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On the Intermittency and Chaos in High Energy Collisions*

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Abstract It is shown that an event sample from the Monte Carlo simulation of a random cascading α model with fixed dynamical fiuctuation strength is intermittent but not chaotic, while the variance of dynamical fluctuation strength in different events will result in both the intermittency and the chaoticity behavior. This shows that fractality and chaoticity are two connected but different features of non-linear dynamics in high energy collisions.

Key Words chaos, entropy index, intermittency, factorial moments, α model

For classical system, the description of non-linear behavior is well established. It has been known by lattice calculation that the classical non-Abelian gauge theory generally exhibits deterministic chaos and that the Lyapunov exponent can be numerically determined[1]. But for quantum system, because of the ambiguity associated with quantum chaos in the realm of quantization, nonconservation of the number of degrees of freedom and lack of a meaningful definition of a trajectory, there is no corresponding description established. The study of non-linear behavior in high energy physics has, therefore, to be started phenomenologically.

The first signal of such a behaviour came from the unexpectedly large local fluctuations in a single event of very high multiplicity recorded by the JACEE collaboration[2]. Such large fluctuations may not be simply due to statistical reason and was taken as a signal of the existence of non-linear dynamical fluctuations. It was soon realized that the idea can be applied to events of any multiplicity provided that a proper average of factorial moments is performed, as done in the pioneer work[3] of Bialas and Peschanski. These authors have been able to show that, if the statistical fluctuations are of Bernouli (fixed multiplicity case) or Poisson (variable multiplicity case) type, the averaged factorial moments F_q are equal to the averaged dynamical probability moments C_q. The anomalous scaling of the latter has taken the name of intermittency (or fractal). This led to extensive experimental studies[4], and the expected anomalous scaling has been observed successfully in the experiments[5].

It should be realized, however, that the averaging procedure, apart from its clear advantages, brings also a danger of losing some important information on the spatial

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patterns from event to event. In particular, some interesting effects, if existing only in part of the events in high-energy collisions, may be missed. A possible example of this kind is the quark-gluon plasma which is expected to be characterized by specific intermittency exponents[6]. It seems therefore important and urgent[7] to study the fluctuation of single-event moments $C_q^{(e)}$ inside an event sample[1]. This fluctuation is related to the chaotic behavior of the system[7]. A quantity μ_q called entropy index can be introduced[8] as an adequate parameter in measuring the chaotic behavior. The positivity of entropy index $\mu_q > 0$ is proved to be a criterion for chaos[9].

Thus, two kind of non-linear phenomena—fractality (intermittency) and chaoticity have been proposed in high energy collisions. In this short note we will study the relation between them using Monte Carlo simulation of a random cascading α model. We will show that the anomalous scaling of the averaged probability moments (fractality or intermittency) and that of the event-space moments of single-event ones (chaoticity) are two connected but different features of non-linear dynamics. The system will exhibite both the fractal and the chaos behaviour only when the dynamical fluctuation strength is not fixed but is distributed over a certain range.

Let us first recall briefly the study of fractality (intermittency) in high energy collisions. This study is performed through the observation of anomalous scaling of averaged factorial moments F_q, which is equal to the averaged probabilty moments C_q

$$F_q(M) = C_q(M) \equiv \frac{1}{M} \sum_{i=1}^{M} \frac{\langle p_i^q \rangle}{\langle p_i \rangle^q} \propto M^{\phi_q} , \tag{1}$$

where a phase space region Δ is divided into M sub-cells, p_i is the probability for a particle to fall in the ith sub-cell.

For a flat inclusive distribution the moment $C_q^{(e)}$ for each event is defined as

$$C_q^{(e)} = M^{q-1} \sum_{i=1}^{M} (p_i^{(e)})^q . \tag{2}$$

We can now consider $C_q^{(e)}$ not only through its average—intended to get a better estimate of the hypothetical anomalous scaling of single-bin moments, cf. eqn. (1)—but also as a pattern-descriptor for particle fluctuations inside bins (just one among the many that could be devised).

$C_q^{(e)}$ may fluctuate greatly from event to event. In a sample consisting of a large number N of events, we get a distribution of $C_q^{(e)}$, denoted by $P(C_q^{(e)})$, which is normalized to unity. The conventionally defined factorial moments, cf eqn. (1), give only an estimate of the mean of $P(C_q^{(e)})$. By taking the normalized moments of $P(C_q^{(e)})$ in event-space defined as

$$C_{p,q} = \langle C_q^{(e)\,p} \rangle / \langle C_q^{(e)} \rangle^p , \tag{3}$$

1) It has been shown[3] that the statistical fluctuations can be eliminated by using the factorial moments averaged over event sample. However, the extension of this method to single-event moments is highly non-trivial. It is easy to show that the elimination of statistical fluctuations in single event factorial moments $F_q^{(e)}$ is incomplete. In order to avoid the complication caused by statistical fluctuations we will in this paper restrict ourself to the study of probability moments $C_q^{(e)}$ directly.

we have a quantification of the fluctuation of the spatial patterns, i.e. we can investigate the full shape of the distribution and, especially, the way it changes with the resolution $\delta = \Delta / M$. The value of p can be any positive real number. If $C_{p,q}(M)$ has a power law behaviour in M, i.e.

$$C_{p,q}(M) \propto M^{\psi_q(p)} , \tag{4}$$

then a new entropy index can be defined as,

$$\mu_q = \frac{\mathrm{d}}{\mathrm{d}p} \psi_q(p) \Big|_{p=1} . \tag{5}$$

It is easy to see that finite, nonvanishing positive values of μ_q corresponds to wide $P(C_q^{(e)})$, which in turn means unpredictable spatial pattern from event to event. By applying the measure to known classical chaotic system, it has been shown[9] that μ_q can be used as a measure of chaos in problems where only the spatial patterns can be observed and the positivity of μ_q is a criterion for chaos.

An alternative way of calculating μ_q[9] is to express $C_{p,q}$ as

$$C_{p,q} = \langle \Phi_q^{(e)^p} \rangle, \tag{6}$$

in which,

$$\Phi_q^{(e)} = C_q^{(e)} / \langle C_q^{(e)} \rangle. \tag{7}$$

With the definition

$$\Sigma_q = \langle \Phi_q^{(e)} \ln \Phi_q^{(e)} \rangle, \tag{8}$$

we can obtain

$$\mu_q = \frac{\partial \Sigma_q}{\partial \ln M} \tag{9}$$

in the scaling region, i.e. where Σ_q exhibits a linear dependence on ln M. We will use this formula in calculating the entropy indices μ_q.

Let us turn now to the consideration of the relation between the fractality and chaoticity in high energy collisions. Since the random cascading α-model[3,10] is often used to study the dynamical fluctuations in these collisions, we will use this simple model as a tool for our investigation.

In the random cascading α-model, the M divisions of a phase space region Δ are made in steps. At the first step, it is divided into two equal parts; at the second step, each part in the first step is further divided into two equal parts, and so on. The steps are repeated until $M = \Delta Y / \delta y = 2^\nu$. How particles are distributed from step-to-step between the two parts of a given phase space cell is defined by independent random variable $\omega_{\nu j_\nu}$, where j_ν is the position of the window ($1 \leqslant j_\nu \leqslant 2^\nu$) and ν is the number of steps. It is given by[10]:

$$\omega_{\nu, 2j-1} = \frac{1}{2}(1 + \alpha r); \ \omega_{\nu, 2j} = \frac{1}{2}(1 - \alpha r), \ j = 1, \cdots, 2^{\nu-1}$$

where r is a random number distributed uniformly in the interval $[-1,1]$. α is a positive number less than unity, which determines the region of the random variable ω and describes the strength of dynamical fluctuations in the model. After ν steps, the probability in the mth window ($m = 1, \cdots, M$) is $p_m = \omega_{1j_1} \omega_{2j_2} \cdots \omega_{\nu j_\nu}$. Then according to

eqn (1), probabilty moment $C_q^{(e)}$ in each event of different division steps are calculated, and the moment $C_{p,q}$ and entropy index μ_q of the sample are obtained using eqn. (3) and eqn.(9).

Our research is done in the following two steps.

(A) Fix the model parameter to a definite value, say $\alpha = 0.34$, the experimental results being around this value. The results of ln C_q, ln $C_{p,q}$ and Σ_q vs. ln M from 6000 MC simulation events are shown in Fig.1 (a),(b),(c) respectively.

In Fig. 1 (a) we see a straight line in bi-logarithm plot, which is an indication of intermittency (or fractality). However, the behavior of ln $C_{p,q}$ vs. ln M in the model, cf. Fig. 1 (b), is quite different from the expected result for chaos[7]. It does not show any scaling behavior or upward bending when M goes larger, as the chaotic behaviour requires[7]. The first going up of $C_{p,q}$ is due to an intrinsic uncertainty of the intermittency parameters[11]. The cascade responsible for intermittent behaviour has different realizations in different events, and the intermittency exponents determined from different realizations of the same random cascade are scattered around the average, i.e. the method has a finite resolution with respect to the parameters of the random cascade. The $C_{p,q}$ saturates when M goes large which means that there isnot any essential fluctuation of spatial pattern from event to event. Therefore, this kind of α model cannot reflect the

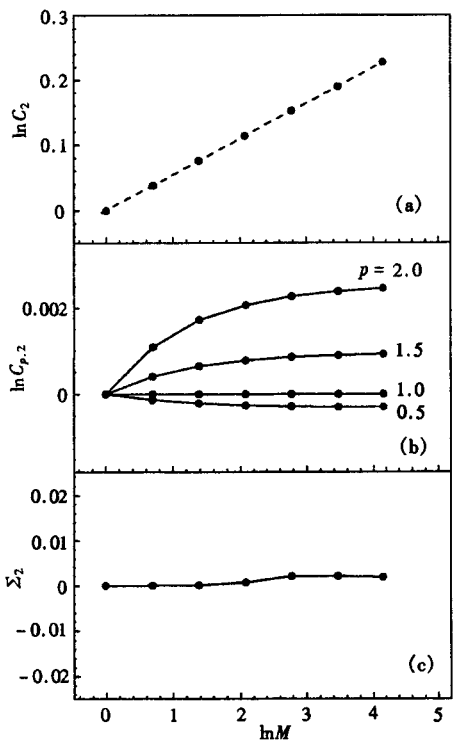

Fig.1 (a) Averaged C_2, (b) $C_{p,2}$, (c) Σ_2 for fixed α Full lines are for guiding the eye
• Monte Carlo; – – – Linear fit

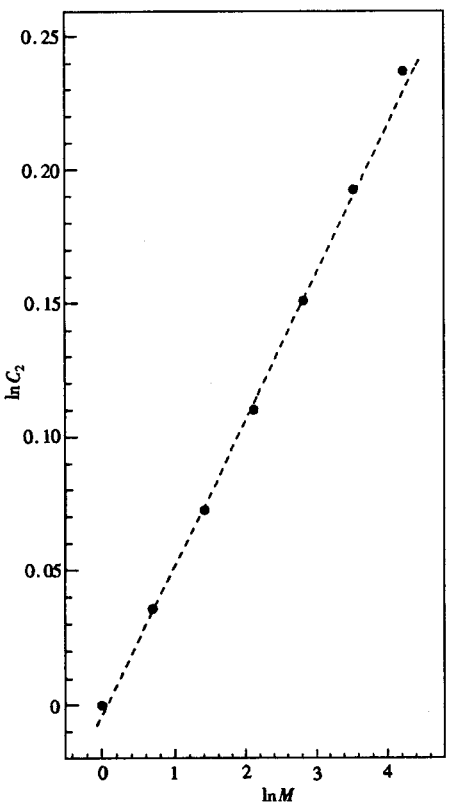

Fig.2 Averaged C_2 for Gaussian-distributed α
• Monte Carlo; – – – Linear fit

feature of chaoticity. From the result showing in Fig. 1 (c), using eqn. (9), we can get for this case $\mu_q \sim 0$. If we do not consider the finite resolution of the model, there isnot any chaotic behavior.

In order to reproduce both intermittency (fractal) and chaos we take the second step.

(B) Instead of giving α a fixed value we let it be a random variable having a Gaussian distribution. The mean value and variance of the Gaussian are both chosen as 0.22. Calculating from 6306 events, the result of ln C_q vs. ln M are shown in Fig. 2. It can be seen from the figure that there is a very good power-law or scaling behavior, which means that though we have changed the method of setting model parameter, the anomalous scaling of the mean value of $C_q^{(e)}$ (intermittency phenomenon) survives. With the method developed in Ref. [12] we can get the effective fluctuation strength in this case as $\alpha_{\text{eff}} = 0.337$. This value of α_{eff} is within the limited range available in actual experiments. However, the behavior of ln $C_{p,q}$ vs. ln M in the present case is much different from the case (A) and shows a typical behaviour of chaoticity, cf. Fig. 3.

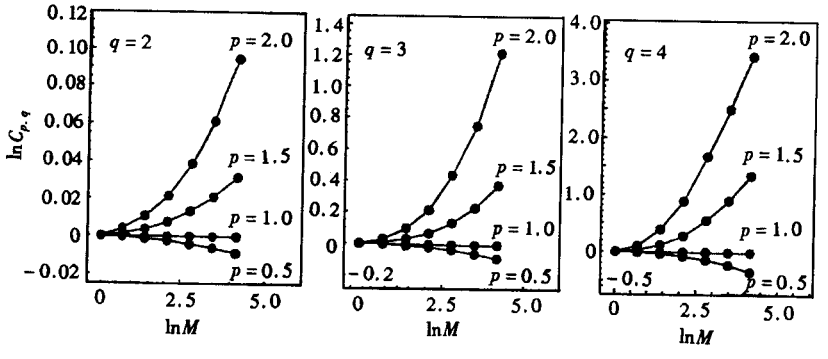

Fig.3 The ln $C_{p,q}$ vs. ln M for the random cascading model with Gaussian-distributed α. Full lines are for guiding the eye

From this result we see that a distribution of α will cause a distribution of single event probabilty moment $C_q^{(e)}$, i. e. for an event sample we have a wide $P(C_q^{(e)})$. For increasing M, along with the expected increase of the average, $P(C_q^{(e)})$ will show a rapid broadening, i. e. a more violent fluctuation of $C_q^{(e)}(M)$ for different events, which will result in a even more unpredictable spatial pattern from event to event.

Using eqn. (8) and eqn. (9), we can calculate entropy indices for this case. The result of Σ_q vs. ln M are shown in Fig. 4(a). By performing a linear fit of Σ_q vs. ln M in the range $M = 8$ to $M = 64$ (i. e. omitting the first three points), μ_q is obtained and plotted in Fig. 4 (b).

From these two steps of MC simulation we can see that the procedure of doing simulation with random cascading α model of fixed strength parameter α, as has been widely used before, captured only one aspect of the non-linear property (intermittency) but cannot reproduce the fluctuation of spatial patterns from event to event. It will cause the loss of information on the spatial patterns in different events and some interesting effects, if existing only in part of the events, may also be lost. If we want to give a more complete description of the non-linear properties using α model, the model parameter

cannot be fixed.

In conclusion, the nonvanishing positive values of μ_q, which is an indication of chaos, correspond to wide $P(C_q^{(e)})$, which in turn means unpredictable spatial pattern from event to event. Such an unpredictability of wide $P(C_q^{(e)})$ is caused by different dynamical fluctuation strength in different events, i. e. by a distribution of dynamical fluctuation strength in an event sample. Events in one sample are all beginning with similar initial condition. During the collision process different event will evolve with a different strength of dynamical fluctuation and this will result in a fluctuation of spatial pattern in the final event space.

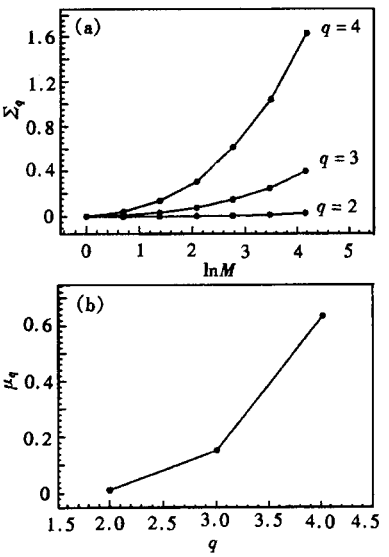

Fig.4 Σ_q and μ_q for Gaussian-distributed α Full lines are for guiding the eye

Dynamical fluctuation strength is directly related to the dynamical mechanism in a particular collision. We take the distribution of dynamical fluctuation strength, i. e. the distribution of model parameter α, to be a Gaussian only because it is the most common distribution of random variables in nature. We have also tried a uniform distribution of α and non-zero positive entropy indices μ_q can be obtained too. (The result is not shown here). Revealing the distribution of dynamical fluctuation strength in different collisions will be a very constructive work and it will certainly help us a lot in studying the mechanism of strong interactions. How the different distributions of dynamical fluctuation strength together with it's mean value and width will influence the entropy index of an event sample is also a problem worthwhile for further investigation.

As has been stressed in the introduction, this study is restricted to the probability moments and the problem of how to eliminate statistical fluctuations in experimental data analysis has not been discussed. To develope an effective method for eliminating the statistical fluctuations for single-event moments is a challenge for future investigation.

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高能碰撞中的间歇与混沌*

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摘要 通过研究随机级联 α 模型，证明了对于固定的动力学起伏强度，多粒子末态相空间有间歇现象，但是没有混沌现象．只有当不同事件的动力学起伏强度不同时才能同时观察到多粒子末态的间歇与混沌行为．这说明，在高能碰撞中，分形与混沌是两个相互联系但又不同的非线性动力学性质．

关键词 混沌 熵指数 间歇 阶乘矩 α模型

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