

Systematic Description of the Superdeformed Bands of the Odd- A Nuclei in the $A \approx 190$ Mass Region^{*}

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Abstract Within the supersymmetry scheme including many-body interactions and a perturbation possessing the $SO(5)$ (or $SU(5)$) symmetry on the rotational symmetry, the superdeformed (SD) bands of the odd- A nuclei in $A \approx 190$ mass region are investigated systematically. Quantitatively good results of the γ -ray energy spectra and the dynamical moments of inertia are obtained. It shows that the supersymmetry approach is powerful in describing not only the generic rotational property, but also the $\Delta I = 4$ bifurcation and the identical bands in SD states simultaneously.

Key words superdeformed bands, energy spectrum, dynamical moment of inertia, supersymmetry model, pairing and antipairing effects

1 Introduction

Since the observation of the superdeformed (SD) bands in ^{191}Hg ^[1], many SD bands have been observed in $A \approx 190$ mass region. Just for the odd- A nuclei in this region, more than 40 SD bands have been established (for a compilation see Ref. [2], for some recent data see Refs. [3,4]). And it has been observed that, for the SD bands in $A \approx 190$ region, there are some fascinating phenomena, such as identical bands^[5], $\Delta I = 4$ bifurcation (or $\Delta I = 2$ staggering)^[6] even though it is not so distinct as that in $^{149}\text{Gd}(1)$ ^[7]. Then different models have been applied to describe the bands and study the underlying physics. In the self-consistent cranked approaches (see, for example, Refs. [8–12]), the single particle configurations and the dynamical moment of inertia of the SD bands in $A \approx 190$ mass region have been calculated. In phenomenological approaches^[13,14], some of the SD bands have also been discussed. In the approach based on the interacting boson model (IBM)^[15], the SD bands of some even-

even nuclei in mass regions $A \approx 190$ and 150 have been studied^[16–19]. With the approach being extended to the supersymmetry^[20–22], the SD bands of the odd- A nuclei in $A \approx 150$ region have been studied systematically^[23]. Meanwhile, the most typical $\Delta I = 2$ staggering SD band $^{149}\text{Gd}(1)$ ^[7], and the identical SD bands $^{152}\text{Dy}(1)$, $^{151}\text{Tb}(2)$, $^{151}\text{Dy}(4)$, $^{153}\text{Dy}(2)$ and $^{152}\text{Dy}(3)$ ^[5], even the identical SD bands with $\Delta I = 4$ bifurcation $\{^{149}\text{Gd}(1), ^{148}\text{Gd}(16), ^{148}\text{Eu}(1)\}$ ^[24], and those in $A \approx 190$ mass region have been reproduced well^[23,25–27]. Furthermore, besides the SD bands in ^{191}Hg and ^{193}Hg , were discussed in Ref. [28], the identical bands in $A \approx 190$ region $\{^{191}\text{Hg}(2), ^{193}\text{Hg}(5)\}$, $\{^{191}\text{Hg}(3), ^{193}\text{Hg}(3)\}$, $\{^{193}\text{Tl}(3), ^{193}\text{Tl}(5)\}$, and $\{^{193}\text{Tl}(1), ^{193}\text{Tl}(2)\}$, and the SD bands with $\Delta I = 4$ bifurcation have also been reproduced^[29,30]. Meanwhile, the SD bands of the odd-odd nuclei in $A \approx 190$, 150 and 130 mass regions have been discussed within the algebraic approach^[31–33], too. However, systematic study on the odd- A SD bands in $A = 190$ region is still lacking. With the approach proposed in Ref. [26], we

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will investigate the SD bands of the odd- A nuclei in $A \approx 190$ mass region systematically in this paper.

The paper is organized as follows. In the next section, the formalism of the approach will be described briefly. In Section 3 the calculated results and some discussions are presented. Section 4 summarizes the discussion and gives some remarks.

2 Formalism

Experimental data show that superdeformed bands exhibit quite good rotational characteristics. The dynamical symmetry group chains to label the states should be the ones ending with $SO(3)$. Basing on the fact that supersymmetry scheme can describe well the normally deformed states in even-even, odd-odd, and odd- A nuclei in a unified way^[20], we assume that SD states can be classified with supersymmetric group chain

$$\begin{aligned} U(m, n) \supset U_B(m) \otimes U_F(n) \supset \cdots \supset \\ [N] \quad [N_B]_m \quad [N_F]_n \\ SO_{B+F}(3) \otimes SU_F(n') \supset Spin(3), \\ L \quad S \quad I \end{aligned}$$

where m is determined by the constituent of the bosons. n is determined by the single particle configuration of the fermion (s), and n' by the total pseudospin. $N = N_B + N_F$ is the total number of particles with N_B , N_F the boson, fermion numbers, respectively. In the framework of supersymmetry^[20], even-even nucleus and its odd- A and odd-odd neighbors are the multiplets of the irreducible representation (irrep) $[N]$ of the supergroup $U(m, n)$.

Since the n and N_F can be fixed with the assignment of the single particle configuration, what we should determine is the m and N_B . Since the bosons to describe positive parity SD states should be s , d and g bosons^[16,17,34], and p , f bosons are essential to depict negative parity states^[35], the constituent of the bosons should be s , d , g , p and f bosons, i.e. $m = 25$. Symmetry analysis^[36,37] showed that the subset of s , d , g bosons holds $SU_{sdg}(5)$, $SU_{sdg}(3)$, and other symmetry limits. Examining the geometric shape, hexadecupole deformation parameter β_4 and energy spectrum indicates that the $SU_{sdg}(5)$ symmetry can describe well-deformed nuclear states as well as the $SU_{sdg}(3)$ symmetry^[38,39]. Other studies manifest that the $SU_{sdg}(5)$ symmetry can generate a geometric shape with C_4 -symmetry^[34] and energy spectra exhibiting ΔI

$= 2$ staggering^[39]. Meanwhile, the potential energy surface of the $SU_{sdg}(5)$ symmetry has two minima displayed with different energies^[38]. Then, the $SU_{sdg}(5)$ symmetry can be taken to describe positive parity SD states. On the other hand, it has been known that the subset of p , f bosons also has $SU(5)$ symmetry (denoted as $SU_{pf}(5)$)^[35]. We have therefore the group chain for the boson part

$$\begin{aligned} U_{sdgpf}(25) \supset U_{sdg}(15) \otimes U_{pf}(10) \supset SU_{sdg}(5) \otimes \\ [N_B] \quad [N_{sdg}] \quad [N_{pf}] \quad IR(5_{pp}) \\ SU_{pf}(5) \supset SU(5) \supset SO(5) \supset SO(3), \\ IR(5_{NP}) \quad [n_1, n_2, n_3, n_4] \quad (\tau_1, \tau_2) \quad L_B \end{aligned}$$

with which SD states can be classified. Here the $IR(5_{pp})$, $IR(5_{NP})$ refer to the irreps $[n_1, n_2, n_3, n_4]_{sdg}$, $[n_1, n_2, n_3, n_4]_{pf}$ of the groups $SU_{sdg}(5)$, $SU_{pf}(5)$, respectively. For a nucleus with definite N_B and N_F , all the possible irreps $[N_{sdg}]$, $[N_{pf}]$, $[n_1, n_2, n_3, n_4]_{sdg}$, $[n_1, n_2, n_3, n_4]_{pf}$, $[n_1, n_2, n_3, n_4]$, (τ_1, τ_2) and L_B can be fixed by the branching rules of the irrep reductions of $U_{sdg}(15) \supset SU_{sdg}(5)$ ^[36,37], $U_{pf}(10) \supset SU_{pf}(10) \supset SU_{pf}(5)$ ^[35], $SU(5) \otimes SU(5) \supset SU(5)$ (See, for example, Ref. [40]), $SU(5) \supset SO(5)$, $SO(5) \supset SO(3)$ ^[36,37] and the trivial one $N_{sdg} = N_B - N_{pf}$ with $N_{pf} \in [0, N_B]$. In practice, it is usual to take $N_{pf} = 0$ for positive parity states and $N_{pf} = 1$ for negative parity states since only "octupole vibration" has been observed in SD states (see for example Ref. [41]). The contribution of the p and f bosons can thus be included on the level of angular momentum coupling. Taking advantage of the spectrum generating principle, we know that every irrep of the $SU(5)$ group contributes a constant to the energies of all the states within this irrep. Consequently, the irreps of the $SU(5)$ groups contribute nothing to the relative excitation energies of the states in a band. The contribution of the bosons to the γ -ray energies (E_γ 's) in a SD band is thus in fact the one with the $SO(5)$ symmetry. We get then

$$\begin{aligned} E = E_0(N_B, N_F) + B[\tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1)] + \\ C_L L(L + 1) + C_S S(S + 1) + C_I I(I + 1), \quad (1) \end{aligned}$$

where (τ_1, τ_2) is the irrep of the $SO(5)$ group. The $L = L_B + L_F$ is the total angular momentum of the effective core (L_F is the pseudo-orbital angular momentum of the fermion). I is the total spin of the nucleus ($I = L + S$ with S being the total pseudospin). For the fully stretched pseudospin configuration (i.e., $I = L + S$), with an effective aligned angular momentum i being introduced as $i =$

$$\frac{C_L S}{C_L + C_I}, \text{ Eq. (1) can be rewritten as}$$

$$E = E_0(N_B, N_F) + B[\tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1)] + CI'(I' + 1), \quad (2)$$

where $I' = I - i$, $C = C_L + C_I$, and $E_0(N_B, N_F)$ is a little different from that in Eq. (1) since a constant is involved.

Obviously, by adjusting the ratio $\frac{C_L}{C_I}$, one can get any value of the alignment i . In particular, taking $C_L = 0$, one has $i = 0$, i.e., the strong coupling limit. If $C_I = 0$, one gets $i = S$, i.e., the pseudospin decoupling limit. Even though the quantum numbers can be fixed with the branching rules of the irrep reduction, it is difficult to determine the (τ_1, τ_2) of the $SO(5)$ due to the complexity of the microscopic configurations of SD states. Given that SD bands are generated by the non-totally symmetric irrep $[2N_B - 2, 2, 0, 0]$ of the $SU(5)$ group, the (τ_1, τ_2) in practical calculation can be simply given as^[23, 25, 26, 29]

$$(\tau_1, \tau_2) = \begin{cases} \left(\left[\frac{L}{2} \right], 0 \right), & \text{if } L = 4k, 4k + 1 (k = 0, 1, \dots) \\ \left(\left[\frac{L}{2} \right] - 1, 2 \right), & \text{if } L = 4k + 2, 4k + 3 (k = 0, 1, \dots) \end{cases}, \quad (3)$$

where $[a]$ denotes the integer part of a .

It is evident that, the variance of the dynamical moment of inertia ($\mathcal{J}^{(2)}$) vs rotational frequency ($\hbar\omega$) can not be reproduced by Eq. (2) if the parameter C is taken as a constant. In light of the variable moment of inertia model^[18, 19, 23, 25, 26, 29, 42, 43], we can write the C as a function of the angular momentum I' . We get thus

$$E = E_0(N_B, N_F) + B[\tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1)] + \frac{C_0}{1 + f_1 I'(I' + 1) + f_2 I'^2(I' + 1)^2} I'(I' + 1), \quad (4)$$

where C_0, f_1 and f_2 are parameters.

3 Calculation and discussion

With Eqs. (4) and (3) under the commonly used assumption^[5, 21, 22] that $E_0(N_B, N_F)$ is a constant for a nucleus, we calculate the γ -ray energies and the dynamical moment of inertia of the 43 odd- A SD bands $^{191}\text{Au}(1-3)$, ^{189}Hg , $^{191}\text{Hg}(1-4)$, $^{193}\text{Hg}(1-5)$, $^{195}\text{Hg}(1-4)$, $^{189}\text{Tl}(1-2)$, $^{191}\text{Tl}(1-2)$, $^{193}\text{Tl}(1-5)$, $^{195}\text{Tl}(1-2)$, $^{193}\text{Pb}(1-6)$, $^{195}\text{Pb}(1-4)$, $^{197}\text{Pb}(1-2)$, ^{195}Bi and $^{197}\text{Bi}(1-2)$. After a nonlinear least square fitting to the experimental γ -ray energies in the strong coupling limit (i.e., $i = 0, I' = I$), we get the E'_γ 's. The best fitted parameters are listed in Table 1. The obtained E'_γ 's of almost all the levels in the SD bands fit experimental data within 2keV if their experimentally observed $\mathcal{J}^{(2)} - \hbar\omega$ plots do not involve humps (i.e., in the microscopic point of view, the single particle configurations for the bands to build upon do not contain any band crossing). The errors of the calculations

$$\sigma = \sqrt{\frac{1}{N} \sum_i \left(1 - \frac{E'_\gamma(\text{cal.})(i)}{E_\gamma(\text{exp.})(i)} \right)^2}, \quad (5)$$

where N is the number of γ -rays in a band, are also listed in Table 1.

Table 1. The fixed parameters in the calculation (B, C_0, f_1 and f_2), the $E_\gamma(I_0 \rightarrow I_0 - 2)$ and the error σ of the calculation.

Band	$B^*/10^{-3}$	C_0^*	$f_1/10^{-5}$	$f_2/10^{-9}$	I_0	$E_\gamma(I_0 \rightarrow I_0 - 2)^*$	$\sigma/10^{-3}$
$^{191}\text{Au}(1)$	-4.209	5.306	4.404	-3.346	9.5	186.8	4.828
$^{191}\text{Au}(2)$	2.467	5.408	4.496	-1.349	19.5	397.8	0.5812
$^{191}\text{Au}(3)$	6.393	5.508	6.200	-5.750	18.5	382.7	0.8395
^{189}Hg	2.118	5.462	2.250	4.900	17.5	366.2	0.6267
$^{191}\text{Hg}(1)$	-0.6686	5.263	2.866	-0.3345	15.5	310.9	0.9263
$^{191}\text{Hg}(2)$	0.9639	5.309	4.026	-2.724	12.5	252.4	0.4614
$^{191}\text{Hg}(3)$	0.8815	5.321	4.770	-3.724	13.5	272.0	0.6537
$^{191}\text{Hg}(4)$	6.876	5.874	4.715	-1.478	12.5	280.9	2.632
$^{193}\text{Hg}(1)$	-4.272	5.426	8.519	-12.33	9.5	192.3	4.073
$^{193}\text{Hg}(2)$	-1.128	5.364	4.995	-3.824	12.5	254.0	0.3135

(Table 1 continued)

Band	$B^*/10^{-3}$	G_0^*	$f_1/10^{-5}$	$f_2/10^{-9}$	I_0	$E_\gamma(I_0 \rightarrow I_0 - 2)^*$	$\sigma/10^{-3}$
$^{193}\text{Hg}(3)$	-4.544	5.369	5.106	-4.132	11.5	233.5	1.094
$^{193}\text{Hg}(5)$	-9.723	4.451	-5.003	15.44	16.5	291.0	4.575
$^{193}\text{Hg}(6)$	-2.334	5.463	2.818	-1.165	11.5	240.5	1.919
$^{195}\text{Hg}(1)$	-2.550	5.377	6.029	5.322	16.5	333.9	0.8506
$^{195}\text{Hg}(2)$	-3.126	5.381	6.037	5.182	13.5	273.9	0.4943
$^{195}\text{Hg}(3)$	8.397	5.046	-0.7697	8.873	14.5	284.5	2.535
$^{195}\text{Hg}(4)$	1.044	5.122	2.992	-1.649	17.5	341.9	0.7346
$^{189}\text{Tl}(1)$	-1.789	5.529	3.823	0.6183	15.5	326.3	0.4812
$^{189}\text{Tl}(2)$	9.740	5.505	4.027	0.8990	14.5	304.5	1.593
$^{191}\text{Tl}(1)$	0.6083	5.390	3.612	1.161	17.5	358.9	0.6705
$^{191}\text{Tl}(2)$	-0.1572	5.822	8.895	12.49	19.5	417.2	0.8429
$^{193}\text{Tl}(1)$	-1.453	5.230	3.923	-4.182	11.5	227.2	1.176
$^{193}\text{Tl}(2)$	-1.354	5.226	4.204	-3.063	10.5	206.6	1.173
$^{193}\text{Tl}(3)$	-0.6049	5.287	2.151	-5.996	9.5	187.9	3.690
$^{193}\text{Tl}(4)$	5.008	5.787	10.33	-23.83	11.5	250.8	2.997
$^{193}\text{Tl}(5)$	2.427	5.335	4.585	-4.886	13.5	271.5	2.019
$^{195}\text{Tl}(1)$	3.363	5.251	3.305	-0.5114	7.5	146.2	1.162
$^{195}\text{Tl}(2)$	-7.792	5.261	0.4113	-0.9868	8.5	167.5	1.348
$^{193}\text{Pb}(1)$	2.007	5.371	4.033	-8.663	13.5	276.9	1.484
$^{193}\text{Pb}(2)$	8.386	5.271	-1.434	11.92	9.5	190.5	1.658
$^{193}\text{Pb}(3)$	-2.466	5.277	3.387	-2.967	12.5	250.6	0.5466
$^{193}\text{Pb}(4)$	1.234	5.349	5.473	-6.459	13.5	273.0	0.4525
$^{193}\text{Pb}(5)$	3.155	5.389	5.759	-8.941	10.5	213.2	0.6071
$^{193}\text{Pb}(6)$	4.273	5.405	6.066	-9.583	11.5	233.0	1.918
$^{195}\text{Pb}(1)$	-0.1677	5.022	2.106	-5.038	9.5	182.13	0.5402
$^{195}\text{Pb}(2)$	-4.557	5.067	-1.237	7.244	8.5	162.58	0.6388
$^{195}\text{Pb}(3)$	9.247	5.442	6.719	-12.84	9.5	198.19	5.902
$^{195}\text{Pb}(4)$	4.946	5.432	5.743	-6.584	10.5	213.58	2.455
$^{197}\text{Pb}(1)$	-0.7075	5.124	1.769	-1.687	9.5	184.4	0.8729
$^{197}\text{Pb}(2)$	2.075	5.122	-0.3773	3.583	10.5	205.5	0.6876
^{195}Bi	20.01	5.546	8.385	-21.66	12.5	261.5	1.496
$^{197}\text{Bi}(1)$	-9.306	5.214	0.5382	3.835	8.5	166.2	1.052
$^{197}\text{Bi}(2)$	1.083	5.209	1.394	8.807	9.5	186.7	1.631

* in keV.

With $\mathcal{J}^{(2)} = 4\hbar^2 / [E_\gamma(I+2) - E_\gamma(I)]$, we get the dynamical moments of inertia of the bands. The obtained results and the comparison with experimental data are shown in Figs. 1—4 for the SD bands in Hg, Tl, Pb isotopes and in ^{191}Au , ^{195}Bi and ^{197}Bi , respectively. The figures show that the dynamical moments of inertia of these SD bands have also been reproduced well.

It is worth to mention that the calculated results have reproduced not only the general trend but also the turnover of $\mathcal{J}^{(2)}$ vs $\hbar\omega$. Extending the suggestion given in Ref. [43] in which only f_1 is taken into account, one can know that the anti-pairing (or pairing favorite) effect is strengthened when the parameters are taken as $f_1 > 0, f_2 > 0$, (or $f_1 < 0, f_2 < 0$).

However, when they are taken as $f_1 > 0, f_2 < 0$, (or $f_1 < 0, f_2 > 0$) both the anti-pairing and pairing favorite effects are included. To show the necessity for us to extend the approach proposed in Ref. [43], we have also evaluated the E_γ 's of these 43 SD bands in that approach (i. e., taking $f_2 \equiv 0$ in Eq.(4)). For the bands without turnover in the $\mathcal{J}^{(2)} - \hbar\omega$ plot, the obtained results in the two approaches are quite close to each other. Nevertheless, the calculated results for the bands with turnover, for instance the bands $^{193}\text{Hg}(1)$, $^{195}\text{Hg}(1)$, $^{195}\text{Hg}(2)$ and $^{193}\text{Tl}(4)$ are quite different. Meanwhile we have also evaluated the E_γ 's with the scheme of Harris expansion^[44] or $I(I+1)$ expansion^[45], and the ab formula^[46,13]. For comparison we re-display the $\mathcal{J}^{(2)}$'s of the bands

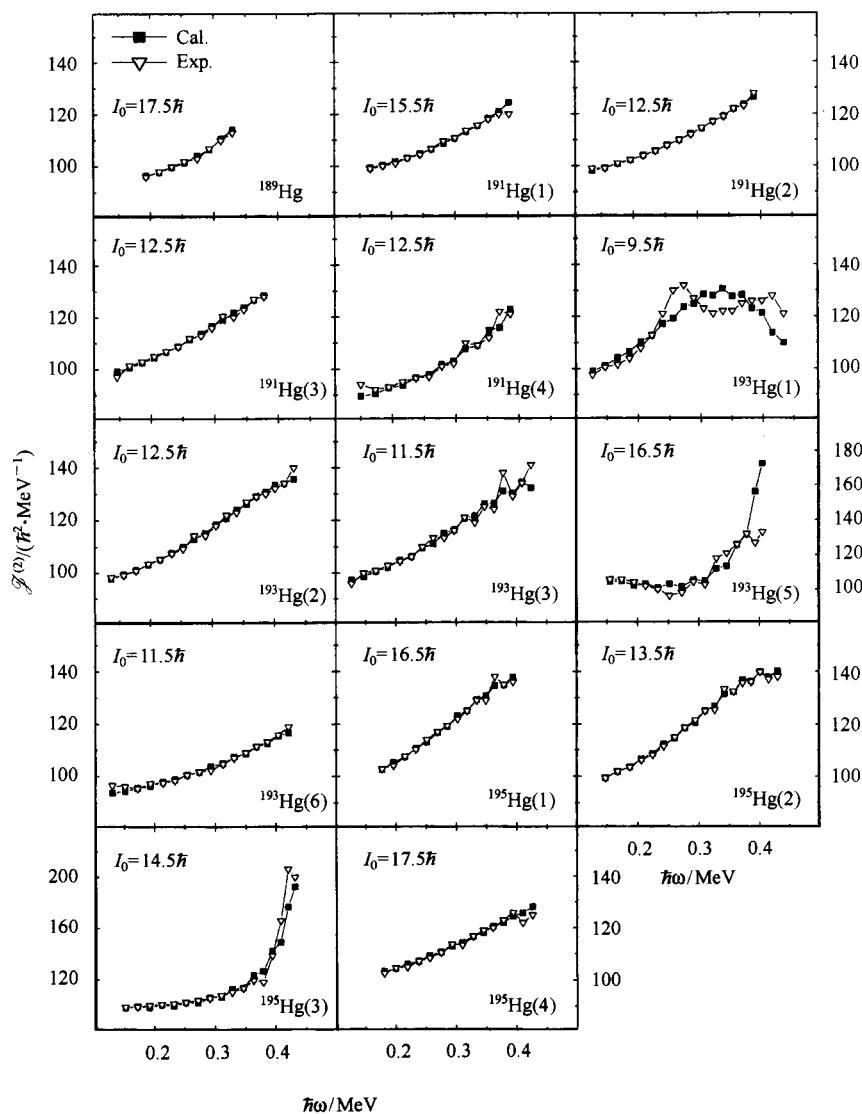


Fig.1. The calculated result of the dynamical moments of inertia as a function of the rotational frequency of the SD bands in the odd- A Hg isotopes and the comparison with experiment. The experimental data are taken from Ref. [2].

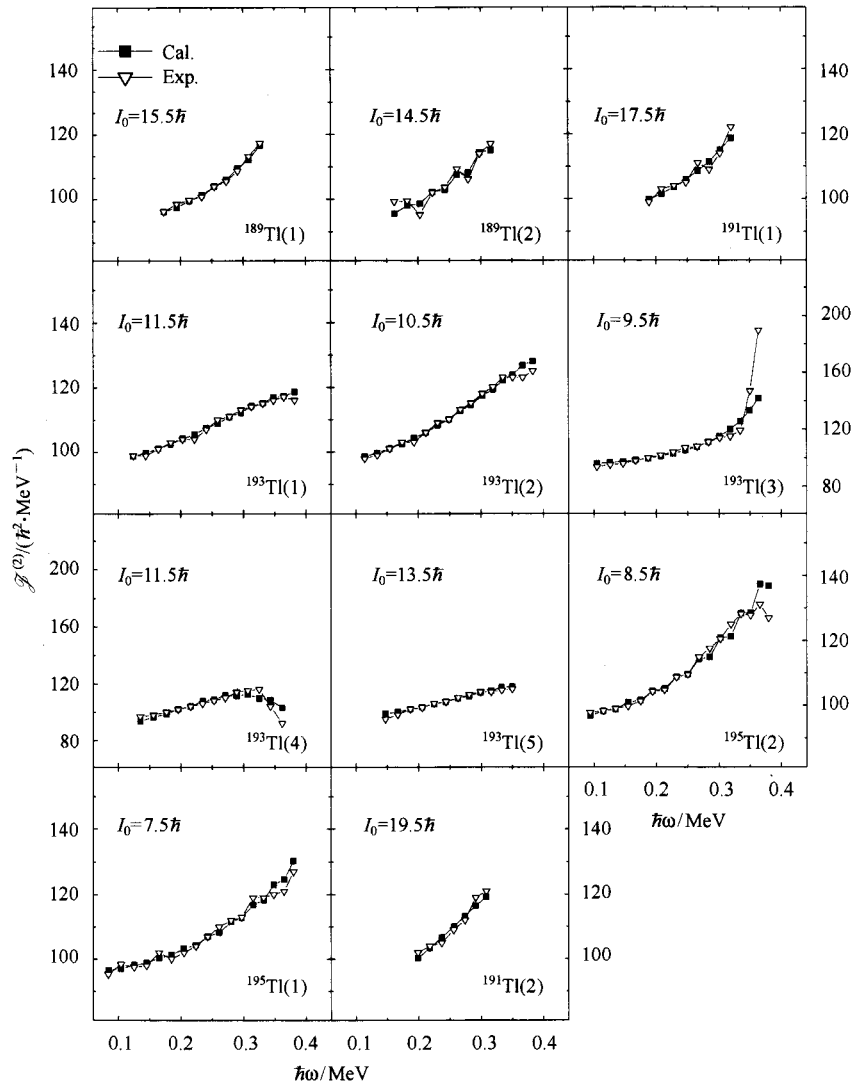


Fig. 2. The same as Fig. 1 but for the SD bands in Tl isotopes.

The experimental data are taken from Refs. [2, 3] and [4].

$^{193}\text{Hg}(1)$, $^{195}\text{Hg}(1)$, $^{195}\text{Hg}(2)$, and $^{193}\text{Tl}(4)$ obtained in the present approach and those gained with the other schemes in Fig. 5. It is apparent that the approach proposed in Ref. [43] and the ab formula produce only a monotonic $\mathcal{E}^{(2)}$'s with the $\hbar\omega$. The three parameter $I(I+1)$ expansion (or Harris expansion) and the present approach generate the turnover of the $\mathcal{E}^{(2)}$'s. Moreover the present approach gives globally little better results than the three parameter $I(I+1)$ expansion (the total calculation error of the 43 bands in the present approach is 67.2871×10^{-3} and that in the three parameter $I(I+1)$ expansion is 68.5988×10^{-3}).

On the other hand, examining the experimentally observed γ -ray energies, we know that there also exists $\Delta I = 4$

bifurcation in the SD bands $^{191}\text{Au}(3)$, $^{195}\text{Hg}(2)$, $^{193}\text{Tl}(5)$ and $^{197}\text{Bi}(1)$ [30], even though the staggering sequence is not as long as that in $^{149}\text{Gd}(1)$ [7] and $^{194}\text{Hg}(1)$ [6], and the amplitude is not so large as that in $^{149}\text{Gd}(1)$. For the completeness to show the efficiency of the present approach and the others, we reexamine the energy difference ΔE_γ between two consecutive γ -ray transitions in the four bands obtained in the above mentioned approaches. In the examination, we take Cederwall's notation [6] to manifest the difference ΔE_γ , where a smooth reference expressed in the fourth-order derivative is subtracted. The obtained results are displayed in Fig. 6. The figure shows obviously that the present approach reproduces the $\Delta I = 4$ bifurcation quite well. However the three parameter

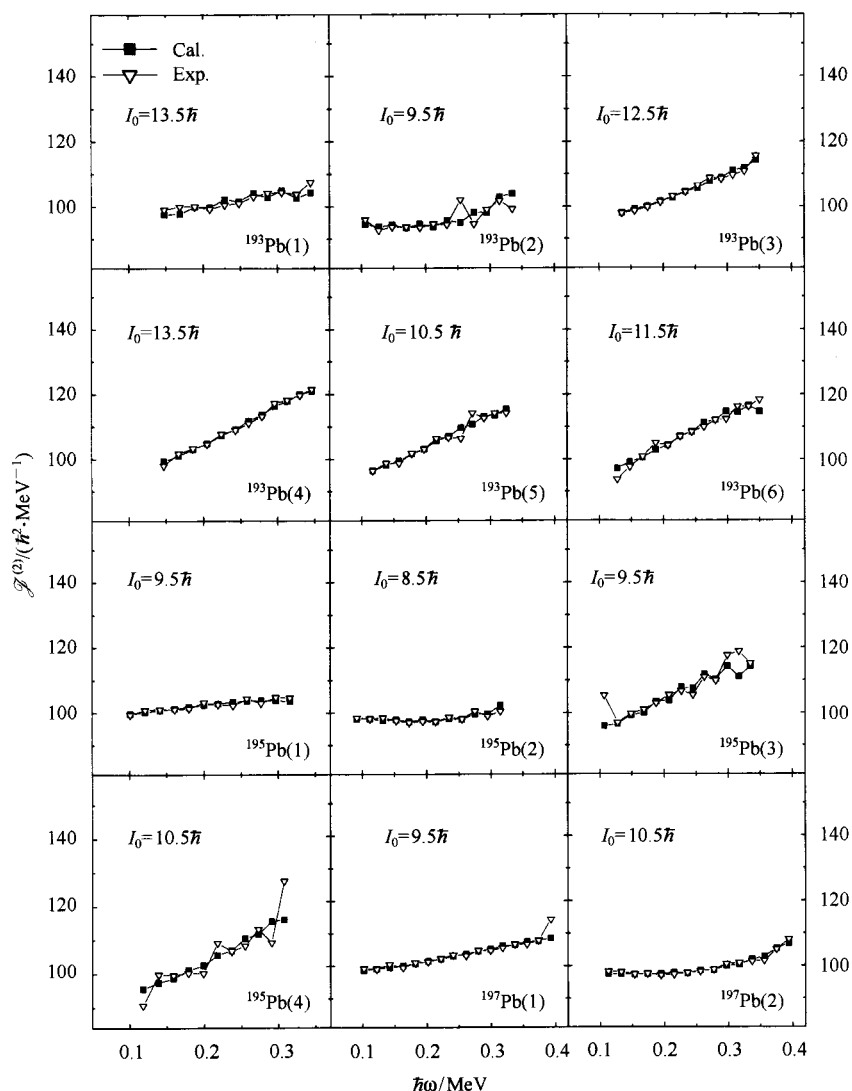


Fig. 3. The same as Fig.1 but for the Pb isotopes. The experimental data are taken from Ref.[2].

$I(I+1)$ expansion does not. The ab formula, the approach proposed in Ref.[43] and the present approach with $B \equiv 0$ do not either (For simplicity, we show only the results in the three parameter $I(I+1)$ expansion in Fig.6). By the way, since the best fitted parameter B in the present approach is very small, the term with $SO(5)$ symmetry contributes only to the $\Delta I = 2$ staggering in γ -ray energies (the total error σ with $B \equiv 0(67.5948 \times 10^{-3})$ is slightly larger than that with $B \neq 0$).

The parameters listed in Table 1 exhibit some systematic properties. For almost all the bands, the C_0 's are about 5.5keV, the f_1 's are positive and the f_2 's are negative. The few exceptions come from that the variation of $\mathcal{J}^{(2)}$ vs $\hbar\omega$ is drastically different from the general behavior of the SD bands

in $A \approx 190$ region. It means that, for the SD bands in $A \approx 190$ region, especially the ones with turnover, the anti-pairing effect plays more important role than the pairing effect in the low rotational frequency region, but the pairing effect recovers in the region of high rotational frequency. It indicates that there exists competition between the pairing and anti-pairing effects in the SD states in $A \approx 190$ mass region (at least for the ones whose $\mathcal{J}^{(2)}$ exhibits turnover with the $\hbar\omega$ increasing). The few exceptions which possess $f_1 > 0$ and $f_2 > 0$ mean that only the anti-pairing effect plays a role in generating the bands, and those with $f_1 < 0$ and $f_2 > 0$ shows that the pairing effect is more important to build the bands. In the other point of view, since the inclusion of the f_1 and f_2 is a way to consider the many-body interactions^[25], the present

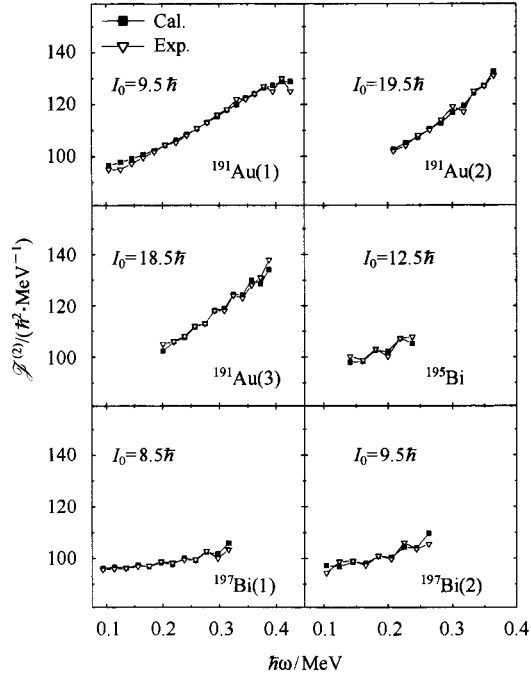


Fig. 4. The same as Fig.1 but for the SD bands ^{191}Au , ^{195}Bi and ^{197}Bi . The experimental data are taken from Ref. [2].

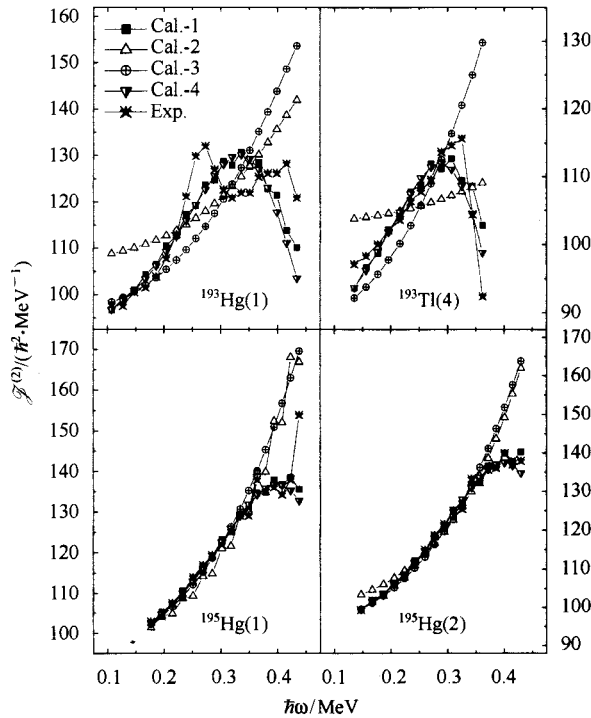


Fig. 5. The same as Fig. 1 but for the bands $^{193}\text{Hg}(1)$, $^{195}\text{Hg}(1)$, $^{195}\text{Hg}(2)$ and $^{193}\text{Tl}(4)$. The ones labeled with Cal.-1, Cal.-2, Cal.-3, Cal.-4 refer to the result obtained with present approach, the approach of Ref. [43], the ab formula and the three parameter $I(I+1)$ expansion. The experimental data are taken from Refs. [2] and [4].

calculation shows that many-body interactions play dominant roles in SD states. In addition, the fitted parameters for the bands in the pairs $\{^{191}\text{Hg}(2), ^{191}\text{Hg}(3)\}$, $\{^{195}\text{Hg}(1), ^{195}\text{Hg}(2)\}$, $\{^{193}\text{Tl}(1), ^{193}\text{Tl}(2)\}$ and $\{^{197}\text{Bi}(1), ^{197}\text{Bi}(2)\}$ are quite close to each other, and the difference between the angular momenta is 1 unit. It manifests that the bands in the pairs are signature partners.

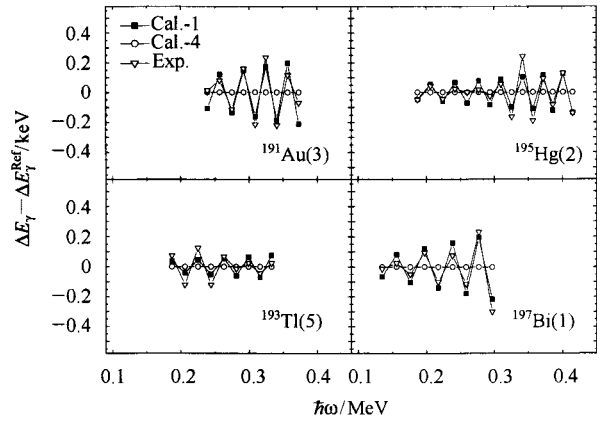


Fig. 6. Calculated results of the energy differences $\Delta E_\gamma - \Delta E_\gamma^{\text{ref}}$ of the SD bands $^{191}\text{Au}(3)$, $^{195}\text{Hg}(2)$, $^{193}\text{Tl}(5)$ and $^{197}\text{Bi}(1)$ in the present approach with $B \neq 0$ (denoted as Cal.-1) and those in the three parameter $I(I+1)$ expansion (denoted as Cal.-4) and comparison with experimental data (taken from Ref. [4] for $^{193}\text{Tl}(5)$ and from Ref. [2] for the others).

4 Summary and remarks

In summary, within the supersymmetry scheme including many-body interactions and a perturbation holding the $SO(5)$ (or $SU(5)$) symmetry on the rotational symmetry, the superdeformed bands in the odd- A nuclei in $A \approx 190$ region are investigated systematically in this paper. The calculated results show that, when the parameters are taken as free ones, not only the generic properties of the SD bands but also the turnover of the dynamical moment of inertia can be described quantitatively well if the observed variation of $\mathcal{J}^{(2)}$ vs $\hbar\omega$ in experiment does not possess humps. The calculated results are also compared with those obtained within the scheme of Harris expansion^[44] or $I(I+1)$ expansion^[45], and the ab formula^[46,13]. The comparison indicates that the present approach is much more powerful to describe the bands with the “turnover” phenomenon and those with $\Delta I = 2$ staggering in E_γ energy. Recalling Ref. [29], one can also know that, with the single particle energy being considered simultaneously,

the supersymmetry approach reproduces the identical SD bands $\{^{191}\text{Hg}(2), ^{193}\text{Hg}(2)\}, \{^{191}\text{Hg}(3), ^{193}\text{Hg}(3)\}$ and $\{^{193}\text{Tl}(3), ^{193}\text{Tl}(5)\}, \{^{193}\text{Tl}(1), ^{193}\text{Tl}(2)\}$ as well. It shows that the supersymmetry scheme approach is powerful to describe not only the global rotational properties of SD bands but also the identical bands.

Investigating the algebraic structure of the groups one knows that the $SO(5)$ and $SU(5)$ symmetries are those of a five-dimensional space. Other investigations have shown that the SD states in the $A \approx 190$ mass region can be described well in the framework of a quantal Hamiltonian with five collective quadrupole coordinates^[47]. Our present calculation confirms the success in another direction. We propose then

that the mean field to generate the SD states may not be the usual three-dimensional space field, but the five-dimensional super-space field spanned by the five collective quadrupole coordinates. Such a super-space may possess the orthogonal rotational symmetry $SO(5)$, even the unitary symmetry $SU(5)$.

It is also worth while mentioning that, even though the meaning of the parameters f_1 and f_2 have been discussed, the microscopic mechanism of involving the many-body interactions and the competition between the pairing and anti-pairing effects in the way described in this paper and Refs. [18, 23, 25, 26, 29, 43] needs to be studied further.

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$A \approx 190$ 质量区奇 A 核超形变带的系统研究*

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摘要 利用包括多体相互作用和具有 $SO(5)$ (或 $SU(5)$)对称性的对转动对称微扰的超对称模型方法,对 $A \approx 190$ 质量区奇 A 核的超形变转动带进行系统的研究,无论是 E2 跃迁 γ 射线能谱,还是动力学转动惯量都得到与实验结果很好定量符合的结果.这表明,超对称方法不仅可以很好地描述超形变带的整体性质,还可以很好地描述超形变带的 $\Delta I = 4$ 分岔和全同带等奇异现象.

关键词 超形变带 能谱 动力学转动惯量 超对称模型 配对和拆对效应

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