第 26 卷 第 3 期 高 能 物 理 与 核 物 理 Vol.26, No.3
2002 年 3 月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS Mar., 2002

量子水平的 Noether 恒等式

李子平
(北京工业大学应用数理学院 北京 100022)

摘要 基于 Green 函数的相空间生成泛函，导出了定域变换下的量子正则 Noether 恒等式；对规范不变系统，导出了位形空间中的量子 Noether 恒等式. 指出在某些情形下由量子 Noether 恒等式可导致系统的量子守恒律，这种求量子守恒律的方法与量子 Noether(第一)定理的程式不同. 用于非 Abel Chern-Simons (CS)理论，求出了 BRS 和 PBRS 守恒荷，这两个守恒荷完全不同.

关键词 路径积分 Noether 恒等式 守恒律 Chern-Simons 理论

1 引言

在经典理论中，系统作用量在有限连续群下的不变性(整体对称性)所联系的守恒律由 Noether 第一定理给出. 作用量在无限连续群下的不变性(定域对称性)涉及 Noether 第二定理，该定域不变性导致系统作用量的泛函微商满足某些微分恒等式(称 Noether 恒等式). Noether 恒等式在场论等诸多物理领域有重要应用[1]. 经典 Noether 恒等式是在位形空间中给出的，近来的工作已将它推广到非不变系统和非定域变换[2]；导出了相空间中的正则 Noether 恒等式[3,4]；用于杨-Mills 场论，求出了有别于 BRST 守恒荷的 PBRST 守恒荷[4,5]. 在导出这些守恒荷时，依据的是经典 Noether 恒等式，并结合了规范不变系统量子化的有效 Lagrange 量，是不彻底的量子理论. 前面的工作已给出了量子水平的正则 Noether 第一定理[6,7]. 本文将建立量子水平的 Noether 恒等式并给出初步应用. 相空间路径积分比位形空间路径积分更普遍[8]. 这里首先从系统 Green 函数的相空间生成泛函出发，导出了系统量子水平的正则 Noether 恒等式，对奇异 Lagrange 量系统，与经典情形不同的是：出现在量子正则 Noether 恒等式中的作用量为量子化后的有效正则作用量，而不是经典正则作用量. 其次，对规范不变系统，利用 Faddeev-popov(FP)方法写出系统 Green 函数的位形空间生成泛函，从该生成泛函出发，导出了位形空间中的量子 Noether 恒等式，与经典结果的差别是有效作用量代替了原始经典作用量. 文中讨论了在某些情形下，由量子 Noether 恒等式可导出量子守恒荷的问题，这与由量子 Noether 第一定理导出守恒荷的程式完全不同，这里实际上给出了一种求系统量子守恒荷的新方法. 最后讨论了理论结

2001-05-18 收稿

果在非 Abel CS 理论中的应用,求出了 BRS 和 PBRS 守恒荷,这两个守恒荷完全不同.

2 量子 Noether 恒等式

设场 $\varphi^a(x)$ 的 Lagrange 量 $\mathscr{L}(\varphi^a, \varphi^a_{,\mu})$ 是正规的[1], $\varphi^a_{,\mu} = \partial_\mu \varphi^a = \frac{\partial}{\partial x^\mu}\varphi^a$, 时空度规为 $g_{\mu\nu} = \mathrm{diag}(1 -1 -1 -1)(\mu, \nu = 0,1,2,3)$. 场 φ^a 的正则动量 $\pi_a = \partial\mathscr{L}/\partial\dot{\varphi}^a$. 在相空间中 φ^a 和 π_a 为独立变量. 系统的量子性质由 Green 函数的相空间生成泛函 Z 来描述, Z 可写为[9]

$$Z[J,K] = \int \mathscr{D}\varphi^a \mathscr{D}\pi_a \exp\left\{ \mathrm{i}\int \mathrm{d}^4 x(\mathscr{L}^P + J_a\varphi^a + K^a\pi_a)\right\}, \tag{2.1}$$

其中

$$\mathscr{L}^P = \pi_a\dot{\varphi}^a - \mathscr{H}_C, \tag{2.2}$$

$\mathscr{H}_C$ 为正则 Hamilton 量密度, J_a 和 K^a 分别为 φ^a 和 π_a 的外源. 这里对动量也引入了外源, 不影响对 Green 函数的计算[9]. 相空间路径积分比位形空间路径积分更基本,后者是前者的特殊情形[8]. 为简化记号,令 $\varphi = (\varphi^a)$, $\pi = (\pi_a)$, $J = (J_a)$, $K = (K^a)$, (2.1)式可写为

$$Z[J,K] = \int \mathscr{D}\varphi \mathscr{D}\pi \exp\left\{ \mathrm{i}\int \mathrm{d}^4 x(\mathscr{L}^P + J\varphi + K\pi)\right\}. \tag{2.3}$$

研究相空间生成泛函在整体对称变换的性质,导出了量子水平的正则 Noether 第一定理[6,7]. 从而得到相应的量子守恒荷. 下面研究系统在相空间中定域变换下的量子性质,考虑无穷小定域变换

$$\begin{cases} x^{\mu'} = x^\mu + \Delta x^\mu = x^\mu + R^\mu_\sigma \varepsilon^\sigma(x), \\ \varphi'(x') = \varphi(x) + \Delta\varphi(x) = \varphi(x) + S_\sigma\varepsilon^\sigma(x), \\ \pi'(x') = \pi(x) + \Delta\pi(x) = \pi(x) + T_\sigma\varepsilon^\sigma(x), \end{cases} \tag{2.4}$$

其中 R^μ_σ, S_σ 和 T_σ 为线性微分算符

$$R^\mu_\sigma = a^{\mu\nu(k)}_\sigma \partial_{\nu(k)}, \quad S_\sigma = b^{\nu(l)}_\sigma \partial_{\nu(l)}, \quad T_\sigma = c^{\nu(m)}_\sigma \partial_{\nu(m)}, \quad \nu(n) = \underbrace{\nu\lambda\cdots\rho\sigma}_{n},$$

$$\partial_{\nu(n)} = \partial_\nu\partial_\lambda\cdots\partial_\rho\partial_\sigma,$$

$a^{\mu\nu(k)}$, $b^{\nu(l)}$, $c^{\nu(m)}$ 均为 x, $\varphi(x)$ 和 $\pi(x)$ 的函数, $\varepsilon^\sigma(x)(\sigma = 1,2,\cdots,r)$ 为无穷小任意函数,它们的值及其微商在四维时空区域的边界上为零. 在(2.4)式变换下, φ 和 π 变换的 Jacobi 行列式记为 $\bar{J} = 1 + J_1[\varphi,\pi,\varepsilon]$. 正则作用量 $I^P = \int \mathrm{d}^4 x \mathscr{L}^P$ 在(2.4)式变换下的变更设为

$$\Delta I^P = \int \mathrm{d}^4 x U_\sigma \varepsilon^\sigma(x), \tag{2.5}$$

其中 $U_\sigma = f^{\nu(n)}_\sigma \partial_{\nu(n)}$, $f^{\nu(n)}$ 为 x, $\varphi(x)$ 和 $\pi(x)$ 的函数. 在(2.4)式变换下,相空间生成泛函可写为

$$\begin{aligned} J[J,K,\varepsilon] = \int \mathscr{D}\varphi\mathscr{D}\pi\Big(1 + J_1 + \mathrm{i}\Delta I^P + \mathrm{i}\int \mathrm{d}^4 x\{J\delta\varphi + K\delta\pi + \partial_\mu[(J\varphi + K\pi)\Delta x^\mu]\}\Big) \cdot \\ \exp\left\{\mathrm{i}\int \mathrm{d}^4 x(\mathscr{L}^P + J\varphi + K\pi)\right\}, \end{aligned} \tag{2.6}$$

其中[1]

$$\Delta I^P=\int d^4x\left\{\frac{\delta I^P}{\delta\varphi}+\frac{\delta I^P}{\delta\pi}\delta\pi+\frac{d}{dt}(\pi\delta\varphi)+\partial_\mu[(\pi\dot{\varphi}-H_C)\Delta x^\mu]\right\},\tag{2.7}$$

$$\frac{\delta I^P}{\delta\varphi}=-\dot{\pi}-\frac{\delta H_C}{\delta\varphi},\quad \frac{\delta I^P}{\delta\pi}=\dot{\varphi}-\frac{\delta H_C}{\delta\pi},\tag{2.8}$$

$$\delta\varphi=\Delta\varphi-\varphi_{,\mu}\Delta x^\mu,\quad \delta\pi=\Delta\pi-\pi_{,\mu}\Delta x^\mu,\tag{2.9}$$

H_C 为正则 Hamilton 量. 注意到 $\varepsilon^\sigma(x)$的边界条件,由(2.5),(2.6)和(2.7)式,有

$$\int\mathscr{D}\varphi\mathscr{D}\pi\left[\frac{\delta I^P}{\delta\varphi}\delta\varphi+\frac{\delta I^P}{\delta\pi}\delta\pi-U_\sigma\varepsilon^\sigma(x)\right]\exp\left\{i\int d^4x(\mathscr{L}^P+J\varphi+K\pi)\right\}=0.\tag{2.10}$$

将(2.4)和(2.9)式代入(2.10)式,对与 R_σ^μ, S_σ, T_σ 和 U_σ 相关的项作分部积分,利用 $\varepsilon^\sigma(x)$ 的边界条件,并对其结果关于 $\varepsilon^\sigma(x)$求泛函微商,得

$$\int\mathscr{D}\varphi\mathscr{D}\pi\left[\widetilde{S}_\sigma\left(\frac{\delta I^P}{\delta\varphi}\right)+\widetilde{T}_\sigma\left(\frac{\delta I^P}{\delta\pi}\right)-\widetilde{R}_\sigma^\mu\left(\varphi_{,\mu}\frac{\delta I^P}{\delta\varphi}+\pi_{,\mu}\frac{\delta I^P}{\delta\pi}\right)-\widetilde{U}_\sigma(1)\right]\cdot$$
$$\exp\left\{i\int d^4x(\mathscr{L}^P+J\varphi+K\pi)\right\}=0,\tag{2.11}$$

其中 $\widetilde{S}_\sigma$, $\widetilde{T}_\sigma$, $\widetilde{R}_\sigma^\mu$ 和 $\widetilde{U}_\sigma$ 分别为 S_σ, T_σ, R_σ^μ 和 U_σ 的伴随算符[10]

将(2.11)式关于 $J(x)$求 n 次泛函微商,得

$$\int\mathscr{D}\varphi\mathscr{D}\pi\left[\widetilde{S}_\sigma\left(\frac{\delta I^P}{\delta\varphi}\right)+\widetilde{T}_\sigma\left(\frac{\delta I^P}{\delta\pi}\right)-\widetilde{R}_\sigma^\mu\left(\varphi_{,\mu}\frac{\delta I^P}{\delta\varphi}+\pi_{,\mu}\frac{\delta I^P}{\delta\pi}\right)-\widetilde{U}_\sigma(1)\right]\varphi(x_1)\cdots\varphi(x_n)\cdot$$
$$\exp\left\{i\int d^4x(\mathscr{L}^P\quad J\varphi+K\pi)\right\}=0.\tag{2.12}$$

在(2.12)式中,让外源为零,$J=K=0$,有

$$\langle 0|T^*\left[\widetilde{S}_\sigma\left(\frac{\delta I^P}{\delta\varphi}\right)+\widetilde{T}_\sigma\left(\frac{\delta I^P}{\delta\pi}\right)-\widetilde{R}_\sigma^\mu\left(\varphi_{,\mu}\frac{\delta I^P}{\delta\varphi}+\pi_{,\mu}\frac{\delta I^P}{\delta\pi}\right)-\widetilde{U}_\sigma(1)\right]\varphi(x_1)\cdots\varphi(x_n)|0\rangle=\tag{2.13}$$

其中$|0\rangle$代表场的真空态,T^* 为一种特定的编时积[11],让 $t_1,t_2,\cdots,t_m\to+\infty$, $t_{m+1},t_{m+2},\cdots,t_n\to-\infty$,由(2.13)式,得

$$\langle \mathrm{out},m\left|\left[\widetilde{S}_\sigma\left(\frac{\delta I^P}{\delta\varphi}\right)+\widetilde{T}_\sigma\left(\frac{\delta I^P}{\delta\pi}\right)-\widetilde{R}_\sigma^\mu\left(\varphi_{,\mu}\frac{\delta I^P}{\delta\varphi}+\pi_{,\mu}\frac{\delta I^P}{\delta\pi}\right)-\widetilde{U}_\sigma(1)\right]\right|n-m,\mathrm{in}\rangle\tag{2.14}$$

由于 m, n 任意,从而得

$$\widetilde{S}_\sigma\left(\frac{\delta I^P}{\delta\varphi}\right)+\widetilde{T}_\sigma\left(\frac{\delta I^P}{\delta\pi}\right)-\widetilde{R}_\sigma^\mu\left(\varphi_{,\mu}\frac{\delta I^P}{\delta\varphi}+\pi_{,\mu}\frac{\delta I^P}{\delta\pi}\right)-\widetilde{U}_\sigma(1)=0.\tag{2.15}$$

(2.15)式为正规 Lagrange 量系统量子情形的正则 Noether 恒等式. 无论变换(2.4)的 Jacobi 行列式是否为 1,此结果均与经典情形的结果形式上相同[12].

设 Lagrange 量 $\mathscr{L}(\varphi^a,\varphi^a_{,\mu})$是奇异的[1],此时正则变量 φ^a, π_a 在相空间中存在固有约束,为约束正则系统. 设 $\Lambda_k(\varphi,\pi)\approx0$($k=1,2,\cdots,K_1$)为第一类约束,$\theta_i(\varphi,\pi)\approx0$($i=1,2,\cdots,I_1$)为第二类约束,与第一类约束相应的规范条件记为 $\Omega_k(\varphi,\pi)\approx0$($k=1,2,\cdots,K_1$). 此约束正则系统 Green 函数的相空间生成泛函为[7,13].

$$Z[J,K] = \int \mathscr{D}\varphi^a \mathscr{D}\pi_a \mathscr{D}\lambda_m \mathscr{D}\overline{C}\mathscr{D}C \exp\left\{ i\int d^4x(\mathscr{L}_{\text{eff}}^P + J_a\varphi^a + K^a\pi_a) \right\}, \tag{2.16}$$

其中

$$\mathscr{L}_{\text{eff}}^P = \mathscr{L}^P + \mathscr{L}_m + \mathscr{L}_{\text{gh}}, \tag{2.17}$$

$$\mathscr{L}_m = \lambda_i\theta_i + \lambda_k\Lambda_k + \lambda_l\Omega_l, \tag{2.18}$$

$$\mathscr{L}_{\text{gh}} = \int d^4y\left[\overline{C}_k(x)\{\Lambda_k(x),\Omega_l(y)\}C_l(y) + \frac{1}{2}\overline{C}_i(x)\{\theta_i(x),\theta_j(y)\}C_j(y) \right], \tag{2.19}$$

$\lambda_m = (\lambda_i, \lambda_k, \lambda_l)$,$\overline{C}(x)$和 $C(x)$为 Grassmann 变量,$\{\cdot,\cdot\}$代表场的 Poisson 括号. 为简化记号,令 $\varphi = (\varphi^a, \lambda, \overline{C}, C)$,$\pi = (\pi_a)$,$J = (J_a)$,$K = (K^a)$,这样(2.16)式可写为

$$Z[J,K] = \int \mathscr{D}\varphi\mathscr{D}\pi \exp\left\{ i\int d^4x(\mathscr{L}_{\text{eff}}^P + J\varphi + K\pi) \right\}. \tag{2.20}$$

上述对正规 Lagrange 量系统的讨论,同样适用于奇异 Lagrange 量系统,只要将相应的 I^P 换为 I_{eff}^P就是了. 在经典理论中,无论是正规 Lagrange 量系统或是奇异 Lagrange 量系统,经典正则 Noether 恒等式是一样的[1]. 在量子水平下,奇异 Lagrange 量系统的正则 Noether 恒等式虽然也具有(2.15)式的形式,但其中的 I^P 应改为 I_{eff}^P.

量子定域正则 Noether 恒等式在某些情形下,可导致系统的量子守恒律. 为了讨论在杨-Mills 场和 CS 理论中的应用,考虑如下无穷小定域变换

$$\begin{cases} \Delta x^\mu = 0 \\ \delta\varphi(x) = b_\sigma\varepsilon^\sigma(x) + b_\sigma^\mu\,\partial_\mu\varepsilon^\sigma(x), \\ \delta\pi(x) = c_\sigma\varepsilon^\sigma(x) + c_\sigma^\mu\,\partial_\mu\varepsilon^\sigma(x), \end{cases} \tag{2.21}$$

其中 b_σ,b_σ^μ,c_σ 和 c_σ^μ 均为 x,φ 和 π 的函数,$\varepsilon^\sigma(x)$为无穷小任意函数. 假设在(2.21)式变换下,有效正则 Lagrange 量 $\mathscr{L}_{\text{eff}}^P$的变更为

$$\delta\mathscr{L}_{\text{eff}}^P = U_\sigma\varepsilon^\sigma(x) = (u_\sigma + u_\sigma^\mu\,\partial_\mu + u_\sigma^{\mu\nu}\,\partial_\mu\partial_\nu)\varepsilon^\sigma(x), \tag{2.22}$$

其中 u_σ,u_σ^μ 和 $u_\sigma^{\mu\nu}$ 为正则变量的函数,例如,某些有质量杨-Mills 场论模型就属于这种情况. 此时量子正则 Noether 恒等式成为

$$b_\sigma\,\frac{\delta I_{\text{eff}}^P}{\delta\varphi} - \partial_\mu\Big(b_\sigma^\mu \qquad\qquad u_\sigma - \partial_\mu u_\sigma^\mu + \partial_\mu\partial_\nu u_\sigma^{\mu\nu}. \tag{2.23}$$

从有效正则作用量的变分

$$\frac{\delta I_{\text{eff}}^P}{\delta\varphi}(b_\sigma + b_\sigma^\mu\,\partial_\mu)\varepsilon^\sigma(x) + \frac{\delta I_{\text{eff}}^P}{\delta\pi}(c_\sigma + c_\sigma^\mu\,\partial_\mu)\varepsilon^\sigma(x) + \frac{d}{dt}[\pi(b_\sigma + b_\sigma^\mu\,\partial_\mu)\varepsilon^\sigma(x)] =$$
$$(u_\sigma + u_\sigma^\mu\,\partial_\mu + u_\sigma^{\mu\nu}\,\partial_\mu\partial_\nu)\varepsilon^\sigma(x). \tag{2.24}$$

用 $\varepsilon^\sigma(x)$乘(2.23)式后与(2.24)式相减,当 $u_\sigma^{\mu\nu}$ 关于 μ,ν 对称时,得

$$\partial_\mu\left[\left(b_\sigma^\mu\,\frac{\delta I_{\text{eff}}^P}{\delta\varphi} + c_\sigma^\mu\,\frac{\delta I_{\text{eff}}^P}{\delta\pi} - u_\sigma^\mu + \partial_\nu u_\sigma^{\mu\nu} - u_\sigma^{\mu\nu}\,\partial_\nu\quad\right)\varepsilon^\sigma(x)\right] +$$
$$\frac{d}{dt}[\pi(b_\sigma + b_\sigma^\mu\,\partial_\mu)\varepsilon^\sigma(x)] = 0. \tag{2.25}$$

将(2.25)式在 $t = \text{const}$ 的类空超曲面 V 上积分,得强守恒律

$$Q = \int_V j_\sigma \varepsilon^\sigma(x) \mathrm{d}^3 x = \mathrm{const}, \tag{2.26}$$

其中

$$j_\sigma = b_\sigma^0 \frac{\delta I_{\mathrm{eff}}^P}{\delta \varphi} + c_\sigma^0 \frac{\delta I_{\mathrm{eff}}^P}{\delta \pi} - u_\sigma^0 + \partial_\nu u_\sigma^{0\nu} - u_\sigma^{0\nu} \partial_\nu + \pi(b_\sigma + b_\sigma^\mu \partial_\mu). \tag{2.27}$$

当变换群有子群,且 $\varepsilon^\sigma(x) = \varepsilon_0^\rho \xi_\rho^\sigma(x)$,其中 ε_0^ρ 为连续群的参数,$\xi_\rho^\sigma(x)$为给定函数.例如 BRS 变换,以及在研究规范不变的能量——动量张量所涉及的变换等均属于这类情况,此时强守恒律为

$$Q_\rho = \int_V j_\sigma \xi_\rho^\sigma \mathrm{d}^3 x = \mathrm{const}. \tag{2.28}$$

导出(2.26)式和(2.28)式未利系统动力学方程.利用系统的量子运动方程[9],$\delta I_{\mathrm{eff}}^P / \delta\varphi = 0$,$\delta I_{\mathrm{eff}}^P / \delta\pi = 0$,由(2.26)或(2.28)式可得系统的量子(弱)守恒律.当系统的有效正则作用量在(2.21)式变换下不变时,(2.28)式恰好给出有限李群(整体变换)对称下的量子守恒律[6].这种导出量子守恒荷的程式与量子正则 Noether(第一)定理完全不同[7].

3 规范不变系统

规范不变系统为约束正则系统[1],该系统的量子化也可用 Faddeev-Popov(FP)方法来实现.设系统规范不变的 Lagrange 量为 $\mathscr{L}(\varphi, \varphi_{,\mu})$,选取适当的规范条件,用 FP 方法得到系统位形空间的生成泛函为[11]

$$Z[J] = \int \mathscr{D}\varphi \exp\left\{ \mathrm{i} \int \mathrm{d}^4 x (\mathscr{L}_{\mathrm{eff}} + J\varphi) \right\}, \tag{3.1}$$

其中 J 为 φ 的外源.有效 Lagrange 量可写为

$$\mathscr{L}_{\mathrm{eff}} = \mathscr{L} + \mathscr{L}_{\mathrm{fix}} + \mathscr{L}_{\mathrm{gh}}, \tag{3.2}$$

$\mathscr{L}_{\mathrm{fix}}$为规范固定项,它与规范条件有关,$\mathscr{L}_{\mathrm{gh}}$为鬼粒子项.对某些场论模型,(3.1)式也可按约束正则系统的相空间生成泛函,作出对正则动量的路径积分来得到(如杨-Mills 场[1]).

考虑无穷小定域变换

$$\begin{cases} x^{\mu'} = x^\mu + \Delta x^\mu = x^\mu + R_\sigma^\mu \varepsilon^\sigma(x), \\ \varphi'(x') = \varphi(x) + \Delta\varphi(x) = \varphi(x) + S_\sigma \varepsilon^\sigma(x), \end{cases} \tag{3.3}$$

其中 R_σ^μ 和 S_σ 为线性微分算符,$\varepsilon^\sigma(x)$为无穷小任意函数,它们的值及其微商在四维时空区域的边界上为零.在(3.3)式变换下,设有效作用量的变更为

$$\Delta I_{\mathrm{eff}} = \Delta \int \mathrm{d}^4 x \mathscr{L}_{\mathrm{eff}} = \int \mathrm{d}^4 x V_\sigma \varepsilon^\sigma(x), \tag{3.4}$$

其中 V_σ 为线性微分算符.在(3.3)式变换,场量变换的 Jacobi 行列式设为 $\bar{J} = 1 + J_1[\varphi, \varepsilon]$.生成泛函(3.1)在(3.3)式变换下为

$$Z[J, \varepsilon] = \int \mathscr{D}\varphi \left\{ 1 + J_1 + \mathrm{i}\Delta I_{\mathrm{eff}} + \mathrm{i} \int \mathrm{d}^4 x [J\delta\varphi + \partial_\mu (J\varphi\Delta x^\mu)] \right\} \cdot \exp\left\{ \mathrm{i} \int \mathrm{d}^4 x (\mathscr{L}_{\mathrm{eff}} + J\varphi) \right\}, \tag{3.5}$$

其中[1]

$$\Delta I_{\mathrm{eff}} = \int \mathrm{d}^4 x \left[\frac{\delta I_{\mathrm{eff}}}{\delta \varphi} \delta \varphi + \partial_\mu \left(\frac{\partial \mathscr{L}_{\mathrm{eff}}}{\partial \varphi_{,\mu}} \delta \varphi \right) + \partial_\mu (\mathscr{L}_{\mathrm{eff}} \Delta x^\mu) \right], \tag{3.6}$$

$$\frac{\delta I_{\mathrm{eff}}}{\delta \varphi} = \frac{\partial \mathscr{L}_{\mathrm{eff}}}{\partial \varphi} - \partial_\mu \frac{\partial \mathscr{L}_{\mathrm{eff}}}{\partial \varphi_{,\mu}}, \tag{3.7}$$

$$\delta \varphi = \Delta \varphi - \varphi_{,\mu} \Delta x^\mu \tag{3.8}$$

注意到 $\varepsilon^\sigma(x)$ 的边界条件，由(3.4)—(3.8)式，得

$$\int \mathscr{D}\varphi \left[\frac{\delta I_{\mathrm{eff}}}{\delta \varphi} (S_\sigma - \varphi_{,\mu} R^\mu_\sigma) \varepsilon^\sigma(x) - V_\sigma \varepsilon^\sigma(x) \right] \exp\left\{ \mathrm{i} \int \mathrm{d}^4 x (\mathscr{L}_{\mathrm{eff}} + J\varphi) \right\} = 0. \tag{3.9}$$

将(3.9)式中与微分算符 S_σ，R^μ_σ 和 V_σ 有关的项作分部积分，利用 $\varepsilon^\sigma(x)$ 的边界条件，然后

$$\frac{\delta I_{\mathrm{eff}}}{\delta \varphi} \Big) - \tilde{V}_\sigma \Big] \exp\left\{ \mathrm{i} \int \mathrm{d}^4 x (\mathscr{L}_{\mathrm{eff}} + J\varphi) \right\} = 0. \tag{3.10}$$

$J = 0$，并让 t 固定，$t_1, t_2, \cdots, t_m \to \infty$，$t_{m+1}$，$t_{m+2}, \cdots, t_n \to -\infty$，类似于上节(2.11)……)式的推导可得

$$\tilde{S}_\sigma \left(\frac{\delta I_{\mathrm{eff}}}{\delta \varphi} \right) - \tilde{R}^\mu_\sigma \left(\varphi_{,\mu} \frac{\delta I_{\mathrm{eff}}}{\delta \varphi} \right) - \tilde{V}_\sigma(1) = 0. \tag{3.11}$$

(3.11)式为规范不变系统在位形空间中定域变换下的量子 Noether 恒等式．与经典结果不同的是：出现在(3.11)式中的是有效作用量 I_{eff}，而不是经典作用量 I．

考虑下列无穷小定域变换

$$\begin{cases} \Delta x^\mu = 0, \\ \Delta \varphi(x) = (b_\sigma + b^\mu_\sigma \partial_\mu) \varepsilon^\sigma(x), \end{cases} \tag{3.12}$$

其中 $\varepsilon^\sigma(x)$ 为无穷小任意函数．假设在(3.12)式变换下，有效 Lagrange 量的变更为

$$\delta \mathscr{L}_{\mathrm{eff}} = V_\sigma \varepsilon^\sigma(x) = (v_\sigma + v^\mu_\sigma + v^{\mu\nu}_\sigma \partial_\mu \partial_\nu) \varepsilon^\sigma(x), \tag{3.13}$$

其中 v_σ，v^μ_σ，$v^{\mu\nu}_\sigma$ 均为 x，φ，$\varphi_{,\mu}$ 的函数．与上节讨论相似，由量子 Noether 恒等式和有效作用量 I_{eff} 变分的基本恒等式可得系统的强守恒律．当 $\varepsilon^\sigma(x) = \varepsilon^0_\rho \zeta^\rho_\sigma$ 时，ε^0_ρ 为参数，ζ^ρ_σ 为场量的函数，此时强守恒荷为

$$Q_\sigma = \int \mathrm{d}^3 x \left[\frac{\partial \mathscr{L}_{\mathrm{eff}}}{\partial \varphi_{,0}} (b_\rho + b^\nu_\rho \partial_\nu) \zeta^\rho_\sigma + b^0_\rho \frac{\delta I_{\mathrm{eff}}}{\delta \varphi} \zeta^\rho_\sigma - v^0_\rho \zeta^\rho_\sigma + (\partial_\nu v^{0\nu}_\rho - v^{0\nu}_\rho \partial_\nu) \zeta^\rho_\sigma \right]. \tag{3.14}$$

利用系统的量子运动方程[11]，$\delta I_{\mathrm{eff}} / \delta \varphi = 0$，由(3.14)式可得量子弱守恒荷．当系统的有效作用量 I_{eff} 在(3.12)式变换下不变时，此时的弱守恒荷恰好是系统整体对称下的量子守恒荷[14]

4　非 Abel CS 理论

非 Abel CS 项与标量场耦合的整体量子对称性已给出[15]，现在研究带 Maxwell 项的非 Abel CS 项与标量场耦合的定域变换下的性质，其 Lagrange 量为

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + (D_\mu\varphi)^+(D^\mu\varphi) + \frac{\kappa}{4\pi}\varepsilon^{\mu\nu\rho}\left(\partial_\mu A^a_\nu A^a_\rho + \frac{1}{3}f^a_{bc}A^a_\mu A^b_\nu A^c_\rho\right), \tag{4.1}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^a_{bc}A^b_\mu A^c_\nu, \tag{4.2}$$

其中 D_μ 代表协变微商. 理论的规范不变性要求 $\kappa = n/4\pi$(n 为整数)[16].

场 A^a_μ,φ 和 φ^+ 的正则动量记为 π^μ_a,π 和 π^+. 系统在相空间的约束为

$$\Lambda^a_1 = \pi^0_a \approx 0, \tag{4.3}$$

$$\Lambda^a_2 = D_i\pi^i_a + \frac{\kappa}{4\pi}\varepsilon^{ij}\partial_i A^a_j \approx 0, \tag{4.4}$$

$\Lambda^a_1 \approx 0$, $\Lambda^a_2 \approx 0$ 为第一类约束. 选取 Coulomb 规范

$$\Omega^a_2 = \partial^i A^a_i \approx 0. \tag{4.5}$$

由 Ω^a_2 的自洽性要求,$\dot{\Omega}^a_2 \approx 0$,可确定另一规范条件

$$\Omega^a_1 = \partial^i\pi^a_i + \nabla^2 A^a_0 - f^a_{bc}A^b_i\,\partial^i A^c_o \approx 0. \tag{4.6}$$

不难求出 $\det\{\Lambda^a, \Omega^b\} = \det M^{ab}_0$,其中

$$M^{ab}_0 = (\delta^{ab}\nabla^2 - f^a_{bc}A^c_i\,\partial^i)\delta(x-y). \tag{4.7}$$

理论的规范无关性,生成泛函中的因子 $\delta(\partial^i A^a_i)\det M^{ab}_0$ 可用因子 $\delta(\partial^\mu A^a_\mu)\det M^{ab}_L$ 来代替[17,18],

$$M^{ab}_L = (\delta^{ab}\partial^2 - f^a_{bc}A^c_\mu\,\partial^\mu)\delta(x-y). \tag{4.8}$$

按 Faddeev-Senjanovic 路径积分量子化方案,相空间生成泛函可写为[19]

$$Z[J] = \int \mathscr{D}A^a_\mu \mathscr{D}\pi^\mu_a \mathscr{D}\varphi \mathscr{D}\pi \mathscr{D}\varphi^+ \mathscr{D}\pi^+ \mathscr{D}\lambda \mathscr{D}\overline{C}^a \mathscr{D}C^a \cdot$$

$$\exp\left\{i\int d^3x\,(\mathscr{L}^P_{\text{eff}} + J^\mu_a A^a_\mu + J^+\varphi + \varphi^+ J + \overline{J}_a C^a + \overline{C}^a J_a)\right\}, \tag{4.9}$$

其中

$$\mathscr{L}^P_{\text{eff}} = \mathscr{L}^P + \mathscr{L}_g + \mathscr{L}_{gh} + \mathscr{L}_m, \tag{4.10}$$

$$\mathscr{L}_g = -\frac{1}{2\alpha_2}(\partial^\mu A^a_\mu)^2, \tag{4.11}$$

$$\mathscr{L}_{gh} = -\partial^\mu\overline{C}^a D^a_{b\mu}C^b \quad (D^a_{b\mu} = \delta^a_b\,\partial_\mu - f^a_{bc}A^c_\mu), \tag{4.12}$$

$$\mathscr{L}_m = \lambda^a_1\Lambda^a_1 + \lambda^a_2\Lambda^a_2 - \frac{1}{2\alpha_1}(\Omega^a_1)^2, \tag{4.13}$$

而 $\mathscr{L}^P$ 为正则 Lagrange 量. 考虑 BRS 变换

$$\begin{cases} \delta\varphi = -i\tau T^a C^a\varphi, \quad \delta\varphi^+ = i\tau\varphi^+ T^a C^a, \\ \delta C^a = \dfrac{1}{2}f^a_{bc}C^b C^c, \quad \delta\overline{C}^a = -\dfrac{1}{\alpha_2}\partial^\mu A^a_\mu, \\ \delta A^a_\mu = -\tau D^a_{b\mu}C^b, \end{cases} \tag{4.14}$$

其中 τ 为 Grassmann 参数($\varepsilon^a(x) = \tau C^a(x)$),$T^a$ 为规范群的生成元. 在 BRS 变换下,$\mathscr{L}^P + \mathscr{L}_g + \mathscr{L}_{gh}$ 不变,A^a_μ 的规范变换不离开第一类约束确定的超曲面[9],$\delta\mathscr{L}_m \approx 0$. 因此在约束超曲面内 I^P_{eff} 在 BRS 变换下不变,由(2.28)式可得量子水平的 BRS(弱)守恒荷:

$$Q_B = \int d^2x\,(\pi^\mu_a\delta A^a_\mu + \pi\delta\varphi + \delta\varphi^+\pi^+ + \overline{R}_a\delta C^a + \delta\overline{C}^a R_a), \tag{4.15}$$

其中 $\overline{R}_a$,R_a 分别为 C^a,$\overline{C}^a$ 的正则动量.

如果仅考虑规范场 A_μ^a 和标量场 φ,φ^+ 变换,而鬼场保持不变,

$$\begin{cases}\delta\varphi = -\mathrm{i}\tau T^a C^a \varphi, & \delta\varphi^+ = \mathrm{i}\tau\varphi^+ T^a C^a, \\ \delta A_\mu^a = -\tau \mathrm{D}_{b\mu}^a C^b, & \delta C^a = \delta \overline{C}^a = 0.\end{cases} \tag{4.16}$$

在约束确定的超曲面内,$\mathscr{L}_{\mathrm{eff}}^P$ 在(4.16)式变换下的变更为

$$\delta\mathscr{L}_{\mathrm{eff}}^P = V_a \varepsilon^a(x) = F_a \varepsilon^a(x) + f_{bc}^a \partial^\mu \overline{C}^a C^b \partial_\mu \varepsilon^c(x), \tag{4.17}$$

其中 F_a 不依赖于 $\varepsilon^a(x)$ 的微商. 按(2.28)式得(弱)守恒荷

$$Q = \int \mathrm{d}^2 x(\pi_a^\mu \mathrm{D}_{b\mu}^a C^b + \pi\delta\varphi + \delta\varphi^+ \pi^+ - f_{bc}^a \dot{\overline{C}}{}^a C^b C^c). \tag{4.18}$$

此守恒荷 Q 与 BRS 守恒荷 Q_B 不同,不妨称 Q 为 PBRS 守恒荷. 在杨-Mills 场论中,也得到类似的结果[5],但导出该结果时,未完全用量子理论.

上述守恒荷 Q_B 和 Q 也可第 3 节所述的位形空间的结果来导出.

参考文献(References)

1 LI Z P. Classical and Quantal Dynamics of Constrained System and Their Symmetry Properties, Beijing, Beijing Polytechnic University Press, 1993(in Chinese)
(李子平. 经典和量子约束系统及其对称性质,北京:北京工业大学出版社,1993)

2 LI Z P. Int. J. Theor. Phys., 1995,**34**:1945

3 LI Z P. Science in China (Scientia Sinica), Series A, 1993,**36**:1212

4 LI Z P. Phys. Rev., 1994, **E50**:876

5 LI Z P. Int. J. Theor. Phys., 1994,**33**:1207

6 LI Z P. Science in China (Scientia Sinica), Series A, 1996, **39**:739

7 LI Z P. Z. Phys., 1997, **C76**:181

8 Mizrahi M M. J. Math. Phys., 1978, **19**:298

9 LI Z P. Int. J. Theor. Phys., 1995, **34**:523

10 LI Z P. Int. J. Theor. Phys., 1987, **26**:853

11 Young B L, Introduction to Quantum Field Theories, Beijing, Science Press, 1987

12 LI Z P. Int. J. Theor. Phys., 1993, **32**:201

13 LI Z P. JIANG J H. Symmetries in Constrained Canonical Systems, Beijing, Science Press, 2002

14 LI Z P, GAO H X. Int. J. Theor. Phys., 1997, **36**:1071

15 LI Z P, Chinese Science Bulletin, 1999, **44**:207

16 Deser S, Jackiw R, Templeton S. Ann. Phys.,1982,**140**:372

17 Sundermeyer K. Lecture Notes in Physics, 169, Berlin, Springer-Verlag, 1982

18 Foussats A, Manavella E, Repetto C et al. Int. J. Theor. Phys., 1995, **34**:1037;J. Math. Phys., 1996, **37**:84

19 LI Z P, LONG Z W. J. Phys., 1999,**A32**:6391

Noether Identities at the Quantum Level

LI Zi-Ping
(College of Applied Science, Beijing Polytechnic University, Beijing 100022, China)

Abstract Based on the phase-space generating functional of Green function, the canonical Noether identities under the local transformation at the quantum level have been derived. For the gauge-invariant system, the quantal Noether identities in configuration space have been also deduced. It is pointed out that in certain cases the quantal Noether identities may be converted to quantal conservation laws of the system. This method for obtaining the quantal conservation laws is significantly different from the first Noether theorem at the quantum level. The application to non-Abelian CS theories is studied, the quantal conserved BRS and PBRS charges are obtained, and these two conserved charges are totally different.

Key words path integral, Noether identity, conservation law, Chern-Simons theories

Received 18 May 2001