Low-Lying Spectra and E2 Transition Rates in Even-even Ce Isotopes in the Interacting Boson Model*

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Abstract The positive parity collective states in the even-even $^{128-150}$Ce isotopes are studied in the framework of the interacting boson model. A schematic Hamiltonian able to describe their spectra and transition is used. It is found that both the light and heavy even Ce isotopes are in the transition from the vibrational limit to the rotational limit. From ^{146}Ce onward, the isotopes are nearly perfect rotors.

Key words spectra, electromagnetic transition, positive parity low-lying collective state

1 Introduction

More than twenty years ago, Arima and Iachello put forward the interacting boson model (IBM). In the IBM, valence nucleon pairs are treated as bosons. It is a very effective phenomenological model for describing low-lying collective properties of nuclei across an entire major shell. For these nuclei in the xenon-barium-cerium mass region, the excitation spectra of the Xe-Ba and ^{132,134}Ce nuclei was approximated by the dynamical symmetry $O(6)$ of the IBM-1[1—5], which is analogous to the gamma-unstable rotor model. It has been recently shown[6,7] that one can describe the low-lying structure of Xe and Ba isotopes by the transition from $U(5)$ to $SU(3)$. This raises the interesting question whether Ce isotopes can also be described by $U(5)$ to $SU(3)$ transition. Meanwhile for the nuclei in the cerium-neodymium with mass numbers around 150 region, it is found that Nd isotopes are in the transition from the vibrational limit to the rotational limit[8—11], and from ^{152}Nd onward, the isotopes are nearly perfect rotors[10]. In this study, we also check if Ce isotopes with mass numbers around 150 can also be described by a transition from $U(5)$ to $SU(3)$.

The structure of these isotopes is studied in the framework of the interacting boson model, the IBM-1, where no distinction is made between neutron boson and proton bosons. As has been showed[8], this simple IBM-1 gives a very good approximation to the symmetric states of the neutron-proton interacting boson model. Usually the low-lying levels are dominantly the symmetric states. Systematics is important in the study of the properties of nuclei[12—14]. We analyzed the systematics of the spectra and electromagnetic transitions of the even Ce isotopes in this work. Finally it was found from this work that the Ce isotopes

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could be well described by a $U(5)$ to $SU(3)$ transition.

The paper is divided as follows. After this short introduction, we describe briefly the model Hamiltonian and the E2 transition operator in sect 2. In sect 3, we give the results and discussion on spectrum and E2 transition properties. Finally, in sect 4, a conclusion is given.

2 Schematic IBM Hamiltonian

The general IBM Hamiltonian contains 7 terms. However, for our study, we take the following schematic Hamiltonian[15].

$$\hat{H} = \varepsilon_d \hat{n}_d + KQ \cdot Q + K_L L \cdot L,$$

where
$$Q_\mu = (s^+ \tilde{d} + d^+ s)^2 + \chi(d^+ \tilde{d})^2_\mu,$$

and
$$L_q = \sqrt{10}(d^+ \tilde{d})^{(1)}_q, \quad \chi = -\frac{\sqrt{7}}{2}.$$

This Hamiltonian is able to give a transition from $U(5)$ to $SU(3)$, if $\varepsilon_d = 0$, then the Hamiltonian reduces to an $SU(3)$ limit Hamiltonian. If $K = 0$, the Hamiltonian becomes a $U(5)$ limit, describing the vibrational collective motion. $K_L(L \cdot L)$ term removes some of the degeneracy for different L values. Therefore the ratio of K/ε_d is a measure of the transition between $U(5)$ and $SU(3)$. If $K/\varepsilon_d = 0$, the Hamiltonian is vibrational, and if this ratio is ∞, the Hamiltonian is rotational. In between, the Hamiltonian is in the transition between $U(5)$ and $SU(3)$. The parameters in the Hamiltonian can be determined by fitting to the experimental spectra. After the determination of the spectra, the wave function is determined. The electric and magnetic transition properties can then be obtained accordingly. For example, the E2 transition operator is

$$T(E2)^2_\mu = e_2[(s^+ \tilde{d} + d^+ s)^2_\mu + \chi(d^+ \tilde{d})^2_\mu]$$

Microscopically, the transition operator can be derived from shell model by the mapping procedure[16—19]. In practice, it is more convenient to treat them as free parameters. Here we adopted the consistent $Q \cdot Q$ Formalism[20]. As is knew, this convention is not an essential requirement of the model, and sometimes, it is even necessary to use a different Q operator in E2 transition calculation to describe the E2 transitions[21,22], for example, the structure for neutron-rich and neutron-deficient Sr nuclei[23—25], neutron-rich Cd nuclei[26], and neutron-deficient and neutron-rich Nd nuclei[10,11]. Noticeably, the reduction in collectivity problem can be solved by using an operator in the transition different from that in the Hamiltonian[27—29]. However in many cases, the consistent $Q \cdot Q$ Formalism can give a good first description of the E2 transition properties. Since there are few experimental data available, we adopt the consistent $Q \cdot Q$ Formalism in a first place. When there are more experimental data on the E2 transition in the future, one can fine-tune the E2 transition operators to reproduce the details.

3 Result and Discussion

In table 1, we give the parameters of the Hamiltonian and of the E2 transition operator in each nucleus studied. From table 1, the values for ε_d, K, K_L, and e_2 are rather constant. For ε_d in the lighter even Ce isotopes, with the exception of ^{130}Ce, the value increases with increasing mass number, until ^{138}Ce. In the heavier even Ce isotopes, ε_d value decreases with increasing mass number. But for K, K_L and e_2 in the lighter even Ce isotopes, these values decrease with increasing mass number until ^{136}Ce and ^{138}Ce. In the heavier even Ce isotopes, these values increase with increasing mass number. It reflects the transition character of the dynamical symmetries in even Ce isotopes.

Table 1 Parameters of energy levels and B(E2) operator for Ce isotopes.

Nucleus	ε_d(MeV)	K(MeV)	K_L(MeV)	e_2(efm^2)
^{128}Ce	0.205	−0.0050	0.0190	15.8
^{130}Ce	0.422	−0.0140	0.0175	14.5
^{132}Ce	0.392	0.0123	0.0194	16.2
^{134}Ce	0.400	0.0050	0.0185	15.0
^{136}Ce	0.500	−0.0025	0.0175	5.00
^{138}Ce	0.724	−0.0044	0.0160	13.3
^{142}Ce	0.600	−0.0050	0.0100	13.2
^{144}Ce	0.295	−0.0050	0.0150	13.8
^{146}Ce	0.224	−0.0050	0.0150	14.2
^{148}Ce	0.165	−0.0050	0.0125	17.6
^{150}Ce	0.090	−0.0050	0.0120	16.8

Table 2 Comparison of B(E2) values in Ce nuclei.

Nucleus	I_i	I_f	Exp(e^{2}fm^4)	Cal(e^{2}fm^4)
^{128}Ce	2_1^+	0_1^+	4360	4350
	4_1^+	2_1^+	7240	7220
	6_1^+	4_1^+	5590	8420
	8_1^+	6_1^+	<5320	8810
	10_1^+	8_1^+	5130	8640
^{130}Ce	2_1^+	0_1^+	3480	3650
	4_1^+	2_1^+	5590	5530
	6_1^+	4_1^+	3590	6120
	8_1^+	6_1^+	>4570	6140
^{132}Ce	2_1^+	0_1^+	3790	3390
	4_1^+	2_1^+	3550	5390
	6_1^+	4_1^+	5580	6050
	8_1^+	6_1^+	2670	6040
	10_1^+	8_1^+	1750	5540
^{134}Ce	2_1^+	0_1^+	2060	2045
^{136}Ce	2_1^+	0_1^+	>2.2	16.1
	4_1^+	2_1^+	330	324
	6_1^+	4_1^+	>0.2	456
	10_1^+	8_1^+	7	488
^{138}Ce	2_1^+	0_1^+	92	92.7
	10_1^+	8_1^+	0.4	88
^{142}Ce	2_1^+	0_1^+	920	930
	4_1^+	2_1^+	1160	1460
	2_2^+	2_1^+	>5	1440
	2_3^+	2_1^+	310	1
	2_3^+	0_1^+	140	0
^{146}Ce	2_1^+	0_1^+	1860	1826
^{148}Ce	2_1^+	0_1^+	3950	3920
^{150}Ce	2_1^+	0_1^+	5580	5680

Using these parameters, we calculate the energy levels and B(E2) value(ratios) for each nucleus. The comparisons between calculated and experimental energy levels and B(E2) value[30—36] for $^{128-150}$Ce are shown in Figs. 1 − 6 and table 2, respectively. In general, the agreement is quite good, especially for the ground state band levels with $J^{\pi} \leqslant 8^{+}$, despite some discrepancies. In general, the lighter even Ce isotopes exhibit staggering in the gamma band. However, the staggering phenomenon in calculation is stronger than that in experiment. The agreement between the calculated and experimental data may be improved by the use of cubic terms[37] in IBM-1 and the quadrupole interaction between like nucleons[38—41] in IBM-2.

3.1 $^{128-136}$Ce

The calculated spectra and experimental spectra are compared in Figs. 1—3. The energy spectra in all five nuclei are reproduced fairly well. The yrast states 2^{+}, 4^{+}, 6^{+}, 8^{+} and second 2^{+} state are in more or less correct positions, although the yrast bands are little bit too stretched. The two-phonon states are slightly split in energy, which may be understood by means of a small anharmonic term in the vibration. The presumed three-phonon states, however, have a much larger energy splitting, with the exception of ^{130}Ce, with the highest and lowest states within a multiplet several hundreds of keV apart. It is difficult to envisage such a large energy splitting as caused by anharmonicties alone.

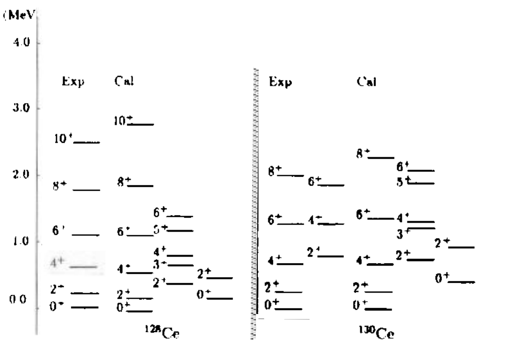

Fig. 1　Spectra for ^{128}Ce and ^{130}Ce.

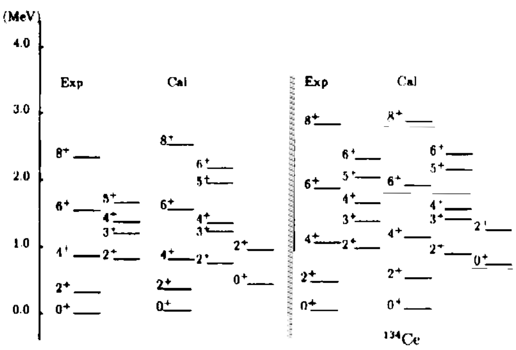

Fig. 2　Spectra for ^{132}Ce and ^{134}Ce.

The low spin states of the lighter even Ce isotopes are interesting for several reasons. In the chart of nuclides the light even Ce isotopes are located in a part of the $A \sim 130$ region where the nuclei may start to form a transition path to strongly quadrupole deformed shapes with a more rigid triaxiality than in xenon and barium nuclei. We compare the low spin spectra of the lighter even Ce isotopes with the spectra of the neighboring Xe and Ba isotopes. The main features of the spectra, the occurrence of a quasi-gamma band, are quite similar. Independently of specific model, this similarity proves the collective character of the low-lying levels in these nuclei. However, the staggering of the levels in the quasi-gamma band is much less pronounced in the light-

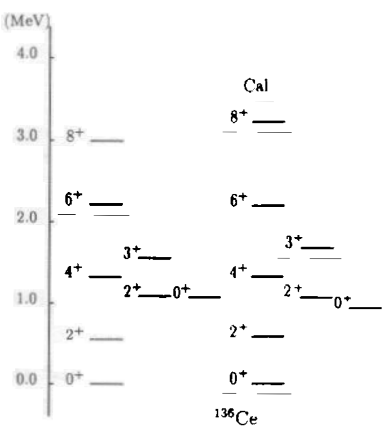

Fig. 3　Spectum for ^{136}Ce.

er even Ce isotopes than in the xenon and barium nuclei. This indicates a stronger triaxial rigidity. It is also interesting to note that in ^{136}Ce, there is a backbending in the ground state band, which maybe a candidate of the collective backbending mechanism put forward in Ref[42].

3.2 $^{138-142}$Ce

The calculated spectra and experimental spectra are compared in Fig.4. In both nuclei, the quality of agreement between theory and experimental data is good. The 6_1^+ and 3_1^+ states become too high and the 0_3^+ is visibly too low for ^{138}Ce. The 0_3^+ state is treated as an intruder state. For ^{142}Ce, we find the calculated energies of the 2_2^+ and 0_3^+ are very smaller than those of the experimental 2_2^+ and 0_3^+ states. As a consequence, they are considered intruder states also. Both nuclei are vibrational. This is also true in our calculation by the relatively large ε_d value.

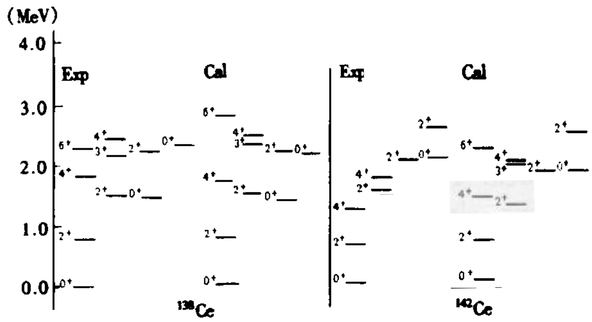

Fig.4 Spectra for ^{138}Ce and ^{142}Ce.

3.3 $^{144-150}$Ce

The calculated spectra and experimental spectra are compared in Fig. 5 and Fig. 6. There is no information for the side bands in ^{150}Ce. The present calculation gives very good reproduction of the ground-state band. The quasi-beta and quasi-gamma are reasonably well reproduced. In particular, the higher spin states in the ground state band up to 10^+ are well reproduced for the $^{146-148}$Ce. In comparison with ^{138,142}Ce, the ε_d values in $^{144-150}$Ce have a big drop. This makes $^{144-150}$Ce closer to the rotational limit. From ^{146}Ce onward, the isotopes are nearly perfect rotors.

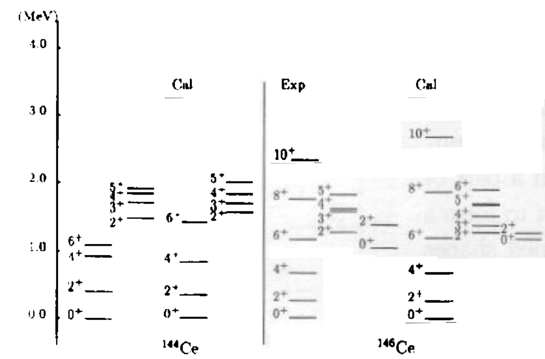

Fig.5 Spectra for ^{144}Ce and ^{146}Ce.

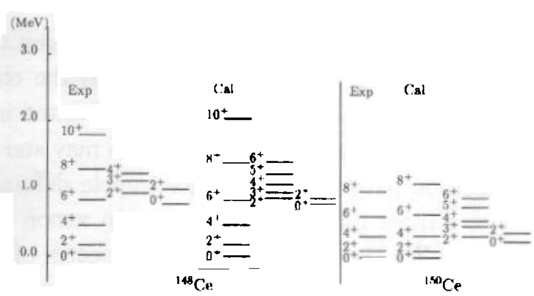

Fig.6 Spectra for ^{148}Ce and ^{150}Ce.

In addition, to explore the transitional properties in the Ce isotopes, we analyzed the systematic of the spectra and electromagnetic transition properties. They are given in Table 3, where the ratios $R = E(2_2^+)/E(2_1^+)$, $R_{4/2} = E(4_1^+)/E(2_1^+)$, $R_{6/2} = E(6_1^+)/E(2_1^+)$, $R_1 = B(\mathrm{E2};4_1^+ \to 2_1^+)/B(\mathrm{E2};2_1^+ \to 0_1^+)$, $R_2 = B(\mathrm{E2};2_2^+ \to 0_1^+)/B(\mathrm{E2};2_2^+ \to 2_1^+)$ are given.

It is obvious that the R reflects vibrator to a stable deformed character in the even Ce isotopes. Similar results are found for $R_{4/2}$ and $R_{6/2}$ values.

Table 3 Comparison of the experimental and calculated values for $^{128-150}$Ce.

	R		$R_{4/2}$		$R_{6/2}$		R_1		R_2	
Nucleus	Exp	Cal	Exp	Cal	Exp	Cal	Exp	Cal	Exp	Cal
^{128}Ce		2.1	2.9	2.9	5.59	5.7	1.66	1.66		0.150
^{130}Ce	3.3	1.9	2.7	2.6	5.2	4.9	1.60	1.52		0.060
^{132}Ce	2.5	2.5	2.6	2.7	4.7	5.2	0.94	1.59		0.150
^{134}Ce	2.3	1.8	2.5	2.3	4.5	4.1		2.34		0.153
^{136}Ce	1.9	1.8	2.3	2.3	4.0	3.7	<150	2.01		0.030
^{138}Ce	1.9	1.9	2.3	2.2	2.9	3.5		1.61		0.001
^{142}Ce	2.4	1.9	1.9	2.1		3.4	1.26	1.57		0.003
^{144}Ce	3.8	4.6	2.4	2.3	2.9	4.0		1.63		0.010
^{146}Ce	4.9	5.0	2.6	2.6	4.5	4.8		1.64		0.040
^{148}Ce	6.2	5.6	2.8	2.9	5.3	5.7		1.61		0.140
^{150}Ce		3.3	3.2	3.3	6.3	6.9		1.39		0.300

Besides the energy levels, we are also interested in the electromagnetic properties of Ce isotopes, particularly in the electric E2 transition. Table 2 and Table 3 give the comparison between calculated and experimental B(E2) values and ratios for the three limiting symmetries in all the nuclei. Results obtained in the present work are in good agreement with experiment. This reflects a transition from $U(5)$ to $SU(3)$.

4 Conclusion

We have given, in schematic way, a detailed study of the energy levels and E2 transition of Ce isotopes in the IBM-1. A good agreement is obtained for both the spectra and the E2 transition. The even Ce isotopes are in the vibrational to rotational transition. From ^{146}Ce onward, the spectra are well described by the $SU(3)$ limit. It is remarkable properties of the even Ce isotopes. More data in future experiment, especially B(E2) values, will be important in verifying our conclusion.

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$^{128-150}$Ce偶偶核的低能谱和电磁跃迁的相互作用玻色子模型*

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摘要 采用相互作用玻色子模型研究了$^{128-150}$Ce同位素核的低能正宇称态的能谱和电磁跃迁. 应用简单的哈密顿量很好的描述它们的能谱和E2跃迁. 研究表明,这些偶偶Ce核是属于$U(5)$到$SU(3)$过渡核,而^{146}Ce以后的核基本是完全的转动核.

关键词 能谱 电磁跃迁 低能正宇称集体态

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