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A New Approach of the Kinematic Fit

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Abstract A new approach of the kinematic fit in event reconstruction is introduced. In the new approach, the method of regula falsi is used to solve the constraint equations, instead of the conventional linear approximation. Applying this algorithm to E705/E771 Monte Carlo data of $J/\psi \to \mu^+ + \mu^-$, the momentum resolution of J/ψ is significantly improved.

Key words kinematic fit, event reconstruction, constraint equations

1 Introduction

Some particles, such as K_s^0, Λ, J/ψ etc, are commonly used to identify their mother particle in high-energy physics experiment, for example, $B \to J/\psi + X$, which is considered as the "golden plated mode"[1]. In reconstruction of the mother particle, we should have a good measurement of daughter particle, so the kinematic fit is commonly performed to improve their momentum resolution.

The procedure of the kinematic fit is briefed as following. According to definite hypotheses about the event type and the corresponding conservation laws, χ^2 and constraint equations are written out. Its minimization is then performed by the Lagrange multiplier method. By requiring the first derivatives of χ^2 to unknown parameters to be zero, a set of equations is obtained. The best estimates of unknown parameters are then obtained by solving this set of equations.

As these equations are in general not linear, they have to be solved by the numerical method. The conventional method is based on the linear approximations, which is to expand the constraint equations to their first derivatives. Although this method looks pretty and is easy to generalize into multidimensional case, unfortunately it may, in some cases, diverge or cannot give a good approximation.

However another approach, the method of regula falsi[2], will always converge and give a better solution. In this paper we will show how we use the method of regula falsi to get the fitted values in the cases that constraint equations are reduced into one-dimension with the form of $f(\alpha) = 0$. The procedure[6] of regula falsi can be illustrated as in Fig.1.

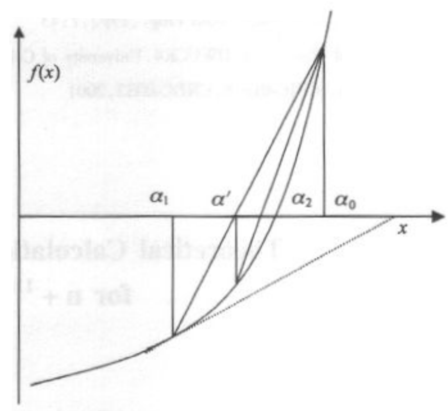

Fig. 1. Comparison of linear approximation method (dashed line) and regula falsi method.

If α_1 and α_2 are such values that $f(\alpha_1)$ and $f(\alpha_2)$ have the opposite signs. Let α' be the value of x at which the line joining the points $(\alpha_1, f(\alpha_1))$ and $(\alpha_2, f(\alpha_2))$ intersects the x-axis. From similar triangles rule (Fig.1)

$$\frac{\alpha' - \alpha_1}{-f(\alpha_1)} = \frac{\alpha_2 - \alpha'}{f(\alpha_2)}.$$

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To solve this equation gives

$$\alpha' = \frac{\alpha_1 f(\alpha_2) - \alpha_2 f(\alpha_1)}{f(\alpha_2) - f(\alpha_1)}.$$

If $f(\alpha') = 0$, the process stops. If $f(\alpha')$ has the same sign as $f(\alpha_1)$, choose $\alpha_1 = \alpha'$. Otherwise choose $\alpha_2 = \alpha'$. The process is then iterated with new pairs (α_1, α_2) until $f(\alpha')$ converges to zero. It is clear that α' is a better approximation than either α_1 or α_2.

Applying our approach to E705/E771 Monte Carlo $J/\psi \to \mu^+ + \mu^-$ data, the momentum resolution of the J/ψ in $J/\psi \to \mu^+ + \mu^-$ has been significantly improved by this new algorithm compared with linear approximation method.

2 Kinematic calculation

Let us recall the kinematic calculation[3,4] of an event that an unstable particle decays into its daughters. We denote the measured variables by $x_i, i = 1,2,3,\cdots, N$, and their error matrix by G^{-1}. In most cases, we can assume that all x_i are uncorrelated, i. e. $(G^{-1})_{i,j} = 0 (i \neq j)$. Because of the measurement errors and the reconstruction errors, the parameters $x_i, i = 1,2,3,\cdots, N$ (such as slopes, intersects and momenta) may not satisfy the physics requirements, For example, the two μs from a J/ψ decay may not converge into one point. We will see that the fit cannot only eliminate the unreasonable results but also improve the error distributions of the reconstructed data.

With constraint equations $f_k(X) = f_k(x_1, x_2, \cdots, x_N) = 0, (k = 1,2,\cdots,K)$, the kinematic fit requires the function

$$\chi^2 = \sum_{i=1}^{N} G_{i,i}(x_i - x_i^0)^2 \quad (1)$$

is minimized. Here superscript 0 refers to measured quantities. It is easy to see that without constraints, the solutions go back to $x_i = x_i^0, i = 1,2,\cdots, N$, which is the original measured values. Introducing the Lagrange multiplier $\alpha_k, k = 1,2,\cdots,K$, we have

$$\chi^2 = \sum_{i=1}^{N} G_{i,i}(x_i - x_i^0)^2 + \sum_{k=1}^{K} 2\alpha_k f_k(X). \quad (2)$$

The minimization of χ^2 can be found by solving the following simultaneous equations:

$$\frac{\partial \chi^2}{\partial x_i} = 2\left\{ G_{i,i}(x_i - x_i^0) + \sum_{k=1}^{K} \alpha_k \frac{\partial f_k(X)}{\partial x_i} \right\} = 0, \quad i = 1,2,\cdots,N; \quad (3)$$

$$\frac{\partial \chi^2}{\partial \alpha_k} = 2 f_k(X) = 0, \quad k = 1,2,\cdots,K. \quad (4)$$

The $\nu + 1$ -th iteration goes on as following:

$$x_i^{\nu+1} = x_i^0 - \frac{1}{G_{i,i}} \sum_{k=1}^{K} \alpha_k^{\nu+1} \frac{\partial f_k(X^\nu)}{\partial x_i}, i = 1,2,\cdots,N. \quad (5)$$

$X^{\nu+1}$ also needs to satisfy Eq. (4), the constraint equations which are generally not linear, so the method of successive substitutions is used to get the fitted values. Newton's method is simply to expand $f_k(X^{\nu+1}) = 0$ to their first derivatives:

$$f_k(X^{\nu+1}) \approx f_k(X^\nu) + \sum_{k=1}^{K} \frac{\partial f_k(X^\nu)}{\partial x_i}(x_i^{\nu+1} - x_i^\nu) = 0, \quad k = 1,2,\cdots,K. \quad (6)$$

Substituting Eq. (5) into Eq. (6), we can get

$$\alpha_k^{\nu+1} = \frac{f_k(X^\nu) + \sum_{i=1}^{N} \frac{\partial f_k(X^\nu)}{\partial x_i}(x_i^0 - x_i^\nu)}{\sum_{i=1}^{N} G_{i,i}^{-1} \left(\frac{\partial f_k(X^\nu)}{\partial x_i} \right)^2}, \quad k = 1,2,\cdots,K. \quad (7)$$

The process goes on until some criteria are fulfilled.

The Newton's method requires some preliminary examination of the equation. It may happen that the equation is of such a character that the second approximation to its root will be worse than the first (as we can see from the Fig. 1, the dashed line), so the $X^{\nu+1}$ will diverge. If we could reduce the constraint equations into one, that is, to eliminate $K - 1$ unknown $x_j, j = N - K + 2, N - K + 3, \cdots, N$, from $K - 1$ constraint equations: $f_k(X^\nu) = 0$ $(k = 1,2,\cdots,K - 1)$,

$$\begin{aligned} x_{N-K+2} &= y_1(x_1, x_2, \cdots, x_M), \\ x_{N-K+3} &= y_2(x_1, x_2, \cdots, x_M), \\ &\vdots \\ x_N &= y_{K-1}(x_1, x_2, \cdots, x_M), \end{aligned} \quad (8)$$

here $M = N - K + 1$, and there are only M independent variables left. Substituting them into the last constraint equation $f_K(X) = 0$, we have

$$F(X') = f_K(X', y_1(X'), y_2(X'), \cdots, y_{K-1}(X')) = 0, \quad (9)$$

here $X' = (x_1, x_2, \cdots, x_M)$, then the iteration equation (5) will be

$$x_i^{\nu+1} = \quad -\frac{1}{G_{i,i}}\alpha^{\nu+1}\frac{\partial F(X^\nu)}{\partial x_i}, i = 1,2,\cdots,M, \tag{10}$$

here $x_i^{\nu+1}$ is only the function of $\alpha^{\nu+1}$ Substituting them into Eq.(9), we have

$$F(\alpha^{\nu+1}) = 0. \tag{11}$$

In such case, the method of regula falsi can be applied. The procedure of method of regula falsi will be shown in detail in the following example.

3 Reduction of constraints

In general there are several constraints. Can they be reduced into one? We will show that, in some cases, the answer is positive. In a J/ψ decay mode of a forward spectrometer, for example

$$J/\psi \to \mu^+ + \mu^-,$$

it can be seen that constraints consist of a definite decay vertex (x, y, z), momentum and energy conservations. If we put the z-axis along the beam direction (e.g. in E705/E771 at Fermi Lab[5]) and y is vertically upward, a right-handed Cartesian coordinate system is then defined. If we denote the momentum vector of $J/\psi, \mu^+$ and μ^- by (p_x, p_y, p_z), (p_{1x}, p_{1y}, p_{1z}) and (p_{2x}, p_{2y}, p_{2z}), respectively, then the track of J/ψ with its slopes of $a_x = p_x/p_z$ and $a_y = p_y/p_z$ and intercepts of b_x and b_y, is described as

$$x = \frac{p_x}{p_z}\cdot z + b_x, \tag{12}$$

$$y = \frac{p_y}{p_z}\cdot z + b_y, \tag{13}$$

and same for μ^+ and μ^-. So the constraint equations can be expressed as

$$\begin{aligned}
f_1 &= \frac{p_{1x}}{p_{1z}}z + \quad - x = 0,\\
f_2 &= \frac{p_{2x}}{p_{2z}}z + b_{2x} - x = 0,\\
f_3 &= \frac{p_{1y}}{p_{1z}}z + b_{1y} - y\\
f_4 &= \frac{p_{2y}}{p_{2z}}z + b_{2y} - y\\
f_5 &= p_x - p_{1x} - p_{2x} = 0,\\
f_6 &= p_y - p_{1y} - p_{2y} = 0,\\
f_7 &= p_z - p_{1z} - p_{2z} = 0,\\
f_8 &= \sqrt{M_\psi^2 + p_x^2 + p_y^2 + p_z^2} - \\
&\sqrt{M_{\mu^+}^2 + p_{1x}^2 + p_{1y}^2 + p_{1z}^2} - \sqrt{M_{\mu^-}^2 + p_{2x}^2 + p_{2y}^2 + p_{2z}^2}\\
&- 0,
\end{aligned} \tag{14}$$

here (b_{1x}, b_{1y}), (b_{2x}, b_{2y}) are intercepts of μ^+ and μ^- tracks, M_ψ and M_μ are masses of J/ψ and $\mu^\pm$ respectively. From f_1 to f_7 it is easy to get

$$\begin{aligned}
y_1 &= p_{1x} = \frac{p_{1z}}{z}(x - b_{1x}),\\
y_2 &= p_{1y} = \frac{p_{1z}}{z}(y - b_{1y}),\\
y_3 &= p_{2x} = \frac{p_{2z}}{z}(x - b_{2x}),\\
y_4 &= p_{2y} = \frac{p_{2z}}{z}(y - b_{2y}),\\
y_5 &= p_x = p_{1x} + p_{2x},\\
y_6 &= p_y = p_{1y} + p_{2y},\\
y_7 &= p_z = p_{1z} + p_{2z}.
\end{aligned} \tag{15}$$

There is only one constraint f_8 and nine independent variables, $p_{1z}, p_{2z}, x, y, z, b_{1x}, b_{2x}, b_{1y}$ and b_{2y} left, so the method of regula falsi can be applied. For the simplicity, we denote the constraint equation as f and the independent variables as $X = (x_1, x_2, \cdots, x_9) = (p_{1z}, p_{2z}, x, y, z, b_{1x}, b_{2x}, b_{1y}, b_{2y})$, $Y = (y_1, y_2, \cdots, y_7)$. Then the iteration Eq. (10) becomes

$$x_i^{\nu+1} = x_i^0 - \frac{1}{G_{i,i}}\alpha^{\nu+1}\cdot \left[\frac{\partial f(X^\nu)}{\partial x_i} + \sum_{k=1}^{7}\frac{\partial f(X^\nu)}{\partial y_k}\frac{\partial y_k}{\partial x_i}\right], i = 1,2,\cdots,9. \tag{16}$$

So $x_i^{\nu+1}$ is only the function of $\alpha^{\nu+1}$. Substituting $x_i^{\nu+1}$, $i = 1,2,\cdots,7$ into the constraint equation f we have

$$F(\alpha^{\nu+1}) = f(X^{\nu+1}(\alpha^{\nu+1}), y(X^{\nu+1}(\alpha^{\nu+1}))) = 0. \tag{17}$$

Now the problem is how to select the points, α_1 and α_2, which make the function $F(\alpha)$ have the opposite signs. The search procedure is as following. Set $\alpha_1 = \alpha^\nu$ and $\alpha_2 = -\alpha^\nu$ to see if they satisfy

$$F(\alpha_1)\cdot F(\alpha_2) < 0. \tag{18}$$

Otherwise set $\alpha_1 = 2\alpha_1$ and $\alpha_2 = 2\alpha_2$ and test again. The process finished if the inequality (18) fulfilled. Usually it will be done in a few steps. For the first iteration, we just take $\alpha_1 = 0$ and $\alpha_2 = 1$ and with the search procedure stated as above find out α_1 and α_2 which satisfy inequality (18). When both α_1 and α_2 are found, we go on with the method of regula falsi as following. Let α' be the abscissa of the intersection point of the x-axis and the line joining the points ($\alpha_1, F(\alpha_1)$) and ($\alpha_2, F(\alpha_2)$), see Fig. 1, that is

$$\alpha' = \frac{\alpha_1 F(\alpha_2) - \alpha_2 F(\alpha_1)}{F(\alpha_2) - F(\alpha_1)}. \tag{19}$$

If $F(\alpha') = 0$, the process terminates. If $F(\alpha')$ has the same sign as $F(\alpha_2)$, replace α_2 with α', otherwise let $\alpha_1 = \alpha'$. The process is then continued to create a sequence of (α_1, α_2) pairs. After a few steps the constraint Eq. (17) can well be approximated. We then go back to Eq. (16) and the iteration is going on as usual.

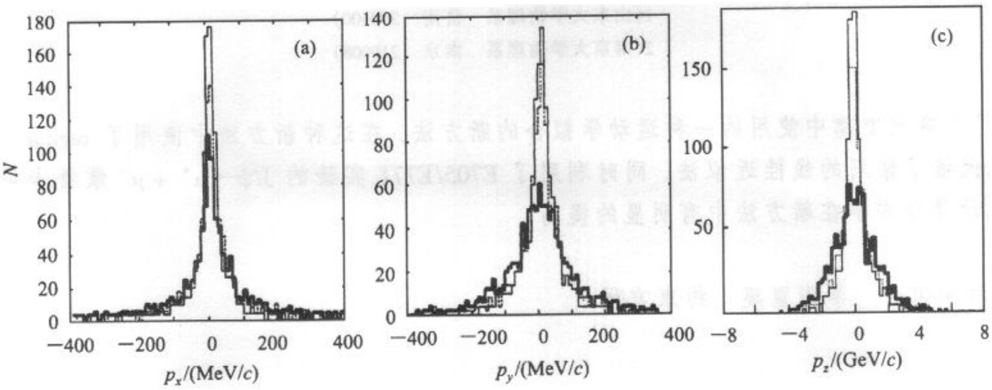

Fig.2. Error distributions of p_x, p_y, p_z (Fig.2(a), 2(b), 2(c), respectively) of the generated events (thick solid lines), corrected by the linear approximation method (dashed lines) and corrected by regula falsi method (thin solid lines).

4 Results

We have generated 1000 events of $J/\psi \to \mu^+ + \mu^-$ with software used in E705/E771 experiment, in which the measurement uncertainties of charged track and J/ψ momentum (10—50GeV) were taken into account in the simulations. The kinematic fit was performed with linear approximation and regula falsi method, respectively, for comparison.

The error distributions of p_x, p_y, p_z of the J/ψ are shown in Fig.2(a), Fig.2(b) and Fig.2(c), respectively. The thick solid line in each plot represents the error distribution of the generated events, the dashed line represents the one corrected by the linear approximation and the thin solid line represents the one corrected by this algorithm, respectively.

The standard deviations of error distribution of reconstructed $p_x p_y p_z$ of J/ψ are shown in Table 1. It can be seen that the Monte-Carlo generated data (simulating experimental data) are most rough. After the kinematic fit, the data were corrected using the constraints and the resolution became better. It is obvious that method of regula falsi gave the even better resolution than the old method.

Table 1. Standard deviations of error distribution of reconstructed $p_x p_y p_z$ of J/ψ.

	σ_{p_x} /(MeV/ c)	σ_{p_y} /(MeV/ c)	σ_{p_z} /(MeV/ c)
Generated events	26	51	1023
Fitted by linear approximation	21	30	426
Fitted by new method	16	23	264

Furthermore, the CPU time of the new algorithm is about half of the linear approximation in our case, because the costly matrix calculations are avoided.

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一种运动学拟合的新方法

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摘要 介绍了在事例重建中使用的一种运动学拟合的新方法. 在这种新方法中使用了 ragula falsi 方法来处理约束方程,代替了常用的线性近似法. 同时利用了 E705/E771 实验的 $J/\psi \to \mu^+ + \mu^-$ 蒙特卡罗数据检验了本方法,发现动量分辨率在新方法中有明显的提高.

关键词 运动学拟合 事例重建 约束方程

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