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# Distribution Functions in QGP Kinetic Equations under Relaxation Time Approximation *

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**Abstract** From QGP kinetic equations with collision integrals, by using the realaxation time approximation, we calculate the distribution functions to the second order correction. We obtain the distribution functions for quarks (and anti-quarks) and gluons under the perturbation of the fluctuation of the color field. Then in the high-temperature-low-density area, we discuss the characteristics of the distribution functions, and use them to get the net baryon density and the energy density.

**Key words** kinetic theory, relativistic heavy ion collision, distribution functions in quasi-equilibrium, relaxation time approximation

## 1 Introduction

One of the main objectives of the future high-energy experiments is to detect a new state of matter called the quark-gluon plasma (QGP), which is expected to be found at large collision machines such as RHIC and LHC. Considerable attention has been paid on the mechanism of the formation and the evolution of the QGP[1—6]. The fundamental theoretic methods dealing with the QGP are the finite-temperature field theory, the kinetic theory and phenomenological analysis. The kinetic theory is a statistical theory that can deal with both the thermal equilibrium and the non-equilibrium phenomena. The kinetic equations for the QGP have been formed under the general frame of the statistical theory and its dynamic basis is the quantum chromodynamics (QCD), which the component particles of the system obey. They are widely used to investigate the properties of the QGP[7,8].

The QGP, if formed in the relativistic heavy ion collisions, is generally believed to be in a thermal non-equilibrium state during initial stage, so it is important to study non-equilibrium phenomena in the QGP. According to statistical mechanics, if the particle distribution function of the system is known, any observable physical quantities can be obtained through standard method. Hence, the non-equilibrium distribution function is the basis for solving non-equilibrium problems. Unfortunately, it is difficult to obtain the non-equilibrium distribution functions by strictly solving the QGP kinetic equations. However, when the fluctuation of the color field in the QGP is at the $gT$ level ($g$ is the coupling constant of perturbative QCD and $T$ is the temperature in the QGP), many properties of the QGP in quasi-equilibrium have been discussed in the QGP kinetic equations without collision integrals[9—11]. Noting the fact that one usually takes collision integral in the relaxation time approximation, we calculate the distribution functions in the QGP kinetic equations under this approximation in this paper, and assume that the perturbation of the color field fluctuation is at the $gT$ level and the fluctuation makes the QGP in a non-equilibrium state.

Since the fluctuation is a small quantity, the system will be in quasi-equilibrium, in other words, it is nearly in equilibrium state. The distribution function of the system can be generally described as

$$f = f^{(0)} + \kappa f^{(1)} + \kappa^2 f^{(2)} + \cdots, \qquad (1)$$

where $f^{(0)}$ is the distribution function in equilibrium, $f^{(1)}$, $f^{(2)}$ are the first and the second order corrections to the distribution functions in equilibrium, $\kappa$ is a scale factor which

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is introduced to describe the order of the corrections. Then the distribution functions in quasi-equilibrium can be obtained by using the step-by-step iteration method.

## 2 Solve the kinetic equation to the second order correction

In a semi-classical limit, the distribution functions are governed by

$$p^{\mu}D_{\mu}f(x,p)+\frac{g}{2}p^{\mu}\frac{\partial}{\partial p_{\nu}}\{F_{\mu\nu},f(x,p)\}=C_f,\tag{2}$$

$$p^{\mu}D_{\mu}\bar{f}(x,p)-\frac{g}{2}p^{\mu}\frac{\partial}{\partial p_{\nu}}\{F_{\mu\nu},\bar{f}(x,p)\}=C_{\bar{f}},\tag{3}$$

$$p^{\mu}\widetilde{D}_{\mu}G(x,p)+\frac{g}{2}p^{\mu}\frac{\partial}{\partial p_{\nu}}\{\widetilde{F}_{\mu\nu},G(x,p)\}=C_G,\tag{4}$$

where $C_i$ ($i=f,\bar{f},G$) are the collision integrals, $f(x,p)$, $\bar{f}(x,p)$, $G(x,p)$ are the distribution functions of quarks, anti-quarks and gluons, respectively.

$A^{\mu}(x)=A^{\mu}_a(x)I^a$, $F^{\mu\nu}(x)=F^{\mu\nu}_a(x)I^a$ are the four-dimension potential and the field tensor of the color field, respectively, and $I^a$ is the generator of group $SU(3)$ in the fundamental representation. The field tensor is defined as

$$F^a_{\mu\nu}(x)=\partial_{\mu}A^a_{\nu}(x)-\partial_{\nu}A^a_{\mu}(x)+gf_{abc}A^b_{\mu}(x)A^c_{\nu}(x),\tag{5}$$

with $f_{abc}$ being the structure constants of the $SU(3)$ group. $\widetilde{A}^{\mu}(x)=A^{\mu}_c(x)T^c$, $\widetilde{F}^{\mu\nu}(x)=F^{\mu\nu}_c(x)T^c$, with $T^c$ being the generator of $SU(3)$ in the adjoint representation.

Correspondingly, the field equation is

$$D_{\mu}F^{\mu\nu}(x)=j^{\nu}(x),$$

$$j^{\nu}(x)=-\frac{g}{2}\int\frac{d^3p}{E_p(2\pi)^3}p^{\nu}\{[f(x,p)-\bar{f}(x,p)]+2iI_af_{abc}G_{bc}(x,p)\}.\tag{6}$$

Eqs.(2)—(4) and Eq.(6) are called the kinetic equations of QGP[11—13].

However, the proper form of the collision integral is not given. In the phenomenological estimation, the collision integral is often taken in the relaxation time approximation,

$$C_i=-\frac{\varepsilon}{\tau}(i-i^{(0)}),$$

where $i=f,\bar{f}$ or $G$ represent the distribution functions, $i^{(0)}=f^{(0)},\bar{f}^{(0)}$ or $G^{(0)}$ are the corresponding distribution functions in equilibrium, $\varepsilon$ and $\tau$ are the single particle energy and the relaxation time, respectively.

Since the QGP kinetic equations are non-Abelian gauge invariant, for convenience, we work in the temporal gauge, $A^{\mu}_a(x)=(0,\boldsymbol{A}_a(t))$. Then the relation between the color electric field and the vector potential becomes simple,

$$E^i_a(x)=-\frac{\partial}{\partial t}A^i_a(x).$$

The QGP kinetic equations in the momentum space and in the temporal gauge are

$$\begin{aligned}&\omega f(\omega,p)+\mathrm{i}g\nu_i\sum_{\omega_1}\frac{1}{\omega_1}[E_i(\omega_1),f(\omega-\omega_1)]-\\&\frac{\mathrm{i}g}{2}\sum_{\omega_1}\left\{E_i(\omega_1),\frac{df(\omega-\omega_1)}{dp_i}\right\}-\\&\frac{\mathrm{i}g}{2}\sum_{\omega_1}\nu_i\left\{E_i(\omega_1),\frac{df(\omega-\omega_1)}{dp_0}\right\}+\frac{g^2}{2}\nu_i\sum_{\omega_1,\omega_2}\frac{1}{\omega_1\omega}\\&\times\{[E_i(\omega_1),E_j(\omega_2)],\partial^j_pf(\omega-\omega_1-\omega_2)\}\\&=-\frac{i}{\tau}(f(\omega,p)-f^{(0)}(\omega,p)),\end{aligned}$$

$$\begin{aligned}&\omega\bar{f}(\omega,p)+\mathrm{i}g\nu_i\sum_{\omega_1}\frac{1}{\omega_1}[E_i(\omega_1),\bar{f}(\omega-\omega_1)]+\\&\frac{\mathrm{i}g}{2}\sum_{\omega_1}\left\{E_i(\omega_1),\frac{d\bar{f}(\omega-\omega_1)}{dp_i}\right\}+\\&\frac{\mathrm{i}g}{2}\sum_{\omega_1}\nu_i\left\{E_i(\omega_1),\frac{d\bar{f}(\omega-\omega_1)}{dp_0}\right\}+\frac{g^2}{2}\nu_i\sum_{\omega_1,\omega_2}\frac{1}{\omega_1\omega_2}\\&\times\{[E_i(\omega_1),E_j(\omega_2)],\partial^j_p\bar{f}(\omega-\omega_1-\omega_2)\}\\&=-\frac{\mathrm{i}}{\tau}(\bar{f}(\omega,p)-\bar{f}^{(0)}(\omega,p)),\end{aligned}$$

$$\begin{aligned}&\omega G(\omega,p)+\mathrm{i}g\nu_i\sum_{\omega_1}\frac{1}{\omega_1}[\widetilde{E}_i(\omega_1),G(\omega-\omega_1)]-\\&\frac{\mathrm{i}g}{2}\sum_{\omega_1}\left\{\widetilde{E}_i(\omega_1),\frac{dG(\omega-\omega_1)}{dp_i}\right\}-\\&\frac{\mathrm{i}g}{2}\sum_{\omega_1}\nu_i\left\{\widetilde{E}_i(\omega_1),\frac{dG(\omega-\omega_1)}{dp_0}\right\}+\frac{g^2}{2}\nu_i\sum_{\omega_1,\omega_2}\\&\times\{[\widetilde{E}_i(\omega_1),\widetilde{E}_j(\omega_2)],\partial^j_pG(\omega-\omega_1-\omega_2)\}\\&=-\frac{i}{\tau}(G(\omega,p)-G^{(0)}(\omega,p)),\end{aligned}$$

$\omega E_i(\omega)$

$$g^2\sum_{\omega_1,\omega_2}\frac{\{[E_i(\omega_1),E_j(\omega_2)],E_j(\omega-\omega_1-\omega_2)\}}{\omega_1\omega_2(\omega-\omega_1-\omega_2)}$$

$$=\frac{1}{i}j^{\prime}(\omega), \tag{10}$$

where $\nu_i = p_i/p_0$.

When the QGP color field is not very strong, in other words, it can be taken as a perturbative quantity, the distribution functions can be expanded as Eq. (1). Considering that the equations of the field are coupled to the distribution functions through the color current, the change of the distribution functions will lead to the change of the amplitude and the frequency of the field. The field intensity and the frequency can be correspondingly expanded as

$$E_\mu = E_\mu^{(1)} + E_\mu^{(2)} + \cdots, \tag{11}$$

$$\omega = \omega^{(0)} + \omega^{(1)} + \omega^{(2)} + \cdots. \tag{12}$$

Inserting them into Eqs. (7)—(10), then the first order correction equations are

$$\omega^{(0)}f^{(1)}(\omega)+\mathrm{i}g\nu_i\sum_{\omega_1}\frac{1}{\omega_1^{(0)}}[E_i^{(1)}(\omega_1),f^{(0)}(\omega-\omega_1)]-$$

$$\frac{\mathrm{i}g}{2}\sum_{\omega_1}\left\{E_i^{(1)}(\omega_1),\frac{\mathrm{d}f^{(0)}(\omega-\omega_1)}{\mathrm{d}p_i}\right\}-$$

$$\frac{\mathrm{i}g}{2}\nu_i\sum_{\omega_1}\left\{E_i^{(1)}(\omega_1),\frac{\mathrm{d}f^{(0)}(\omega-\omega_1)}{\mathrm{d}p_{(0)}}\right\}=$$

$$-\frac{\mathrm{i}}{\tau}(f(\omega)-f^{(0)}(\omega)), \tag{13}$$

$$\omega^{(0)}\bar{f}^{(1)}(\omega)+\mathrm{i}g\nu_i\sum_{\omega_1}\frac{1}{\omega_1^{(0)}}[E_i^{(1)}(\omega_1),\bar{f}^{(0)}(\omega-\omega_1)]+$$

$$\frac{\mathrm{i}g}{2}\sum_{\omega_1}\left\{E_i^{(1)}(\omega_1),\frac{\mathrm{d}\bar{f}^{(0)}(\omega-\omega_1)}{\mathrm{d}p_i}\right\}+$$

$$\frac{\mathrm{i}g}{2}\nu_i\sum_{\omega_1}\left\{E_i^{(1)}(\omega_1),\frac{\mathrm{d}\bar{f}^{(0)}(\omega-\omega_1)}{\mathrm{d}p_{(0)}}\right\}=$$

$$-\frac{\mathrm{i}}{\tau}(\bar{f}(\omega)-\bar{f}^{(0)}(\omega)), \tag{14}$$

$$\omega^{(0)}G^{(1)}(\omega)+\mathrm{i}g\nu_i\sum_{\omega_1}\frac{1}{\omega_1^{(0)}}[\widetilde{E}_i^{(1)}(\omega_1),G^{(0)}(\omega-\omega_1)]-$$

$$\frac{\mathrm{i}g}{2}\sum_{\omega_1}\left\{\widetilde{E}_i^{(1)}(\omega_1),\frac{\mathrm{d}G^{(0)}(\omega-\omega_1)}{\mathrm{d}p_i}\right\}-$$

$$\frac{\mathrm{i}g}{2}\nu_i\sum_{\omega_1}\left\{\widetilde{E}_i^{(1)}(\omega_1),\frac{\mathrm{d}G^{(0)}(\omega-\omega_1)}{\mathrm{d}p_{(0)}}\right\}=$$

$$-\frac{\mathrm{i}}{\tau}(G(\omega)-G^{(0)}(\omega)),$$

$$\omega^{(0)}E_i^{(1)}(\omega)=\frac{1}{\mathrm{i}}j_i^{(1)}(\omega). \tag{16}$$

After solving Eqs. (13)—(16), we get

$$f^{(1)}=\mathrm{i}\frac{g}{\omega+\mathrm{i}/\omega}\frac{\mathrm{d}f^{(0)}}{\mathrm{d}p_i}E_i^{(1)}(\omega)+$$

$$\mathrm{i}g\frac{\nu_i}{\omega^{(0)}+\mathrm{i}/\tau}\frac{\mathrm{d}f^{(0)}}{\mathrm{d}p_0}E_i^{(1)}(\omega), \tag{17}$$

$$\bar{f}^{(1)}=-\mathrm{i}\frac{g}{\omega-\mathrm{i}/\omega}\frac{\mathrm{d}\bar{f}^{(0)}}{\mathrm{d}p_i}E_i^{(1)}(\omega)-$$

$$\mathrm{i}g\frac{\nu_i}{\omega^{(0)}-\mathrm{i}/\tau}\frac{\mathrm{d}\bar{f}^{(0)}}{\mathrm{d}p_0}E_i^{(1)}(\omega), \tag{18}$$

$$G^{(1)}=\mathrm{i}\frac{g}{\omega+\mathrm{i}/\omega}\frac{\mathrm{d}G^{(0)}}{\mathrm{d}p_i}\widetilde{E}_i^{(1)}(\omega)+$$

$$\mathrm{i}g\frac{\nu_i}{\omega^{(0)}+\mathrm{i}/\tau}\frac{\mathrm{d}G^{(0)}}{\mathrm{d}p_0}\widetilde{E}_i^{(1)}(\omega), \tag{19}$$

$$\omega^{(0)2}=-\frac{g^2}{2\pi^2}\int_0^\infty p^2[N_f(\mathrm{d}f^{(0)}+\mathrm{d}\bar{f}^{(0)})+2N_c\mathrm{d}G^{(0)}]. \tag{20}$$

where $N_f$ is the flavor number and $N_c$ is the color number.

The equilibrium distribution is colorless. We take Fermi-Dirac distribution as quark (and anti-quark) distribution and Bose-Einstein distribution as gluon's, that is

$$f^{(0)}=\{\exp[\beta(\varepsilon-\mu)]+1\}^{-1}, \tag{21}$$

$$\bar{f}^{(0)}=\{\exp[\beta(\varepsilon+\mu)]+1\}^{-1}, \tag{22}$$

$$G^{(0)}=[\exp(\beta\varepsilon)-1]^{-1}, \tag{23}$$

where $\varepsilon$ is the single particle energy which satisfies the on-shell relation $\varepsilon=\sqrt{p^2+m^2}$, $\beta$ is the reciprocal of the temperature and $\mu$ is the chemical potential.

Inserting Eqs. (21)—(23) into Eqs. (17)—(19), we get

$$f^{(1)}=-\frac{2\mathrm{i}g}{\omega^{(0)}+\mathrm{i}/\tau}\frac{\beta\nu_i}{\exp[\beta(\varepsilon-\mu)]+1}\times$$

$$\exp[\beta(\varepsilon-\mu)]f^{(0)}E_i^{(1)}(\omega), \tag{24}$$

$$\bar{f}^{(1)}=\frac{2\mathrm{i}g}{\omega^{(0)}-\mathrm{i}/\tau}\frac{\beta\nu_i}{\exp[\beta(\varepsilon+\mu)]+1}\times$$

$$\exp[\beta(\varepsilon+\mu)]\bar{f}^{(0)}E_i^{(1)}(\omega), \tag{25}$$

$$G^{(1)}=-\frac{2\mathrm{i}g}{\omega^{(0)}+\mathrm{i}/\tau}\frac{\beta\nu_i}{\exp(\beta\varepsilon)-1}\times$$

$$\exp(\beta\varepsilon)G^{(0)}\widetilde{E}_i^{(1)}(\omega). \tag{26}$$

In the similar way, the second order correction distributions are

$$f^{(2)}=2g^2\frac{1}{\omega^{(0)}+\mathrm{i}/\tau}E_i^{(1)a}(\omega_1)E_j^{(1)}(\omega-\omega_1)\delta_{ab}\beta^2\nu_i\nu_j\times$$

$$\frac{\exp[\beta(\varepsilon-\mu)]\{1-\exp[\beta(\varepsilon-\mu)]\}}{\{1+\exp[\beta(\varepsilon-\mu)]\}^2}f^{(0)}, \tag{27}$$

$$\bar{f}^{(2)}=2g^2\frac{1}{\omega^{(0)}-\mathrm{i}/\tau}E_i^{(1)a}(\omega_1)E_j^{(1)}(\omega-\omega_1)\delta_{ab}\beta^2\nu_i\nu_j\times$$

$$\frac{\exp[\beta(\varepsilon+\mu)]\{1-\exp[\beta(\varepsilon+\mu)]\}}{\{1+\exp[\beta(\varepsilon+\mu)]\}^2}\bar{f}^{(0)} \tag{28}$$

$$G^{(2)} = 2g^2 N_c^2 \frac{1}{\omega^{(0)}+i/\tau} \widetilde{E}_i^{(1)a}(\omega_1)\widetilde{E}_j^{(1)}(\omega-\omega_1)\delta_{ab}\beta^2 v_i v_j \times \frac{\exp(\beta\varepsilon)[-1-\exp(\beta\varepsilon)]}{[\exp(\beta\varepsilon)-1]^2} G^{(0)} \tag{29}$$

where $\widetilde{E}_i(\omega) = N_c E_i(\omega)$.

Therefore, computing to the second order correction, the distribution functions are

$$f = \left(1+2g^2\frac{\omega^{(0)2}}{\omega^{(0)2}+(1/\tau)^2}\frac{\exp[\beta(\varepsilon-\mu)]}{\exp[\beta(\varepsilon-\mu)]+1} - 16g^4\frac{\omega^{(0)2}(\omega^{(0)2}-(1/\tau)^2)}{(\omega^{(0)2}+(1/\tau)^2)^2}\cdot\frac{\exp[\beta(\varepsilon-\mu)]\{1-\exp[\beta(\varepsilon-\mu)]\}}{\{\exp[\beta(\varepsilon-\mu)]+1\}^2}\right)\cdot\frac{1}{\exp[\beta(\varepsilon-\mu)]+1}, \tag{30}$$

$$\bar{f} = \left(1-2g^2\frac{\omega^{(0)2}}{\omega^{(0)2}+(1/\tau)^2}\frac{\exp[\beta(\varepsilon+\mu)]}{\exp[\beta(\varepsilon+\mu)]+1} - 16g^4\frac{\omega^{(0)2}(\omega^{(0)2}-(1/\tau)^2)}{(\omega^{(0)2}+(1/\tau)^2)^2}\cdot\frac{\exp[\beta(\varepsilon+\mu)]\{1-\exp[\beta(\varepsilon+\mu)]\}}{\{\exp[\beta(\varepsilon+\mu)]+1\}^2}\right)\cdot\frac{1}{\exp[\beta(\varepsilon+\mu)]+1}, \tag{31}$$

$$G = \left(1+2g^2N_c\frac{\omega^{(0)2}}{\omega^{(0)2}+(1/\tau)^2}\frac{\exp(\beta\varepsilon)}{\exp(\beta\varepsilon)+1} - 16g^4N_c^2\frac{(\omega^{(0)2}-(1/\tau)^2)}{(\omega^{(0)2}+(1/\tau)^2)^2}\cdot\frac{\exp(\beta\varepsilon)[-1-\exp(\beta\varepsilon)]}{[\exp(\beta\varepsilon)-1]^2}\right)\cdot\frac{1}{\exp(\beta\varepsilon)-1}, \tag{32}$$

where the relativistic limit $v_i \to 1$ and $\omega_1^{(0)} = \omega_2^{(0)} = \omega^{(0)}$ $A_i = gT$ are used.

Considering two kinds of light quarks, $N_f = 2$, and from Eq.(20), we get

$$\omega^{(0)} = \left[\frac{4}{3}g^2T^2 + \frac{\mu^2 g^2}{\pi^2}\right]^{1/2} \tag{33}$$

Inserting Eq.(33) into Eqs.(30)—(32), we can get the distribution functions of the quarks, anti-quarks and gluons.

## 3 Results and discussions

It is generally believed that the QGP, if formed in the central area of the rapidity in high-energy heavy ion collisions, has high temperature (more than 175MeV at least) and low chemical potential (less than 50 MeV). Now we are in a position to discuss the characteristics of the distribution functions of component particles in the QGP. In the computation, the coupling constant $g$ is taken as 0.3 and the relaxation time is taken less than 1fm. Noting that gluons can attain equilibrium more easily than quarks in the same condition, the relaxation time for gluons is less than that for quarks[14]. The results from the numerical calculation are shown in Figs.1—4.

### 3.1 The quark distribution function in the high-temperature-low-density area

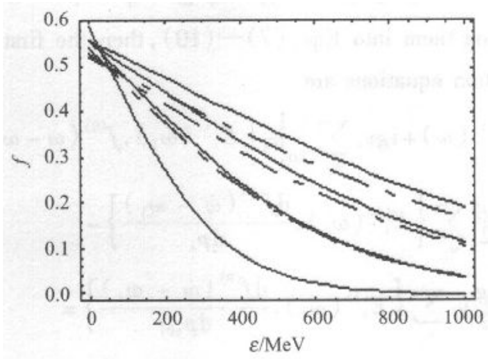


Fig.1. Comparison of the quark distribution functions in quasi-equilibrium (the solid lines) and the corresponding functions in equilibrium (the dashing lines). The relaxation time is 1fm, and from up to down, the temperature is 600, 450, 300, 150MeV respectively.

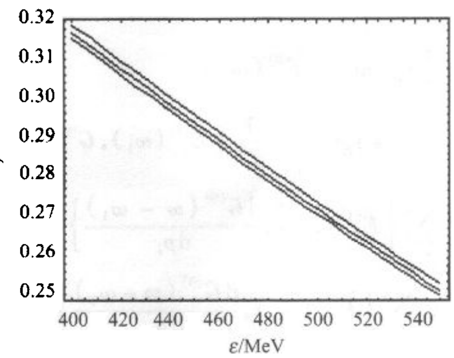


Fig.2. Quark distribution functions under different relaxation time in quasi-equilibrium. The temperature is 450MeV. From up to down, the relaxation time is 1, 0.6, 0.3fm, respectivly. In order to distinguish them clearly they are plotted in a small energy span(400—550MeV).

Fig.1 shows that at any given single particle energy $\varepsilon$, the value of the quark distribution functions in quasi-equilibrium (the solid line) is larger than that of the corresponding distribution functions in equilibrium (the dashing line). The higher the temperature is, the larger the difference between the two kinds of distribution functions is. Fig.1 also shows that the distinct difference exists near the Fermi surface. We all know that in equilibrium, when $\varepsilon = \mu$, the value of the distribution function is 0.5, no matter how high the temperature is. However, in quasi-equilibrium, the value we get is larger than 0.5. From Fig.2, one can see that the curve transfers upward if the relaxation time of the system becomes longer. It means that the difference between the distribution functions in quasi-equilibrium and in equilibrium is larger if the system needs more time to attain the equilibrium state.

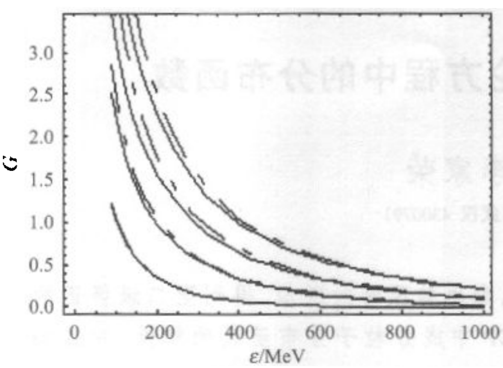


Fig.3. Comparison of the gluon distribution functions in quasi-equilibrium (the solid lines) and the corresponding functions in equilibrium (the dashing lines). From up to down, the temperature is 600, 450, 300, 150MeV, respectively, and the relaxation is 1fm.

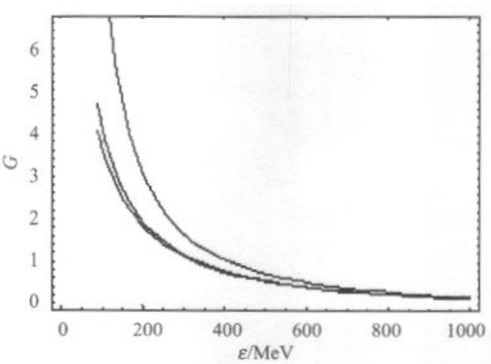


Fig.4. Gluon distribution functions under different relaxation time in quasi-equilibrium. The temperature is 450MeV. From up to down, the relaxation time is 0.3, 0.6, 1fm, respectively.

### 3.2 Gluon distribution functions in the high-temperature-low-density area

In Fig.3, it is easy to see that the curves of the gluon distribution functions in quasi-equilibrium (the solid line) are away from that of the corresponding distribution functions in equilibrium (the dashing line). The higher the temperature is, the more obvious the tendency of deviation is. In addition, the smaller the gluon energy is, the larger the deviation is. From Fig.4, it is obvious that the curve transfers downward if the relaxation time becomes longer. It means that when the system takes longer relaxation time, the gluon distribution function is farther to its corresponding equilibrium function.

According to the statistical physics, if the particle distribution functions in a system are known, the net particle number density and the energy density can be obtained. They are

$$N = \sum_i \int \frac{d^3 p}{(2\pi)^3} (f^i - \bar{f}^i), \tag{34}$$

$$E = \sum_i \int \frac{d^3 p}{(2\pi)^3} p^0 (f^i + \bar{f}^i + G), \tag{35}$$

where $N$ and $E$ are the net baryon density and the energy density.

Inserting Eqs. (30)—(32) into Eqs. (34), (35), using $\varepsilon = \sqrt{p^2 + m^2}$, and taking $\mathrm{fm}^3$ as the unit of volume instead of $\mathrm{GeV}^{-3}$, the net baryon density and energy density can be gotten in the high-temperature and low-density area. At different temperature, the results are given in Table 1 (the relaxation time is 1fm and the chemical potential is 50MeV).

Table 1.

| $T$/MeV | 300 | 450 |
|---|---|---|
| $N/(1/\mathrm{fm}^3)$ | 0.13422 | 0.43657 |
| $E/(\mathrm{GeV}/\mathrm{fm}^3)$ | 1.47615 | 8.29649 |

In Table 1 we can see that the net baryon density and the energy density are consistent with the phenomenological estimation within the order-of-magnitude. For example, at the energy scaling of RHIC, according to the estimation of the Bjorken formula, the energy density is $3.5—7.5\mathrm{GeV}/\mathrm{fm}^3$.

In short, based on the kinetic theory, we solve the QGP kinetic equation in the relaxation time approximation. Calculating to the second order correction, we obtain

the quark (anti-quark) and gluon distribution functions in quasi-equilibrium. Then in the high-temperature-low-density case, we especially analyse the characteristics of the distribution functions and calculate the net baryon number and the energy density. The results are found to be content with the phenomenological analysis of heavy ion collision experiments.

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# 在弛豫时间近似下QGP动力论方程中的分布函数

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**摘要**　从有碰撞项的QGP动力论方程出发,在色涨落扰动下,利用弛豫时间近似,得到至二级修正的夸克和胶子分布函数,通过数值分析重点讨论了高温低密情况下QGP中成分粒子分布函数的特性,并且由分布函数得到净重子数密度和能量密度.

**关键词**　动力论　相对论重离子碰撞　近平衡分布函数　弛豫时间近似

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