B→KK Decays with the Soft-Gluon Corrections*

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Abstract We analyze the B→KK decays with the soft-gluon corrections by using the QCD light-cone sum rules(LCSR). Although one can calculate the leading order factorization parts and the radiative corrections from hard-gluon exchanges at α_s order in QCD factorization approach, it is worthwhile to estimate the nonfactorizable soft-gluon contributions from all the tree and penguin diagrams systematically. Our results show that the soft-gluon effects always decrease the branching ratios and give a few percentage corrections at most in the B→KK decays.

Key words B meson decays, QCD light-cone sum rules, QCD factorization approach

1 Introduction

Recently, A. Khodjamirian[1] has presented an approach to calculate the hadronic matrix elements of nonleptonic B meson decays within the framework of the light-cone sum rules, where the nonfactorizable soft contributions can be effectively dealt with. As we know, QCD factorization approach[2] provided that the hadronic matrix elements for B→ππ, Kπ decays can be expanded in the powers of α_s and $\frac{\Lambda_{QCD}}{m_b}$ and exhibited a considerablly strong predicative potential. However, this approach cannot calculate nonfactorizable contributions from the soft-gluon exchanges. Thus it is interesting to evaluate the corrections from the soft-gluon exchanges by using light-cone QCD sum rules.

In the previous paper[3], the role of the soft-gluon exchanges in B→ππ has been studied by using the light-cone QCD sum rules. Compared to Ref.[1], the calculations are carried out not only for the tree operators but also for the penguin ones. Ref.[3] showed that the $\frac{\Lambda_{QCD}}{m_b}$ corrections from the soft-gluon exchanges are not always negligible in the process B→ππ and the nonfactorizable soft contributions are almost as important as the O(α_s) correction parts, and in some cases even have the same order effects as that of the factorization amplitude. Therefore it is worthwhile to evaluate the nonfactorizable soft-gluon contributions in the process B→KK.

2 Correlator and sum rules

Similar to the case of B→ππ, we can calculate the contributions from the soft-gluon exchanges in B→KK including the tree and penguin operators. We begin with the effective Hamiltonian H_{eff} which is responsible for the B→KK decays[4]:

$$H_{eff}=\frac{G_F}{\sqrt{2}}\Big[V_{ub}V_{uq}^*(c_1(\mu)O_1(\mu)+c_2(\mu)O_2(\mu))-V_{tb}V_{tq}^*\sum_{i=3}^{10}c_iO_i\Big]+\text{h.c.},\qquad(1)$$

where $O_{1,2}$ are the tree operators and O_3—O_{10} denote the penguin ones. By applying the Fierz transformation, the operators which is related to the soft corrections to B→KK can be clearly presented in effective weak Hamiltonian:

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$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}}\Big[V_{\text{ub}}V_{\text{ud}}^*\Big(c_1(\mu) + \frac{c_2(\mu)}{3}\Big)\times O_1(\mu) + 2c_2(\mu)\tilde{O}_1(\mu) + \cdots\Big], \tag{2}$$

where

$$O_1 = (\bar{s}\Gamma_\mu u)(\bar{u}\Gamma^\mu b), \tag{3}$$

and

$$\tilde{O}_1 = \Big(\bar{s}\Gamma_\mu \frac{\lambda^a}{2}u\Big)\Big(\bar{u}\Gamma^\mu \frac{\lambda^a}{2}b\Big), \tag{4}$$

the penguin operators are denoted by ellipses. In the above $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$, $Tr(\lambda^a\lambda^b) = 2\delta^{ab}$. To the operator O_1, we employ the results of QCD factorization approach directly to the contributions from the factorization and α_s corrections since the result of LCSR is consistent with the prediction of the QCD factorization approach.

In order to calculate the nonfactorizable matrix elements induced by the operator $\tilde{O}_1$, we choose a proper vaccum-kaon correlation function:

$$F_\alpha^{(\tilde{O}_1)}(p,q,k) = -\int d^4x e^{-i(p-q)x}\int d^4y e^{i(p-k)} \times \langle 0|\, T\{j_{\alpha5}^{(K)}(y)\tilde{O}_1(0)j_5^{(B)}(x)\}\,|\, K^-(q)\rangle, \tag{5}$$

where $j_{\alpha5}^{(K)} = \bar{u}\gamma_\alpha\gamma_5 s$ and $j_5^{(B)} = m_b\bar{b}i\gamma_5 d$ are the quark currents interpolating K and B mesons, respectively. The decomposition of the correlation function Eq.(5) in terms of independent momenta is straightforward and contains four invariant amplitudes:

$$F_\alpha^{(\tilde{O}_1)} = (p-k)_\alpha F^{(\tilde{O}_1)} + q_\alpha\tilde{F}_1^{(\tilde{O}_1)} + k_\alpha\tilde{F}_2^{(\tilde{O}_1)} + \epsilon_{\alpha\beta\lambda\rho}q^\beta p^\lambda k^\rho\tilde{F}_3^{(\tilde{O}_1)}. \tag{6}$$

In what follows only the amplitude $F^{(\tilde{O}_1)}$ is relevant. To obtain $F^{(\tilde{O}_1)}$, we calculate the correlation function by expanding the T-product of three operators, two currents and $\tilde{O}_1$, near the light-cone $x^2 \sim y^2 \sim (x-y)^2 \sim 0$. To stay away from hadronic thresholds in both channels of K and B currents, we choose the following kinematical region in Eq.(5):

$$q^2 = p^2 = k^2 = 0 \text{ and } |(p-k)^2| \sim |(p-q)^2| \sim |P^2| \gg \Lambda_{\text{QCD}}^2, \tag{7}$$

where $P \equiv p - k - q$.

Following the standard procedure for QCD sum rule calculation, we can obtain

$$A^{(\tilde{O}_1)}(B\to KK) = \langle K(p), K(-q)\,|\,\tilde{O}_1\,|\,B(p-q)\rangle = \frac{-i}{\pi^2 f_K f_B m_B^2}\int_0^{s_0^K} ds\, e^{\frac{-s}{M^2}}\int_{m_b^2}^{R(s,m_b^2,m_B^2,s_0^B)} ds' \times e^{\frac{m_B^2-s'}{M'^2}}\, Im_{s'}Im_s F_{\text{QCD}}^{(\tilde{O}_1)}(s,s',m_B^2), \tag{8}$$

where s_0^K and s_0^B are effective threshold parameters. A straightforward calculation gives the following results for the twist-3 and twist-4 contributions:

$$F_{\text{QCD}}^{(\tilde{O}_1)} = F_{\text{tw3}}^{(\tilde{O}_1)} + F_{\text{tw4}}^{(\tilde{O}_1)}, \tag{9}$$

with

$$F_{\text{tw3}}^{(\tilde{O}_1)} = \frac{m_b f_{3K}}{4\pi^2}\int_0^1 dv\int D\alpha_i \times \frac{\varphi_{3K}(\alpha_i)}{(m_b^2-(p-q)^2(1-\alpha_1))(-P^2v\alpha_3-(p-k)^2(1-v\alpha_3))}\times [(2-v)(q\cdot k)+2(1-v)q\cdot(p-k)](q\cdot(p-k)), \tag{10}$$

and

$$F_{\text{tw4}}^{(\tilde{O}_1)} = -\frac{m_b^2 f_K}{4\pi^2}\int_0^1 dv\int D\alpha_i\tilde{\varphi}_{\perp}(\alpha_i)\frac{1}{m_b^2-(p-q+q\alpha_1)^2}\times \frac{(4v-6)(p-k)q}{(p-k-qv\alpha_3)^2} + \frac{m_b^2 f_K}{2\pi^2}\int_0^1 dv\int d\alpha_1 d\alpha_3\Phi_1(\alpha_1,\alpha_3)\times \frac{1}{[m_b^2-(p-q+q\alpha_1)^2]^2}\frac{(2pq-2vqk)(p-k)q}{(p-k-qv\alpha_3)^2} - \frac{m_b^2 f_K}{2\pi^2}\int_0^1 dv\int d\alpha_3\Phi_2(\alpha_3)\frac{1}{[m_b^2-(p-q\alpha_3)^2]^2}\times \frac{(2pq-2vqk)(p-k)q}{(p-k-qv\alpha_3)^2} + \frac{m_b^2 f_K}{2\pi^2}\int_0^1 dv 2v^2\int d\alpha_3\Phi_2(\alpha_3)\times \frac{1}{pq[m_b^2-(p-q\alpha_3)^2]}\frac{[(p-k)q]^3}{(p-k-qv\alpha_3)^4} - \frac{m_b^2 f_K}{2\pi^2}\int_0^1 dv(2v-2)v\int d\alpha_3\Phi_2(\alpha_3)\times \frac{1}{m_b^2-(p-q\alpha_3)^2}\frac{[(p-k)q]^2}{(p-k-qv\alpha_3)^4},$$

where the definitions of $\varphi_{3K}(\alpha_i)$, $\varphi_{\perp}(\alpha_i)$ and $\varphi_{\parallel}(\alpha_i)$ can be found in Ref.[5].

By taking the duality approximation and applying Borel transformation, we get the following sum rule:

$$A^{(\tilde{O}_1)}(B\to KK) = i m_B^2\Big(\frac{1}{4\pi^2 f_K}\int_0^{s_0^K} ds\, e^{-\frac{s}{M^2}}\Big)\times \Big(\frac{m_b^2}{2f_B m_B^4}\int_{u_0^B}^1 \frac{du}{u}e^{\frac{m_B^2}{M'^2}-\frac{m_b^2}{uM'^2}}\Big[\frac{m_b f_{3K}}{u}\int_0^u \frac{dv}{v}\varphi_{3K}\times (1-u,u-v,v) + f_K\int_0^u\frac{dv}{v}\times \Big[3\tilde{\varphi}_\perp(1-u,u-v,v) - \Big(\frac{m_b^2}{uM'^2}-1\Big)\frac{\Phi_1(1-u,v)}{u}\Big] + f_K\Big(\frac{m_b^2}{uM'^2}-2\Big)\frac{\Phi_2(u)}{u^2}\Big]\Big),$$

where the light-cone wave functions are introduced as the following:

$$\frac{\partial \Phi_1(w,v)}{\partial v}=\tilde{\varphi}_{\perp}(w,1-w-v,v)+\tilde{\varphi}_{\parallel}(w,1-w-v,v)$$

$$\frac{\partial \Phi_2(v)}{\partial v}=\Phi_1(1-v,v). \quad (13)$$

For the kaon distribution amplitudes in Eq.(12), we employ their asymptotic forms which were given by Ref.[5]:

$$\varphi_{3K}(\alpha_i)=360\alpha_1\alpha_2\alpha_3^2,\quad \tilde{\varphi}_{\perp}(\alpha_i)=10\delta^2\alpha_3^2(1-\alpha_3),$$

$$\tilde{\varphi}_{\parallel}(\alpha_i)=-40\delta^2\alpha_1\alpha_2\alpha_3. \quad (14)$$

3 Decay amplitudes with the soft-gluon corrections

The hadronic matrix elements of the penguin operator can be obtained from the same procedure. In fact, they can be represented by the tree diagram operators exactly. So the calculations are simplified relatively. Here, to compare with the calculated results, we write down the B→KK decay amplitudes for all decay channels in terms of the sum of three parts: the factorization part M_f, the α_s correction term M_{α_s} and the soft-gluon contribution M_{nf}:

$$M_{f+\alpha_s}(\bar{B}_s^0\to K^0\bar{K}^0)=-i\frac{G_F}{\sqrt{2}}f_K F_0^{B\to K}(0)(m_B^2-m_K^2)\times \left\{V_{tb}V_{ts}^*\left[a_4-\frac{1}{2}a_{10}+(a_8-2a_6)R_1\right]\right\}, \quad (15)$$

$$M_{nf}(\bar{B}_s^0\to K^0\bar{K}^0)=-\frac{G_F}{\sqrt{2}}[V_{tb}V_{ts}^*(2c_3-c_9)]A^{(\tilde{o}_1)}, \quad (16)$$

with $R_1=\dfrac{m_K^2}{(m_s+m_d)(m_d-m_b)}$.

$$M_{f+\alpha_s}(B_s^0\to K^+K^-)=i\frac{G_F}{\sqrt{2}}f_K F_0^{B\to K}(0)(m_B^2-m_K^2)\times \{V_{ub}V_{us}^*a_1-V_{tb}V_{ts}^*[a_4-2(a_6+a_8)R_2+a_{10}]\},$$

$$M_{nf}(B_s^0\to K^+K^-)=\sqrt{2}G_F V_{ub}V_{us}^*c_2A^{(\tilde{o}_1)}-\sqrt{2}G_F V_{tb}V_{ts}^*(c_3+c_9)A^{(\tilde{o}_1)}, \quad (18)$$

with $R_2=\dfrac{m_K^2}{(m_u-m_b)(m_s+m_u)}$.

$$M_{f+\alpha_s}(B^-\to K^-K^0)=-i\frac{G_F}{\sqrt{2}}f_K F_0^{B\to K}(0)(m_B^2-m_K^2)\times \left\{V_{tb}V_{td}^*\left[a_4-\frac{1}{2}a_{10}+(-2a_6+a_8)R_3\right]\right\}, \quad (19)$$

$$M_{nf}(B^-\to K^-K^0)=-\frac{G_F}{\sqrt{2}}[V_{tb}V_{td}^*(2c_3-c_9)]A^{(\tilde{o}_1)} \quad (20)$$

with $R_3=\dfrac{m_K^2}{(m_s-m_b)(m_s+m_u)}$. In order to do the numerical calculation, we take[1] $f_K=160\text{MeV}$, $s_0^K=1.62\text{GeV}^2$ and $M^2=0.5-1.2\text{GeV}^2$ for the parameters of the kaon channel. For the B meson, we put $f_B=180\text{MeV}$, $m_b=4.7\text{GeV}$, $s_0^B=35\text{GeV}^2$, $\mu_b=\sqrt{m_B^2-m_b^2}\approx 2.4\text{GeV}$, $M'^2=8-12\text{GeV}^2$, $f_{3K}(\mu_b)=0.0035\text{GeV}^2$, $\delta^2(\mu_b)=0.17\text{GeV}^2$, $f_{BK}^+=0.32$[6], and $f_{B_sK}^+=0.27$[7]. The values of Wilson coefficients c_i, coefficients a_i, and scale parameter $\mu=\dfrac{m_b}{2}$ are taken from Ref.[8].

Focusing on a numerical comparison of M_f, M_{α_s} and M_{nf}, we write down the numerical results for the decay amplitudes $M=M^T+M^P$ by defining the tree amplitudes $M^T=M_f^T+M_{\alpha_s}^T+M_{nf}^T$ and the penguin amplitudes $M^P=M_f^P+M_{\alpha_s}^P+M_{nf}^P$:

$$M(\bar{B}_s^0\to K^0\bar{K}^0)=M^P(\bar{B}_s^0\to K^0\bar{K}^0)=V_{tb}V_{ts}^*\times \{[8.34568\times10^{-7}i]+[-2.96515\times10^{-7}-1.39319\times10^{-7}i]+[-1.08188\times10^{-8}i]\}, \quad (21)$$

$$M(\bar{B}_s^0\to K^+K^-)=M^T(\bar{B}_s^0\to K^+K^-)+M^P(\bar{B}_s^0\to K^+K^-)=V_{ub}V_{us}^*\times \{[1.02213\times10^{-5}i]+[-3.24748\times10^{-7}+3.77232\times10^{-7}i]+[-1.22115\times10^{-7}i]\}+V_{tb}V_{ts}^*\{[8.4483\times10^{-7}i]+[-2.97568\times10^{-7}-1.56528\times10^{-7}i]+[-4.44113\times10^{-9}i]\}, \quad (22)$$

$$M(B^-\to K^0K^-)=M^P(B^-\to K^0K^-)=V_{tb}V_{td}^*\times \{[1.02012\times10^{-6}i]+[-3.59909\times10^{-7}-1.68011\times10^{-7}i]+[-1.08188\times10^{-8}i]\}. \quad (23)$$

For comparison, we plot the branching ratios (Br) for these decay modes as a function of γ in Fig.1—Fig.3. Eqs.(21)—(23) show that the soft-gluon contributions to the decay amplitudes are much smaller than the factorization parts in B→KK decays. The contributions depend on different decay modes. In the case of $\bar{B}_s^0\to K^0\bar{K}^0$, there are only penguin diagram contributions and they come mainly from the factorization and α_s correction parts; soft-gluon contribution is suppressed by the order of 10^{-1} with respect to the former. The soft-gluon effects make the branching ratio smaller and the results are shown in Fig.1.

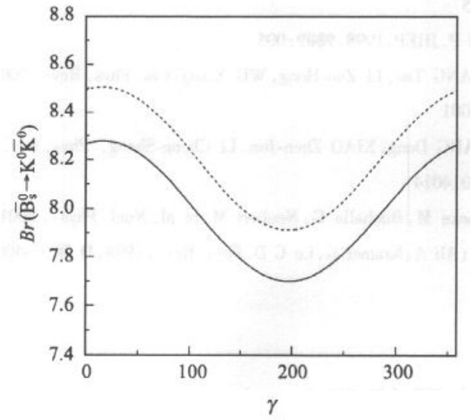

Fig. 1. Dependence of the branching ratio on the weak phase γ in the $\bar{B}_s^0 \to K^0\bar{K}^0$ channel. The dashed and solid lines correspond to the values obtained with $M_f + M_{\alpha_s}$ and $M_f + M_{\alpha_s} + M_{nf}$, respectively.

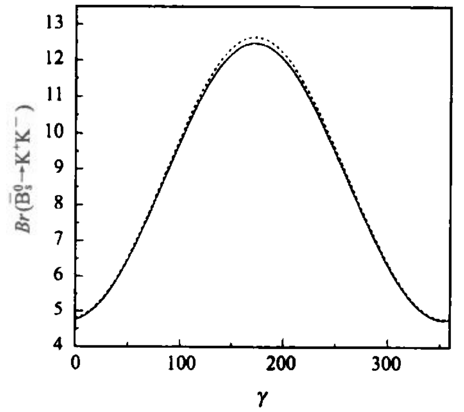

Fig. 2. Dependence of the branching ratio on the weak phase γ in the $B_s^0 \to K^+K^-$ channel. The dashed and solid lines correspond to the values obtained with $M_f + M_{\alpha_s}$ and $M_f + M_{\alpha_s} + M_{nf}$, respectively.

The dashed and solid curves correspond to the values obtained from $M_f + M_{\alpha_s}$ and $M_f + M_{\alpha_s} + M_{nf}$, respectively. In the case of $B_s^0 \to K^+K^-$, the soft contribution has the same order as the α_s correction parts in the tree amplitudes. In the penguin amplitudes, it has the amplitude of order 10^{-9}, which is smaller than those of factorization and α_s correction parts (of order 10^{-7}). It is shown from Fig. 2 that the total branching ratio is suppressed by soft-gluon effects. In the case of $B^- \to K^0K^-$, the soft-gluon amplitude is of order 10^{-8}, which is obviously lower than that of the factorization (of order 10^{-6}) and α_s correction (of order 10^{-7}). Fig. 3 shows that the total branching ratio is decreased by soft-gluon effects, too.

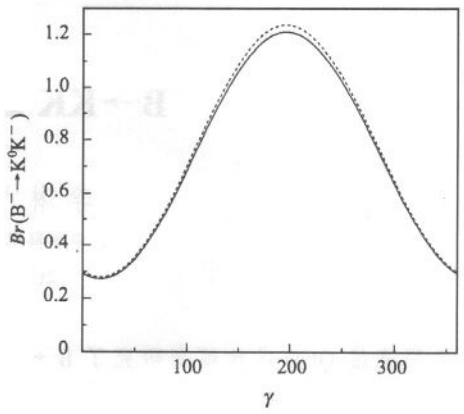

Fig. 3. Dependence of the branching ratio on the weak phase γ in the $B^- \to K^0K^-$ channel. The dashed and solid lines correspond to the values obtained with $M_f + M_{\alpha_s}$ and $M_f + M_{\alpha_s} + M_{nf}$, respectively.

4 Summary

In this paper, we have analyzed the B→KK decays with the soft-gluon corrections by the QCD light-cone sum rules. The soft contributions depend on decay modes, and in most situations they have the amplitudes which are suppressed by the order of 10^{-1} or 10^{-2} of factorization and α_s correction amplitudes. Only in the case of $B_s^0 \to K^+K^-$, they have the same order amplitude with α_s correction parts which is smaller than the factorization amplitude. Our results show that the soft-gluon effects on B→KK are small and they always suppress the branching ratio values with 2%—3% corrections.

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B→KK衰变中的软胶子修正*

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摘要 应用光维QCD求和规则研究了B→KK衰变的软胶子交换修正. 虽然QCD因子化方法已经计算了领头阶的因子化和硬胶子交换的α_s阶辐射修正部分,然而系统地估算所有树图和企鹅图的非因子化软胶子贡献是有价值的. 我们的结果表明在B→KK衰变中软胶子效应总是使分支比值减小,约为几个百分点.

关键词 B介子衰变 软胶子修正 光锥QCD求和规则

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