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Thermal Property of Protoneutron Star Matter*

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Abstract We study the composition, the temperature and the equation of state of isoentropic protoneutron star matter in the mean field approximation of the relativistic σ-ω-ρ model. It is shown that, fixing the baryon density, the fraction of neutron at $S=2$ is smaller than that at $S=1$ and the fractions of proton, electron, and muon at $S=2$ are larger than those at $S=1$, respectively, especially in the region of low baryon density. Keeping baryon density invariant, the fractions of hyperons at $S=2$ are larger(smaller) than those at $S=1$ in the region of relative low(high) density of baryons. Also the temperature, the energy density and the pressure at $S=2$ are larger than those at $S=1$, respectively. In addition, we demonstrate that the finite entropy impose more influence on the fractions of particles as well as the temperature than on the equation of state of the protoneutron star matter. As a consequence, the contributions of antiparticles are very small under our consideration.

Key words protoneutron star, thermal dynamics, entropy

1 Introduction

After seconds of supernova explosion, a hot compact remnant which is the so-called protoneutron star(PNS)is left. The PNS evolutes to a cold neutron star in two stages. In the first stage which lasts for about 12s[1], the PNS is not transparent to the thermal neutrinos and must cool through the neutrino diffusion to the surface. And in the second stage which lasts perhaps one million years, neutrino emission dominates the cooling but the star is observable only through its thermal photonic emission.

The entropy in the interior of PNS is moderately high during the early evolution, and the entropy per baryon is about 1—2(in units of Boltzman constant), which corresponds to a temperature in the range T=(20—50)MeV. Many thermodynamic quantities of PNS matter may vary with the temperature, such as the pressure, the abundance of various baryons and the chemical potential. Although finite temperature effects on these quantities are practically negligible when considering the overall structure of evolved neutron stars, they are important in assessing the thermal history of these celestial bodies and properties of their interior matter[2, 3].

In view of the uncertainties in the actual temperature profiles within the hot interior of PNS, an isoentropic or an isothermal state was usually assumed[4, 5]. The first case is characterized by a constant entropy per baryon S=const. And the second case which corresponds to a vanishing heat flux in a static star is more complicated(for detail, see the Ref. [6]). In the current theory, the isothermal state within the hot interior will be reached on a time scale corresponding to thermal equilibrium which is much longer than the lifetime of a PNS, so that it is usually discarded in studying the thermal properties of PNS matter. In the present paper, we study the properties of isoentropic PNS matter including hyperons Λ, Σ, Ξ, Δ(PNHS), and also the matter of the PNS composed

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only of the nucleons, electrons and μ^-(PNNS) for comparison. The contributions of antiparticles are considered in the work. Because the internal constitution of the hot PNS depends on the nature of strong interaction, we employ the relativistic σ-ω-ρ model[7, 8], in which the strong interaction is mediated by exchanging σ, ω and ρ mesons.

In Sec. 2, we present the basic theory. The results of numerical calculation are given in Sec. 3, and the summary appears in Sec. 4.

2 The basic theory

The Lagrangian density in the relativistic σ-ω-ρ model is written as[9],

$$\begin{aligned} L = & \sum_B \bar{\psi}_{\mathrm{B}}(\mathrm{i}\gamma_\mu\partial^\mu + m_{\mathrm{B}} - g_{\sigma\mathrm{B}}\sigma - g_{\omega\mathrm{B}}\gamma_\mu\omega^\mu - \\ & \frac{1}{2}g_{\rho\mathrm{B}}\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu)\psi_{\mathrm{B}} + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \\ & \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - U(\sigma) - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \\ & \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu + \sum_l \bar{\psi}_{\mathrm{l}}(\mathrm{i}\gamma_\mu\partial^\mu - m_{\mathrm{l}})\psi_{\mathrm{l}}, \end{aligned} \tag{1}$$

where

$$U(\sigma) = \frac{1}{3!}c\sigma^3 + \frac{1}{4!}\mathrm{d}\sigma^4, \tag{2}$$

$$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu, \tag{3}$$

$$\rho_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu, \tag{4}$$

ψ_{B} and ψ_{l} are the field operators of baryon B(B=n, p, Λ, Σ, Ξ, Δ) and lepton l(l=e, μ), respectively, σ, ω^μ and $\boldsymbol{\rho}^\mu$ the field operators of σ, ω and ρ meson, respectively, $g_{\sigma\mathrm{B}}$, $g_{\omega\mathrm{B}}$ and $g_{\rho\mathrm{B}}$ the coupling constants between σ, ω, ρ meson and baryon B, respectively, and m_{B}, m_{l} and m_{i}(i = σ, ω, ρ) the mass of baryon, lepton and σ, ω, ρ meson, respectively. $\boldsymbol{\tau}$ is the isospin operator. In general, the coupling constants between different meson(σ, ω and ρ) and nucleon are equal. They are determined by either the saturated property or the symmetry energy of nuclear matter. The coupling constants between mesons and hyperons have to be assumed in accordance with a few experimental data[10] and the nucleon coupling constants due to our poor knowledge. As for leptons, we assume them to be free fermi gas.

According to the thermodynamical statistical physics[11], the thermodynamic potential is denoted by

$$\Omega = -\frac{1}{\beta}\ln Z_{\mathrm{G}} = -\frac{1}{\beta}\ln\mathrm{Tr}\exp[-\beta(\hat{H} - \mu\hat{N})], \tag{5}$$

where Z_{G} is the grand partition function, $\beta = \frac{1}{kT}$, k the Boltzman constant, T the temperature. Halmitonian $\hat{H}$ is given by,

$$\hat{H} = \int\mathrm{d}^3xT^{00} = \int\mathrm{d}^3x\left(\partial^0\phi\frac{\partial L}{\partial(\partial^0\phi)} - g^{00}L\right), \tag{6}$$

where $\hat{N}$ is the particle number operator. The particle number density n is obtained by

$$n = -\left(\frac{\partial\Omega}{\partial\mu}\right)_{TV}, \tag{7}$$

so the number densities of baryons and leptons, denoted by n_{B} and n_{l}, respectively, can be calculated by

$$\begin{aligned} n_{\mathrm{B}} = & \frac{\gamma_{\mathrm{B}}}{(2\pi)^3}\int\mathrm{d}^3k\left[\frac{1}{1+\exp\beta(\tilde{E}_{\mathrm{B}}^+ - \mu_{\mathrm{B}})} - \right. \\ & \left.\frac{1}{1+\exp\beta(\tilde{E}_{\mathrm{B}}^- + \mu_{\mathrm{B}})}\right], \end{aligned} \tag{8}$$

$$\tilde{E}_{\mathrm{B}}^\pm = \sqrt{\tilde{M}_{\mathrm{B}}^2 + k^2} \pm g_{\omega\mathrm{B}}\tilde{V}_\omega \pm \frac{1}{2}I_{3\mathrm{B}}g_{\rho\mathrm{B}}\tilde{V}_\rho, \tag{9}$$

$$n_{\mathrm{l}} = \frac{\gamma_{\mathrm{l}}}{(2\pi)^3}\int\mathrm{d}^3k\left[\frac{1}{1+\exp\beta(E_{\mathrm{l}} - \mu_{\mathrm{l}})} - \frac{1}{1+\exp\beta(E_{\mathrm{l}} + \mu_{\mathrm{l}})}\right], \tag{10}$$

$$E_{\mathrm{l}} = \sqrt{m_{\mathrm{l}}^2 + k^2}, \tag{11}$$

with the energy density ε and pressure p being

$$\begin{aligned} \varepsilon = & \sum_{\mathrm{B}}\mu_{\mathrm{B}}n_{\mathrm{B}} - \frac{\partial\ln Z_{\mathrm{G}}}{\partial\beta} = \\ & \sum_{\mathrm{B}}\frac{\gamma_{\mathrm{B}}}{(2\pi)^3}\int\mathrm{d}^3k\left[\frac{E_{\mathrm{B}}}{1+\exp\beta(\tilde{E}_{\mathrm{B}}^+ - \mu_{\mathrm{B}})} + \right. \\ & \left.\frac{E_{\mathrm{B}}}{1+\exp\beta(\tilde{E}_{\mathrm{B}}^- + \mu_{\mathrm{B}})}\right] + \frac{1}{2}\tilde{m}_\sigma^2\tilde{v}^2 - \frac{1}{2}m_\omega^2\tilde{V}_\omega^2 + \\ & \tilde{V}_\omega\sum g_{\omega\mathrm{B}}n_{\mathrm{B}} - \frac{1}{2}m_\rho^2\tilde{V}_\rho^2 + \\ & \tilde{V}_\rho\sum I_{3\mathrm{B}}g_{\rho\mathrm{B}}n_{\mathrm{B}} + U(\sigma), \end{aligned} \tag{12}$$

$$\begin{aligned} p = & -\left(\frac{\partial\Omega}{\partial V}\right)_{T\mu} = \\ & \sum_{\mathrm{B}}\frac{1}{\beta}\frac{\gamma_{\mathrm{B}}}{(2\pi)^3}\int\mathrm{d}^3k\left[\ln(1+\exp[-\beta(\tilde{E}_{\mathrm{B}}^+ - \mu_{\mathrm{B}})]) + \right. \\ & \left.\ln(1+\exp[-\beta(\tilde{E}_{\mathrm{B}}^- + \mu_{\mathrm{B}})])\right] - \frac{1}{2}\tilde{m}_\sigma^2\tilde{v}^2 + \\ & \frac{1}{2}m_\omega^2\tilde{V}_\omega^2 + \frac{1}{2}m_\rho^2\tilde{V}_\rho^2 - U(\sigma). \end{aligned} \tag{13}$$

Then, the entropy density s is given by

$$Ts = p + \epsilon - \sum_{\mathrm{B}} \mu_{\mathrm{B}} n_{\mathrm{B}}. \qquad (14)$$

For convenience, we further define S as the entropy per baryon, namely,

$$S = \frac{s}{\rho}, \quad \rho = \sum_{\mathrm{B}} n_{\mathrm{B}}. \qquad (15)$$

Due to the chemical equilibrium and the neutrality in PNS, the chemical potential and the fraction of each particle are constrained by following relations:

$$\mu_{\mathrm{B}} = b\mu_{\mathrm{n}} - q_{\mathrm{B}}\mu_{\mathrm{e}}, \qquad (16)$$

$$\sum_{\mathrm{B}} q_{\mathrm{B}} n_{\mathrm{B}} + \sum_{\mathrm{l}} q_{\mathrm{l}} n_{\mathrm{l}} = 0, \qquad (17)$$

where μ_{B}, μ_{n} and μ_{e} are the chemical potentials of baryon B, neutron and electron, respectively, b and q_{B} are the baryonic number and the charge of the baryon B, respectively, and q_{l} is the charge of the lepton l.

3 Numerical results

In the calculation, we take $g_{\mathrm{s}} = 8.43$, $g_{\omega} = 8.70$, $g_{\rho} = 8.55$, $c = 3912.31\mathrm{MeV}$, $d = 402.73$, and assume $\frac{g_{\mathrm{Hs}}}{g_{\mathrm{s}}} = \frac{g_{\mathrm{H}\rho}}{g_{\rho}} = 0.6$ and $\frac{g_{\mathrm{H}\omega}}{g_{\omega}} = 0.66(\mathrm{H}=\Lambda, \Sigma, \Xi, \Delta)$[12]. Fig. 1 and Fig. 2 depict the relation between the fractions of various particles and the baryon density at $S=1$ and $S=2$ in PNHS. At a specific baryon density, the fraction of neutron at $S=2$ is smaller than that at $S=1$ and the fractions of proton, electron, and muon at $S=2$ are larger than those at $S=1$, respectively, especially in the region of low baryon density. Keeping the baryon density invariant, the fractions of hyperons at $S = 2$ are larger(smaller) than those at $S=1$ in the region of relative low(high) density of baryons, respectively. The fractions of Δ^{++}, Δ^{+}, Δ^{0} are so small that we do not plot them in Fig. 2. We plot the energy densities of the isoentropic PNNS and PNHS matter with respect to the baryon density in Fig. 3. The energy density of the isoentropic PNNS(or PNHS) matter at $S=2$ is a little larger than that at $S=1$ when the baryon density is fixed, especially in the high baryon density region. And at the same baryon density and S, the PNNS matter has a larger energy density than the PNHS matter does. In Fig. 4, we present the relation between the pressure and the baryon density. Its character is similar to that of the energy density shown in Fig. 3. We can see from this figure that the curve for the PNNS matter is much stiffer than that for

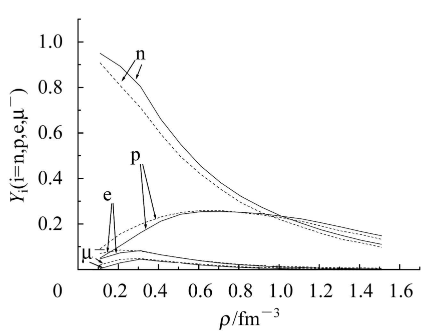

Fig. 1. The fractions of neutrons, protons, electrons and μ^- with respect to the baryon density in PNHS. The solid and dashed curves represent the cases $S = 1$ and $S = 2$, respectively.

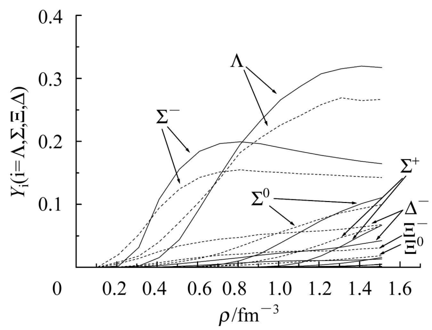

Fig. 2. The fractions of Λ, Σ, Δ, Ξ with respect to baryon density, the solid and dashed curves represent the cases $S = 1$ and $S = 2$, respectively.

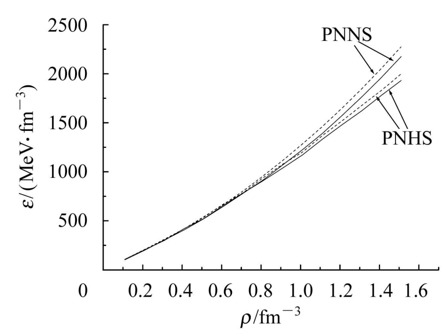

Fig. 3. The energy densities of isoentropic PNNS and PNHS matter with respect to baryon density, the solid and dashed curves represent the cases $S = 1$ and $S = 2$, respectively.

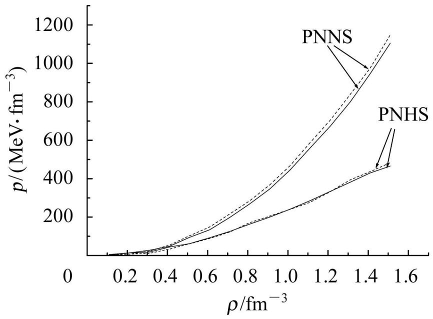

Fig. 4. The pressures of isoentropic PNNS and PNHS matter with respect to baryon density, the solid and dashed curves represent the cases $S=1$ and $S=2$, respectively.

the PNHS matter. In Fig. 5, the temperature of isoentropic PNNS(or PNHS) matter at $S=2$ is much higher than that at $S=1$ when the baryon density is fixed. The temperature has a maximum value at the center of the star and decreases with decreasing density. Besides, the PNNS matter is hotter than the PNHS matter at a fixed S.

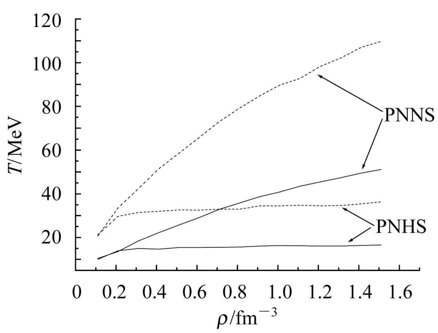

Fig. 5. The temperature in the interior of isoentropic PNNS and PNHS, the solid and dashed curves represent the cases $S=1$ and $S=2$, respectively.

4 Summary

We study the thermal properties of PNS matter at a finite entropy. At a specific baryon density, the fraction of neutron at $S=2$ is smaller than that at $S=1$ and the fractions of proton, electron, and muon at $S=2$ are larger than those at $S=1$, respectively, especially in the region of low baryon density. Also the fractions of hyperons at $S=2$ are larger(smaller) than those at $S=1$ in the region of relative low(high) density of baryons, respectively. It follows that the temperature of isoentropic PNNS(or PNHS) matter at $S=2$ is much higher than that at $S=1$. And the temperature has a maximum value at the center of the star and decreases as decreasing density. In addition, the PNNS matter is hotter than the PNHS matter when their entropy per baryon are equal. Finally, the energy density or pressure of PNNS(or PNHS)at $S=2$ is slightly larger than that at $S=1$ when the baryon density is fixed. And the PNNS matter has larger energy density and pressure than the PNHS matter does when their entropy per baryon are the same. We can see from above analysis that the finite entropy imposes more influence on the particle abundance and temperature than on the energy density and pressure(namely the equation of state). The contributions of antiparticles are also studied. The result shows that they are very small. In the calculation, the coupling constants between mesons and baryons are assumed to be constants. They can be varied according with baryon density[13]. And we will study the influence of the density dependency of coupling constants on the thermal properties of protoneutron star matter in our next work.

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质子中子星物质的热力学性质*

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摘要　在相对论σ-ω-ρ模型的平均场近似下, 研究了质子中子星物质在均熵状态下的组成、温度和物态方程. 如给定每一个重子的熵，一些热力学量的值将随重子密度的增加而增加, 当考虑超子时, 这些值会减小. 给定重子密度, 中子在$S=2$时的组分比$S=1$时的小, 而质子、电子、μ子在$S=2$时的组分比$S=1$时的大, 特别是在低密度区域. S是每个重子的熵. 保持重子密度不变, 在低密度区域, 超子在$S=2$时的组分比$S=1$时的大, 在高密度区域则相反. 同样, 在同一重子密度处, $S=2$时的温度、能量密度及压强分别比$S=1$时的大. 另外, 有限熵对粒子组分和温度的影响比对质子中子星物质的物态方程的影响大. 还研究了反粒子的贡献, 他们确实很小.

关键词　质子中子星　热力学　熵

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