}{4m+m_{\overline{\rm q}}}\alpha^2\tau^2/2}\right], \tag{14}$$

respectively.

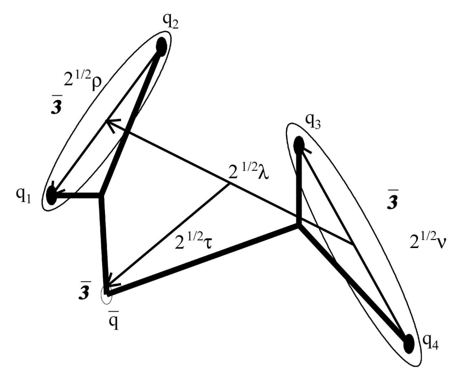

Fig. 2. The Jacobian coordinates for $\rm qq-\overline{q}-qq$ configuration of Θ^+[29].

The total trial wave functions of Θ^+ then can respectively be formed by combining the color, flavor and spin wave functions with the spatial trial wave functions. The Young diagrams of the possible quark pair combinations in the color, flavor, spin and spatial spaces for ${\rm q}^4$ and the corresponding symmetry under the interchanges of the two q-q pairs are shown in Fig. 3, respectively. The possible ${\rm q}^4$ states coupled by quark pairs, given in Fig. 3, are presented in Fig. 4. It is shown that there is only one combination

$$|\Phi(J,M_J,P,\alpha,\zeta)\rangle|_{J=\frac{1}{2},M_J=\frac{1}{2},P=+} = |\Psi^S_{\frac{1}{2},\frac{1}{2}}\rangle = |(\overline{\boldsymbol{3}}_{12},\overline{\boldsymbol{3}}_{34})_{\boldsymbol{3}_{1234}},\overline{\boldsymbol{3}}_5\rangle^C\otimes|(\boldsymbol{6}_{12},\boldsymbol{6}_{34})_{\overline{\boldsymbol{6}}_{1234}},\overline{\boldsymbol{3}}_5\rangle^F\otimes |(\boldsymbol{3}_{12},\boldsymbol{3}_{34})_{\boldsymbol{3}_{1234}},\boldsymbol{2}_5\rangle^S_{\frac{1}{2},\frac{1}{2}}\otimes|\psi^S_{00}(\alpha,\xi)\rangle^O, \tag{15}$$

for the negative parity state, where P and ζ denote the parity of the state and the aggregate of all the variables in color, flavor, spin and spatial spaces, respectively, and three possible combinations

$$|\Psi^{\lambda(1)}_{\frac{1}{2},\frac{1}{2}}\rangle = |(\overline{\boldsymbol{3}}_{12},\overline{\boldsymbol{3}}_{34})_{\boldsymbol{3}_{1234}},\overline{\boldsymbol{3}}_5\rangle^C\otimes\sum(1\ \tfrac{1}{2}\ M_L\ M_S;\tfrac{1}{2}\ \tfrac{1}{2})\otimes\frac{1}{\sqrt{2}}|(\overline{\boldsymbol{3}}_{12},\overline{\boldsymbol{3}}_{34})_{\overline{\boldsymbol{6}}_{1234}},\overline{\boldsymbol{3}}_5\rangle^F\otimes |(\boldsymbol{1}_{12},\boldsymbol{1}_{34})_{\boldsymbol{1}_{1234}},\boldsymbol{2}_5\rangle^S_{\frac{1}{2},M_S}+|(\boldsymbol{6}_{12},\boldsymbol{6}_{34})_{\overline{\boldsymbol{6}}_{1234}},\bar{\boldsymbol{3}}_5\rangle^F\otimes|(\boldsymbol{3}_{12},\boldsymbol{3}_{34})_{\boldsymbol{1}_{1234}},\boldsymbol{2}_5\rangle^S_{\frac{1}{2},M_S})\otimes|\psi^\lambda_{1M_L}(\alpha)\rangle^O, \tag{16}$$

$$|\Psi^{\lambda(2)}_{\frac12,\frac12}\rangle=|(\overline{\mathbf 3}_{12},\overline{\mathbf 3}_{34})_{\mathbf 3_{1234}},\overline{\mathbf 3}_5\rangle^C\otimes|(\overline{\mathbf 3}_{12},\overline{\mathbf 3}_{34})_{\overline{\mathbf 6}_{1234}},\overline{\mathbf 3}_5\rangle^F\otimes\sum\left(1\frac32 M_L M_S;\frac12\ \frac12\right)\otimes|(\mathbf 3_{12},\mathbf 3_{34})_{\mathbf 5_{1234}},\mathbf 2_5\rangle^S_{\frac32,M_S}\otimes|\psi^\lambda_{1M_L}(\alpha)\rangle^O, \tag{17}$$

and

$$|\Psi^{\tau}_{\frac12,\frac12}\rangle = |(\overline{\mathbf 3}_{12},\overline{\mathbf 3}_{34})_{\mathbf 3_{1234}},\overline{\mathbf 3}_5\rangle^C\otimes|(\mathbf 6_{12},\mathbf 6_{34})_{\overline{\mathbf 6}_{1234}},\bar{\mathbf 3}_5\rangle^F\otimes\sqrt{\frac12}\Big(\sum(1\ \frac32 M_L M_S;\frac12\ \frac12)\otimes|(\mathbf 3_{12},\mathbf 3_{34})_{\mathbf 3_{1234}},\mathbf 2_5\rangle^S_{\frac32,M_S}\otimes |\psi^\tau_{1M_L}(\alpha)\rangle^O+\sum(1\ \frac12 M_L M_S;\frac12\ \frac12)\otimes|(\mathbf 3_{12},\mathbf 3_{34})_{\mathbf 3_{1234}},\mathbf 2_5\rangle^S_{\frac12,M_S}\otimes|\psi^\tau_{1M_L}(\alpha)\rangle^O\Big). \tag{18}$$

for the positive parity state. Then, the total trial wave function with the λ excitation mode can be expressed by

$$|\Psi^{\lambda}_{\frac12,\frac12}\rangle=a\ |\Psi^{\lambda(1)}_{\frac12,\frac12}\rangle+b\ |\Psi^{\lambda(2)}_{\frac12,\frac12}\rangle, \tag{19}$$

where a and b are mixing constants. Since there is no spin and flavor operators in the Hamiltonian (1), the eigenenergies of two λ modes are degenerated. Thus we take $a=b=1/\sqrt{2}$. The total trial wave function for the positive parity state can be written as

$$|\Phi(J,M_J,P,\alpha,\zeta)\rangle_{|J=\frac12,M_J=\frac12,P=-}= c\ |\Psi^{\lambda}_{\frac12,\frac12}\rangle+\sqrt{1-c^2}\ |\Psi^{\tau}_{\frac12,\frac12}\rangle, \tag{20}$$

where c stands for the variational parameter.

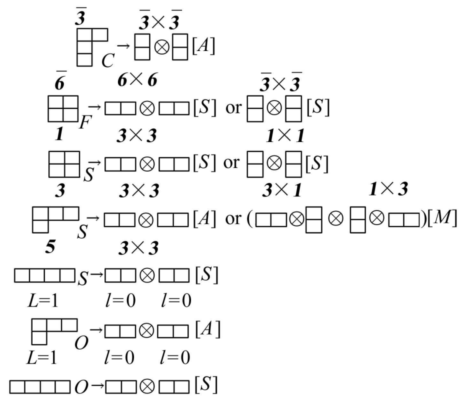

Fig. 3. The possible quark pair combinations for q^4 together with the symmetries under interchanging two q-q pairs.

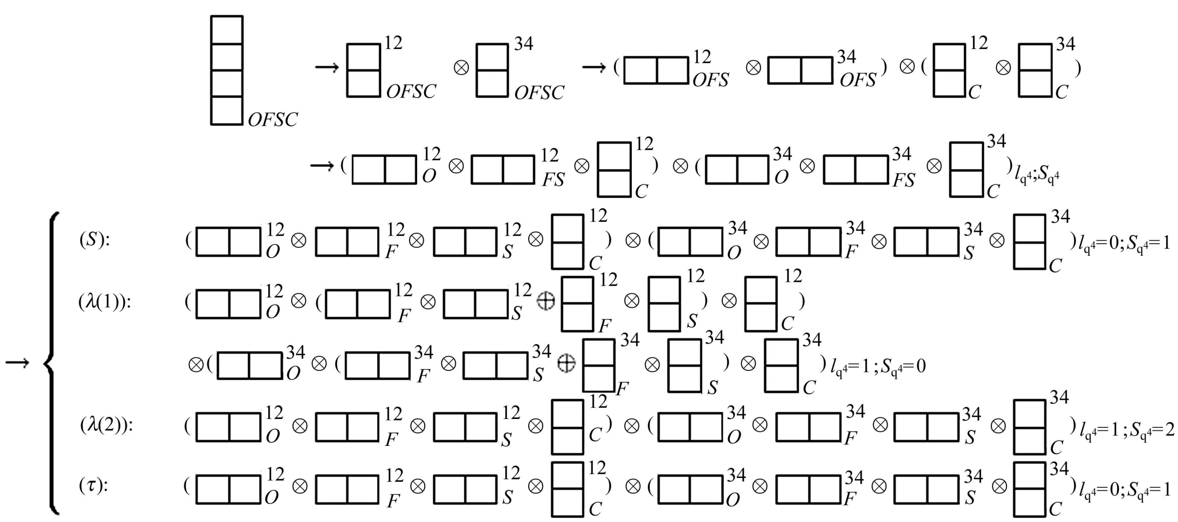

Fig. 4. The possible q^4 states coupled by quark pairs given in Fig. 3.

3 Result and discussion

Before numerical calculation, the model parameters should be pre-determined. According to the result from LQCD calculation, the quark-gluon strong coupling constant α_s and the quark masses $m_{u(d)}$ and $m_{\bar s}$ are chosen to be those used in the baryon spectrum calculation, namely $\alpha_s = 0.75$, $m_{u(d)} = 0.313$GeV and $m_s = 0.470$GeV, respectively[26], and $b^{5q}_Y = b^{3q}_Y = 0.90\text{GeV}^{-2}$ and $C^{5q}_Y = 5/3C^{3q}_Y$.

In the variational calculation, the trial wave function is chosen in a more flexible form

$$|\Psi(J,M_J,P,\zeta)\rangle = \sum_{i=1}^{5} a_i \Phi(J,M_J,P,\alpha_i,\zeta), \qquad (21)$$

where $\Phi(J,M_J,P,\alpha_i,\zeta)$ takes the forms in Eqs. (16) and (20) accordingly, and a_i's and α_i's are variational parameters. The mass of Θ^+ can be calculated by

$$M_{\Theta^+} = 4m_{\rm q} + m_{\rm s} + E_{5\rm q}. \qquad (22)$$

where $E_{5\rm q}$ is the variational energy for Θ^+.

To determine $C_{\rm Y}^{5\rm q}$, we should find out the value of $C_{\rm Y}^{3\rm q}$ first. We calculate the mass of nucleon

$$M_{\rm N} = 3m_{\rm q} + E_{3\rm q}, \qquad (23)$$

where $E_{3\rm q}$ is the variational energy for nucleon, with the Hamiltonian in the same form in Eq. (1), except the double Y-shape confining potential is replaced by a Y-shape confining potential, and the same $\alpha_{\rm s}$, $m_{\rm u(d)}$ and $m_{\rm s}$ values mentioned above. The resultant $C_{\rm Y}^{3\rm q}$ and $C_{\rm Y}^{5\rm q}$ values, the masses of Θ^+ in the negative parity and positive parity states and corresponding variational parameters are tabulated in Table 1.

From Table 1, one sees that the mass of Θ^+ in the negative parity state is about 100MeV lower than that in the positive parity state, but is still more than 300MeV higher than the experimental value of 1.540GeV. The mass of Θ^+ with positive parity is 2.082GeV, which is comparable with the LQCD prediction.

Table 1. The masses of Θ^+ in the negative parity and positive parity states and nucleon, and the corresponding variational parameters. The model parameters used are: $b_{\rm Y}^{3\rm q} = b_{\rm Y}^{5\rm q} = 0.9\text{GeV}^{-2}$, $\alpha_{\rm s} = 0.75$ and $x = 0.95$.

	$\rm N(J^P=\frac{1}{2}^+)$	$\Theta^+(J^P=\frac{1}{2}^-)$	$\Theta^+(J^P=\frac{1}{2}^+)$
α_1/fm^{-1}	0.753	0.748	0.753
α_2/fm^{-1}	0.945	0.944	1.242
α_3/fm^{-1}	1.544	1.510	1.509
α_4/fm^{-1}	1.589	1.550	1.526
α_5/fm^{-1}	1.948	1.947	2.145
a_1	−0.1039	−0.1158	−0.1224
a_2	−0.1424	−0.1363	−0.0412
a_3	0.5258	0.5263	0.2621
a_4	0.4649	0.4739	0.2995
a_5	−0.0549	−0.0554	−0.1547
$C_{\rm Y}$	−0.820	−1.367	−1.367
M/GeV	0.939	1.935	2.082

In summary, we extend the calculation of baryon with the Y-mode flux-tube confining potential to the Θ^+ case, and calculate the mass of Θ^+ in either positive parity or negative parity states. The potential used is in the form of the double Y-shape confining potential, originated from LQCD, supplemented by a Coulomb potential. The resultant mass of Θ^+ with negative parity is about 1.935GeV. This value is about more than 300MeV higher than the value reported by experiments. The calculated mass of Θ^+ with positive parity is about 2.082GeV. This value is consistent to the LQCD prediction. Nevertheless, our results do not support the reported the peak at 1.540GeV as a pentaquark state.

References

1 Nakano T et al(LEPS Collaboration). Phys. Rev. Lett., 2003, **91**: 012002

2 Barmin V V et al(DIANA Collaboration). Phys. Atom. Nucl., 2003, **66**: 1715

3 Stephanyan S et al(CLAS Collaboration). Phys. Rev. Lett., 2003, **91**: 252001

4 Barth J et al(SAPHIR Collaboration). Phys. Lett., 2003, **B572**: 127

5 Abt I et al(HERA-B Collaboration). Phys. Rev. Lett., 2004, **93**: 212003

6 Schael S et al(ALEPH Collaboration). Phys. Lett., 2004, **B599**: 1

7 BAI J X et al(BES Collaboration). Phys. Rev., 2004, **D70**: 012004

8 Smith E. 'Review on Jlab Results' in Hadron05(cbpf). 2005

9 Strottman D. Phys. Rev., 1979, **D20**: 748

10 Diakonov D et al. Z. Phys., 1997, **A359**: 305

11 Jaffe R, Wilczek F. Phys. Rev. Lett., 2003, **91**: 232003

12 Karliner M, Lipkin H J. hep-ph/0307243

13 ZHU S L. Phys. Rev. Lett., 2003, **91**: 232002

14 Sugiyama J, Doi T, Oka M. Phys. Lett., 2004, **B581**: 167

15 Stancu Fl. Phys. Lett., 2004, **B595**: 269

16 HUANG F. Phys. Lett., 2004, **B586**: 69

17 Alexandrou C, Koutsou G. Phys. Rev., 2005, **D71**: 014504

18 Takahashi T T et al. Phys. Rev. Lett., 2001, **86**: 18. hep-ph/0507187

19 Mathur N, Lee E X et al. Phys. Rev., **D70**: 074508

20 Holland K, Juge K J. hep-lat/0504007

21 Alexandrou C, de Forcrand P. Nucl. Phys. Proc. Suppl., 2003, **119**: 667

22 SONG X C et al. Mod. Phys. Lett., 2001, **86**: 18

23 ZHAO H M, ZENG Z Q et al. nul-th/0504053

24 Isgur N, Karl G. Phys. Rev., 1978, **D18**: 418; Isgur N,

Karl G. Phys. Rev., 1979, **D19**: 2653; Capstick S, Isgur N. Phys. Rev., 1986, **D34**: 2809; **D28**: 2807
25 Bali G S. Phys. Rep., 2001, **343**: 1
26 ZHAO H M et al. Commun. Theor. Phys., 2005, accepted
27 Okiharu F, Suganuma H, Takahashi T T. Phys. Rev. Lett., 2005, **94**: 192001
28 Capstick S, Isgur N. Phys. Rev., 1986, **D34**: 2809; **D28**: 2807
29 Dudek J J. hep-ph/0403235

具有双 Y-型结构的 Θ^+ 的研究*

赵洪明[1,2;1)]　曾卓全[1,2]　沈彭年[5,4,1]　丁亦兵[2]　李学潜[3,4]

1 (中国科学院高能物理研究所　北京　100049)
2 (中国科学院研究生院物理学部　北京　100049)
3 (南开大学物理学院　天津　300071)
4 (中国科学院理论物理所　北京　100080)
5 (兰州重离子加速器国家实验室原子核理论中心　兰州　730000)

摘要　采用由单胶子交换产生的库仑势加上流管模型给出的双 Y-型禁闭势之和的相互作用势, 分别计算了正、负宇称态的 Θ^+ 的质量, 给出负宇称态 Θ^+ 的质量为 1.935GeV, 正宇称态 Θ^+ 的质量为 2.082GeV. 其中较低的负宇称态仍比实验给出的 Θ^+(1540) 的共振峰高出近 400MeV.

关键词　五夸克态　流管模型　Y-型禁闭势

2005 – 11 – 21 收稿,　2005 – 12 – 20 收修改稿

*国家自然科学基金(10375090, 10475042, 10475089, 10435080), 中国科学院知识创新重大项目(KJCX2-SW-N02), 高等学校博士学科点专项基金和中国科学院高能物理研究所创新基金(U529)资助

1) E-mail: zhaohm@mail.ihep.ac.cn