第 27 卷 第 6 期
2003 年 6 月

高 能 物 理 与 核 物 理
HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS

Vol.27, No.6
Jun., 2003

光孤子约束系统的量子场论*

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摘要　光孤子系统可用奇异 Lagrange 量描述,系统含 Dirac 约束. 通常按对应原理写出系统对易关系和量子运动方程时,未计及约束. 文中对该系统进行严格的 Dirac 括号量子化,给出了系统的对易关系和量子运动方程,还对系统进行了路径积分量子化,并根据量子水平的 Noether 定理,导出了系统在时空平移变换不变性下的量子能量和动量守恒. 系统还具有相位变换下的不变性,相应导出了系统的粒子数守恒.

关键词　量子场论　约束系统　光孤子　守恒律

1　引言

光孤子通信近来受到人们极大的关注[1—4]. 描述光孤子传输的非线性薛定谔方程,可由 Lagrange 量导出. 在光孤子传输经典理论不断发展和完善的同时,孤子传输的量子理论也得到较大的发展. 它的量子效应的研究一般是在位形空间中给出的. 文献[3]在相应的正则形式中研究了光孤子. 在位形空间中,描述光孤子的场量满足量子非线性薛定谔方程[2]. 它是非线性薛定谔方程的量子改形,是按对应原理写出的,给出的对易关系和量子运动方程未计及系统存在约束,这种处理不能认为是严格的. 按 Dirac 约束理论,用奇异 Lagrange 量描述的光孤子系统是约束正则(哈密顿)系统,它的量子化应该用 Dirac 括号量子化(或约束正则系统的路径积分量子化). 本文首次应用 Dirac 约束理论,对该光孤子系统实行了严格的量子化,给出了系统的对易关系和量子运动方程. 在某些情况下,与按对应原理写出的结果相同[2]. 此外,光孤子在传输过程中形状、振幅等保持不变的原因是系统存在许多运动常数(即守恒量),因此研究光孤子的守恒量具有重要意义. 本文从路径积分量子化出发导出了光孤子系统的量子能量、动量和粒子数守恒.

2　光孤子系统的正则量子化

光孤子概念自 1973 年被提出[5],1980 年在实验中被发现[6],现在可以无形变地传输 14000km,它为光纤通信提供了最优载体. 对光孤子量子效应的研究一般是通过量子化满足非线性薛定谔方程的场. 实际光孤子传输系统,并不能用标准非线性薛定谔方程进行精确描述,而必须考虑一些高阶色散及高阶非线性的影响,传输方程为如下的高阶非线性薛定谔方程(本文采用自然单位制)[1]

$$\mathrm{i}\frac{\partial\Phi}{\partial t}+\frac{\partial^2\Phi}{\partial x^2}+2C\Phi|\Phi^2| = -\mathrm{i}d\frac{\partial^3\Phi}{\partial x^3}-\mathrm{i}pC|\Phi^2|\frac{\partial\Phi}{\partial x},\qquad(1)$$

Φ 为光波包络函数,C,p 和 d 为常数[1]. 当 $p=6d$ 时,有孤子解,相应系统的 Lagrange 密度为

$$\mathscr{L}=\mathrm{i}\Phi^*\frac{\partial\Phi}{\partial t}-\frac{\partial\Phi^*}{\partial x}\frac{\partial\Phi}{\partial x}+C\Phi^*\Phi^*\Phi\Phi-\mathrm{i}d\frac{\partial\Phi^*}{\partial x}\frac{\partial^2\Phi}{\partial x^2}+\mathrm{i}3dC\Phi^*\Phi^*\Phi\frac{\partial\Phi}{\partial x}.\qquad(2)$$

Lagrange 量(2)式是奇异的,下面分析系统在相空间

2002 - 11 - 11 收稿
* 北京市自然科学基金(1942005)资助

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的约束,并给出其量子化. 系统的正则动量为

$$\pi = \frac{\partial \mathscr{L}}{\partial \dot{\Phi}} = \mathrm{i}\Phi^*,$$

$$\pi^* = \frac{\partial \mathscr{L}}{\partial \dot{\Phi}^*} = 0. \tag{4}$$

相应系统的正则 Hamilton 量为

$$H_c = \int \mathrm{d}x[\pi\dot{\Phi} + \pi^*\dot{\Phi}^* - \mathscr{L}] = \int \mathrm{d}x \mathscr{H}_c, \tag{5}$$

式中 $\mathscr{H}_c = \frac{\partial \Phi^*}{\partial x}\frac{\partial \Phi}{\partial x} - C\Phi^*\Phi^*\Phi\Phi + \mathrm{i}d\frac{\partial \Phi^*}{\partial x}\frac{\partial^2\Phi}{\partial x^2} - \mathrm{i}3dC\Phi^*\Phi^*\Phi\frac{\partial \Phi}{\partial x}$ 为正则 Hamilton 量密度. 按 Dirac 的约束理论,由(3)和(4)式知系统有两个初级约束:

$$\phi^{01} = \pi - \mathrm{i}\Phi^* \approx 0, \tag{6}$$

$$\phi^{02} = \pi^* \approx 0, \tag{7}$$

系统的总 Hamilton 量为

$$H_T = \int \mathrm{d}x[\mathscr{H}_c + \lambda_{01}\phi^{01} + \lambda_{02}\phi^{02}]. \tag{8}$$

初级约束的自恰性条件

$$\dot{\phi}^{01} = \{\phi^{01}, H_T\} \approx 0, \tag{9}$$

$$\dot{\phi}^{02} = \{\phi^{02}, H_T\} \approx 0, \tag{10}$$

确定两个 Lagrange 乘子 λ_{01} 和 λ_{02},不给出新的约束. 这里符号"$\approx$"为 Dirac 意义下的弱等,$\{\cdot,\cdot\}$代表场的 Poisson 括号. 容易看出系统的约束均为第二类,记 $\theta^1 = \phi^{01}, \theta^2 = \phi^{02}$. 设 F 和 G 是正则变量的函数,F 和 G 的 Dirac 括号定义为

$$\{F(t,x), G(t,x')\}_D = \{F(t,x), G(t,x')\} - \int \mathrm{d}^3u\mathrm{d}^3v\{F(t,x), \theta_i(t,u)\}C_{ij}^{-1}(t,u,v)\cdot\{\theta_j(t,v), G(t,x')\}, \tag{11}$$

其中 C 是以第二类约束函数的 Poisson 括号为元素构成的矩阵

$$C = \begin{bmatrix}\{\theta^1,\theta^1\} & \{\theta^1,\theta^2\}\\ \{\theta^2,\theta^1\} & \{\theta^2,\theta^2\}\end{bmatrix} = \begin{bmatrix}0 & -\mathrm{i}\delta(x-x')\\ \mathrm{i}\delta(x-x') & 0\end{bmatrix}. \tag{12}$$

它的逆矩阵为

$$C^{-1} = \begin{bmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{bmatrix}\delta(x-x'). \tag{13}$$

经计算,场量的 Dirac 括号为

$$\{\Phi(x), \pi(x')\}_D = \{\Phi,\pi\} - \{\Phi,\theta_i\}C_{ij}^{-1}\{\theta_j,\pi\} = \delta(x-x'), \tag{14}$$

$$\{\Phi^*(x), \pi^*(x')\}_D = \{\Phi^*,\pi^*\} - \{\Phi^*,\theta_i\}C_{ij}^{-1}\cdot\{\theta_j,\pi^*\} = \delta(x-x'), \tag{15}$$

$$\{\Phi(x), \Phi^*(x')\}_D = \{\Phi,\Phi^*\} - \{\Phi,\theta_i\}C_{ij}^{-1}\{\theta_j,\Phi^*\} = -\mathrm{i}\delta(x-x'),$$

$$\{\Phi(x),\Phi(x')\}_D = \{\Phi^*(x),\Phi^*(x')\}_D = \{\pi(x),\pi^*(x')\}_D = 0.$$

因此,按 Dirac 量子化规则$\{\cdot,\cdot\}_D \to -\mathrm{i}[\cdot,\cdot]$,相应的场算符满足对易关系

$$[\hat{\Phi}(x), \hat{\pi}(x')] = \mathrm{i}\delta(x-x'), \tag{18}$$

$$[\hat{\Phi}^*(x), \hat{\pi}^*(x')] = \mathrm{i}\delta(x-x'), \tag{19}$$

$$[\hat{\Phi}(x), \hat{\Phi}^*(x')] = \delta(x-x'), \tag{20}$$

$$[\hat{\Phi}(x),\hat{\Phi}(x')] = [\hat{\Phi}^*(x),\hat{\Phi}^*(x')] = [\hat{\pi}(x'),\hat{\pi}(x')] = 0. \tag{21}$$

对仅含第二类约束的系统,其经典运动方程为

$$\dot{F} = \{F, H_c\}_D. \tag{22}$$

对光孤子系统,有

$$\dot{\Phi} = \{\Phi, H_c\}_D = \{\Phi, H_c\} - \{\Phi,\theta_i\}C_{ij}^{-1}\{\theta_j, H_c\} = -\mathrm{i}\Big[-\frac{\partial^2\Phi}{\partial x^2} - 2C|\Phi^2|\Phi - \mathrm{i}d\frac{\partial^3\Phi}{\partial x^3} - \mathrm{i}6dC|\Phi^2|\frac{\partial \Phi}{\partial x}\Big].$$

过渡到量子情形,相应的量子运动方程为

$$\mathrm{i}\dot{\hat{\Phi}} = -\frac{\partial^2\hat{\Phi}}{\partial x^2} - 2C|\hat{\Phi}^2|\hat{\Phi} - \mathrm{i}d\frac{\partial^3\hat{\Phi}}{\partial x^3} - \mathrm{i}6dC|\hat{\Phi}^2|\frac{\partial \hat{\Phi}}{\partial x}.$$

当 $d = p = 0$ 时,这种严格推导方法所得结果和用对应原理得到的结果一致[2]

3 光孤子系统的路径积分量子化和量子守恒量

根据 Senjanovic 路径积分量子化方法[7],对仅含第二类约束系统,其量子跃迁振幅为

$$Z[0] = \int \mathscr{D}\Phi\mathscr{D}\pi\mathscr{D}\Phi^*\mathscr{D}\pi^*\delta(\theta_i)\cdot[\det|\{\theta_i,\theta_j\}|]^{1/2}\cdot\exp\Big\{\mathrm{i}\int \mathrm{d}^2x[\pi\dot{\Phi} + \pi^*\dot{\Phi}^* - \mathscr{H}_c]\Big\} = \int \mathscr{D}\Phi\mathscr{D}\pi\mathscr{D}\Phi^*\mathscr{D}\pi^*\delta(\pi - \mathrm{i}\Phi^*)\delta(\pi^*)\cdot\exp\Big\{\mathrm{i}\int \mathrm{d}^2x[\mathrm{i}\pi\dot{\pi}^* - \mathrm{i}\pi^*\dot{\pi} - \mathscr{H}_c]\Big\}, \tag{25}$$

由于$\{\theta_i,\theta_j\}$与场量无关,可从生成泛函中略去. 利用 δ 函数的性质,系统在相空间中 Green 函数的生成泛函为[7]

$$Z[J,J^*] = \int \mathscr{D}\Phi\mathscr{D}\pi\mathscr{D}(\Phi^*\mathscr{D}\pi^*)\mathscr{D}\mu_i\cdot$$

$$\exp\left\{\mathrm{i}\int \mathrm{d}^2 x[\mathscr{L}_{\mathrm{eff}}^{P} + J\Phi + J^* \Phi^*]\right\} =$$

$$\int \mathscr{D}\Phi\mathscr{D}\pi\mathscr{D}(\Phi^*\mathscr{D}\pi^*)\mathscr{D}\lambda_i \cdot$$

$$\exp\left\{\mathrm{i}I_{\mathrm{eff}}^{P} + \int \mathrm{d}^2 x[J\Phi + J^* \Phi^*]\right\}, \qquad (26)$$

这里 $I_{\mathrm{eff}}^{P} = \int \mathrm{d}^2 x \mathscr{L}_{\mathrm{eff}}^{P} = \int \mathrm{d}^2 x[\mathrm{i}\pi\dot{\pi}^* - \mathrm{i}\pi^*\dot{\pi} - \mathscr{H}_{C} + \mu_i\theta^i]$ 为有效正则作用量,$\mathscr{L}_{\mathrm{eff}}^{P}$ 为有效正则 Lagrange 密度,μ_i 为与 λ_{0i} 不同的乘子.

根据量子水平的正则 Noether 定理[8,9],如果系统相空间中 Green 函数生成泛函(26)中的有效正则作用量在下列无穷小整体变换

$$\begin{cases} x'^{\mu} = x^{\mu} + \Delta x^{\mu} = x^{\mu} + \varepsilon_{\sigma}\tau^{\mu\sigma}(x,\varphi,\pi) \\ \varphi'(x') = \varphi(x) + \Delta\varphi(x) = \varphi(x) + \varepsilon_{\sigma}\xi^{\sigma}(x,\varphi,\pi) \\ \pi'(x') = \pi(x) + \Delta\pi(x) = \pi(x) + \varepsilon_{\sigma}\eta^{\sigma}(x,\varphi,\pi) \end{cases}$$

下不变[这里 $\varphi(x)$,$\pi(x)$ 代表 $\varphi(x) = (\Phi,\Phi^*)$,$\pi(x) = (\pi,\pi^*)$],且相应变换的 Jacobi 行列式与场量无关,则系统存在如下量子守恒量[8]

$$Q^{\sigma} = \int \mathrm{d}^3 x[\pi(\xi^{\sigma} - \varphi_{,k}\tau^{k\sigma}) - \mathscr{H}_{\mathrm{eff}}\tau^{0\sigma}], \quad (27)$$

其中 $\mathscr{H}_{\mathrm{eff}}$ 为与 $\mathscr{L}_{\mathrm{eff}}^{P}$ 相应的有效 Hamiltonian 密度.

光孤子系统的有效正则作用量 I_{eff}^{P} 在时间平移变换

$$\begin{cases} t' = t + \varepsilon \\ \varphi'(x') = \varphi(x) \\ \pi'(x') = \pi(x) \end{cases} \qquad (28)$$

下具有不变性,且相应变换的 Jacobi 行列式与场量无关.由(28)知,$\tau^{0\sigma} = 1$,$\tau^{k\sigma} = 0$,$\xi^{\sigma} = 0$,代入(27)式中得系统量子水平的能量守恒:

$$E = \int \mathrm{d}x\Big[\frac{\partial\Phi^*}{\partial x}\frac{\partial\Phi}{\partial x} - C\Phi^*\Phi^*\Phi\Phi +$$

$$\mathrm{i}d\frac{\partial\Phi^*}{\partial x}\frac{\partial^2\Phi}{\partial x^2} - \mathrm{i}3dC\Phi^*|\Phi^2|\frac{\partial\Phi}{\partial x}\Big]. \qquad (29)$$

光孤子系统的 I_{eff}^{P} 在空间平移变换

$$\begin{cases} x'^{i} = x^{i} + \varepsilon^{i} \\ \varphi'(x') = \varphi(x) \\ \pi'(x') = \pi(x) \end{cases} \qquad (30)$$

下具有不变性,且相应变换的 Jacobi 行列式与场量无关.由(30)知,$\tau^{0\sigma} = 0$,$\tau^{k\sigma} = 1$,$\xi^{\sigma} = 0$,代入(27)式中得系统量子水平的动量守恒:

$$p = \int \mathrm{d}x(\pi\partial_j\Phi + \pi^*\partial_j\Phi^*) =$$

$$\int \mathrm{d}x\pi\partial_j\Phi. \qquad (31)$$

光孤子系统的 I_{eff}^{P} 在无穷小相位变换

$$\begin{cases} \Phi'(x') = \Phi(x)\mathrm{e}^{\mathrm{i}\theta} \cong \Phi(x) + \mathrm{i}\theta\Phi(x) \\ \Phi^{*\prime}(x') = \Phi^*(x)\mathrm{e}^{-\mathrm{i}\theta} \cong \Phi^*(x) - \mathrm{i}\theta\Phi^*(x) \end{cases} \qquad (32)$$

下具有不变性,且相应变换的 Jacobi 行列式与场量无关.由(32)知,$\tau^{0\sigma} = 0$,$\tau^{k\sigma} = 0$,$\xi = \mathrm{i}\Phi$,$\xi^* = -\mathrm{i}\Phi^*$,代入(27)式中得系统量子水平的粒子数守恒[10]:

$$Q = \int \mathrm{d}x[\mathrm{i}\pi\Phi - \mathrm{i}\pi^*\Phi^*] =$$

$$\int \mathrm{d}x\mathrm{i}\pi\Phi = -\int \mathrm{d}x\Phi^*\Phi. \qquad (33)$$

由于(25)式中的动量积分为 Gauss 型,作出对动量的积分后,得出位形空间的生成泛函,从位形空间中的对称性分析也可导出这些量子守恒量.在相空间中讨论的优点是无须对动量进行积分.一般情形下,作出动量的路径积分是困难的,甚至是不可能的.上述结果也可从经典 KDV 方程守恒量的分析得出[11].可以证明,由(2)式描述的光孤子系统,无论在经典水平或量子水平,均存在守恒量(29),(31)和(33)式.

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Quantum Field Theory of Optical Soliton Constrained Systems

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Abstract The system of a optical soliton can be described by a singular Lagrangian. To our knowledge, the commutation relations and quantum equations of motion are given by using corresponding principle, it's not satisfactory since the constraints are ignored. In this paper, the commutation relations and quantum equations of motion are derived based on the Dirac theory of constrained systems. The conserved energy, momentum and the number of particles for this system are discussed at the quantum level.

Key words quantum field theory, constrained systems, optical soliton, conserved quantity

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Received 11 November 2002

* Supported by Beijing Municipal Natural Science Foundation (1942005)

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