# Particle-group Correlations and Collectivity for 2.1A GeV Ne+NaF Collision

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A concept of particle-group correlations is proposed in this paper. In accordance with the new concept, a new method of collective flow measurement is constructed. Using the new method, the particle-group correlations arising from collective flow are studied with  $4\pi$  data for 2.1A GeV Ne+NaF collisions at the Bevalac streamer chamber. Through comparison with the Monte Carlo results, the collectivity of the particles in the final state of this collision is inferred in the range of 75 to 95%.

Key words: collective flow, collectivity, transverse motion correlations, particle-group correlations.

#### 1. INTRODUCTION

One of the major goals of relativistic heavy-ion collisions is to extract information about properties of nuclear equation at high temperature and high density so as to obtain the nuclear equation of state (EOS). In 1974, W. Schied *et al.* first predicted the existence of collective flow in the theory and pointed out that information about EOS could be obtained through the study of collective flow. Since then, based on the momentum distribution of particles in the final state many methods have been proposed for collective flow analysis, such as ellipsoid tensor [1, 2], transverse momentum analysis [3–6], azimuthal distribution function analysis [7, 8], particle-pair correlation function [9, 10],

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high-order collective-flow correlations function [11], transverse motion correlation function [12, 13], and the method proposed by Beckmann *et al.* [14–16], etc. Among the methods listed above, flow parameters reflect the properties of the "strength" of collective flow — the magnitude of the collective flow — from different aspects. In 1992, J. Jiang *et al.* extended a particle-pair correlation function [9, 10] to a high-order collective-flow correlation function [11], and revealed another essential characteristic of collective flow through the definition of the "collectivity" of particles — the ratio of the number of particles in the collective motion to the multiplicity of the event — in the final state. In addition, they first quantitatively inferred the "collectivity" of collective flow for 0.44 GeV Ar + Pb collisions in the Bevalac streamer chamber.

The strength of collective flow reflects quantitatively the magnitude of collective flow; the "collectivity" of collective flow reflects more deeply the essence of collective flow. The strength and collectivity, describing the collective flow from two complementary aspects, are closely related to the anisotropic transverse motion [11-13] which involves both the azimuthal distribution and the distribution of transverse momentum magnitude of particles. This description can be done through quantitative description of particle-pair correlation function [9, 10], transverse momentum magnitude correlation function [6, 12, 13], and transverse motion correlation function [12, 13]. To analyze the "strength" and the "collectivity" arising from collective flow involves both the selection of an appropriate parameter capable of fully utilizing all types of information of particles in the final state and the study on the characteristic of collective correlation of particles in the final state. What has just been discussed shows that some correlations, such as azimuthal correlations, transverse momentum magnitude correlations, and transverse motion correlations, etc., exist among the final particles in relativistic nucleus-nucleus collisions with a non-zero impact parameter. Do the particle-group correlations exist among particle groups? If there is this kind of correlation, do they provide the possibility to conduct any analysis of "strength" and "collectivity" arising from collective flow? It is of significance to investigate these questions in order to make further study on the essence of collective flow, and of the quantitative analysis of the "strength" and "collectivity" of collective flow.

In this paper, the method of papers [3, 14] is first analyzed and a concept of particle-group correlations is proposed. Second, the correlation function of two particle groups in the collision is defined and the experimental events are analyzed. Third, correlations among N-particle groups are studied and the strength of collective flow of the experimental events is analyzed. Finally, through comparison with the Monte Carlo results, the collectivity of the collective flow of the experimental events is inferred.

# 2. A BRIEF INTRODUCTION TO THE EXPERIMENT

The experimental data samples for this investigation come from the Bevalac streamer chamber  $4\pi$  experiments, in which the system 2.1A GeV Ne+NaF was studied. A description in greater detail can be found in [17, 18]. There are 2707 events with multiplicity  $M \ge 13$  for the case of Ne + NaF. Assuming a simple geometrical picture, the impact parameters are between 0-0.5 fm. To avoid the effect of experimental factors, e.g., misidentification of particles, absorption in the targets, and loss of particle energy, we apply polar angle cut for experimental data. Assuming  $\theta_{lab} \ge 4^{\circ}$  [18], then the range of multiplicity after the cut is between 4 to 23. Figure 1 shows the distribution of the nucleon multiplicity M for Ne + NaF experimental events after the polar angle cut.

## 3. CORRELATION BETWEEN TWO PARTICLE GROUPS

The particles in the collision event with multiplicity M are randomly and as equally as possible divided into two particle groups. Vector  $Q_i$  is formed by the transverse momentum  $p_j^i$  of each particle in either particle group.

$$Q_i = \sum_{j=1}^{\infty} \omega_j p_j^{t}, \quad (i = 1, 2),$$
 (1)

where  $\omega_j = \pm 1$ , for baryon with rapidity  $y_{lab} - y_{cm} > \delta$  or  $< -\delta$ ; otherwise  $\omega_j = 0$ , with rapidity  $|y_{lab} - y_{cm}| \le \delta$ . Here  $y_{cm}$  is the center-mass rapidity of the system. The choice of  $\delta$  is meant to remove the particles adjacent to the center-mass rapidity so as to avoid any effect on the result of collective flow analysis. The  $\delta$  value is 0.2 for the samples of the experimental events in this paper, which means the approximate removal of 23% of nucleon detected in the reaction. The magnitude and direction of vector  $Q_i$  indicate a certain directional concentration of the transverse momentum distribution of corresponding particle groups.

In collision events of multiplicity M, the distribution probability of angle  $\psi$  between vectors  $Q_1$  and  $Q_2$  of the two particle groups is as follows:

$$PS(\psi) = dN / d\psi, \tag{2}$$

where  $\psi = \cos^{-1}[(Q_1 \cdot Q_2)/(|Q_1| \cdot |Q_2|)]$ ,  $0 \le \psi \le \pi$ . Adapting the approach of azimuthal correlation function, we define the correlation function between particle groups:

$$CS(\psi) = PS(\psi) / PSM(\psi),$$
 (3)

where  $PSM(\psi)$  is the distribution probability of the angle  $\psi$  for Monte Carlo events. In accordance with the approach in paper [3], Monte Carlo events are generated [3, 10, 11] by randomly mixing particles from different events with in the same multiplicity range and the obtained distribution of the angle  $\psi$  should be isotropic. The correlation function between two particle groups is the ratio of the number of correlated particle-group pairs to uncorrelated particle-group pairs within the same bin of the azimuthal difference  $\psi$ . The solid circles with error in Fig. 2 show the calculated values of  $CS(\psi)$  for the experimental samples.

It can be seen from the distribution of  $CS(\psi)$  in Fig. 2, the value of the correlation function  $CS(\psi)$  is larger in the small  $\psi$  region whereas the value is smaller in the larger  $\psi$  region. There are the correlations among the particle groups in the final state. This type of analysis is essentially a reduction of the information of particle transverse motion — the magnitude and azimuthal of transverse momentum — in the collision events, to produce transverse momentum collective vectors  $Q_1$  and  $Q_2$ . The properties of the particle-group correlations can be studied through analyzing vectors  $Q_1$  and  $Q_2$ .

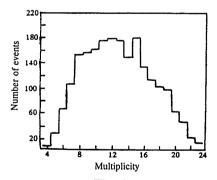
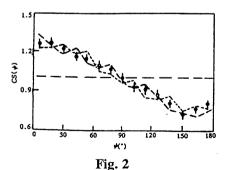


Fig. 1
The distribution of the nucleus multiplicity M for Ne + NaF events after the polar cut.



The collection function of two particle groups  $CS(\psi)$ . Solid circles show the experimental events with error bars, the Monte Carlo events of simulated collective flow are shown by dotted line ( $\alpha = 75\%$ ) and dashed line ( $\alpha = 95\%$ ).

This method can be merged into transverse motion correlation function [6, 12, 13], which is similar to the approach by Danielewicz and Odyniec [3], who studied collective flow in 1985.

Similarly, Beckmann et al. [14] divided the particles in collision events into two particle groups — one forward emitted and the other backward emitted — in the center-mass system, and studied the correlations among particle groups by means of vector  $W_s$ . The definition and more detailed description of  $W_s$  can be found in Ref. [14]. The "strength" of collective flow in experimental events can be tested by means of the approach shown in Ref. [3, 14]; however, more difficulties arise when the "collectivity" information of collective flow is taken into account. The reasons for this are, first, that the variable selected includes the summation of transverse momentum of all particles and, second, that the information of experimental events could not be fully utilized when only the correlation between two particle groups are considered in the event. To solve this problem, all information in the experimental events should be fully utilized and the sensitive variable of collective flow should be studied and tested. It has been proved that the study of the correlations among multiple particle groups (N particle groups) is an efficient solution to this problem.

## 4. CORRELATION AMONG N PARTICLE GROUPS

We now extend the correlation between two particle groups to that of N particle groups.

Assuming the multiplicity of the collision event is M, and then the M particles are randomly and as equally as possible divided into N particle groups. Each particle group is a subset  $\Omega_i$  in this event, where  $i = 1, 2, \dots, N$ . We define a vector of each subset  $\Omega_i$  as

$$Q_i = \sum_{j=1}^{\infty} \omega_j p_j^{t}, \quad (i = 1, 2, \dots, N),$$
(4)

where  $p_j^t$  is the transverse momentum for the j-th particle in the i-th particle group.  $\omega_j$  has the same meaning as in Eq. (1). Now the N vectors  $Q_i$  correspond to N azimuthal angles  $\varphi(Q_i)$ . Then we define the correlative angle variable among N particle groups as

$$\psi_N = \left(\prod_{k=1}^K \Delta \varphi\right)^{1/K}, \quad K = (1/2)N(N-1), \tag{5}$$

where  $\Delta \varphi$  is the angle between vector  $\mathbf{Q}_i$  and  $\mathbf{Q}_j$  formed by two random particle groups in the same collision event and  $\Delta \varphi = \cos^{-1} \left[ (\mathbf{Q}_i \cdot \mathbf{Q}_j) / (|\mathbf{Q}_i| \cdot |\mathbf{Q}_j|) \right], \ 0 \le \Delta \varphi \le \pi$ , the product symbol  $\Pi$  runs over all K azimuthal separations  $\Delta \varphi$  formed from N particle groups.

Similar to the definition of the correlation function between two particle groups, the correlation function of N particle groups is defined as

$$CS(\psi_N) = PS(\psi_N) / PSM(\psi_N), \tag{6}$$

where  $PS(\psi_N)$  and  $PSM(\psi_N)$  indicate, according to  $\psi_N$ , the distribution probabilities for experimental events and Monte Carlo events, respectively. It can be seen from the definition of  $\psi_N$  that  $\psi_N$  is related not only to the azimuthal of the particles, but also to the transverse momentum magnitude for each collision event.  $\psi_N$  contains more information. The "strength" and "collectivity" of collective flow can be quantitatively analyzed through studying the correlation function  $CS(\psi_N)$  in the whole region of  $0 \le \psi_N \le \pi$ . The solid circles with error in Fig. 3 show the calculated values of  $CS(\psi_N)$  for the experimental samples. For N=3-10, the mean value of nucleon multiplicity  $\langle M \rangle$  is 16. Note that correlation function  $CS(\psi_N)$  in the region of  $0 \le \psi_N \le \pi/2$  shows more clearly the effect of particle

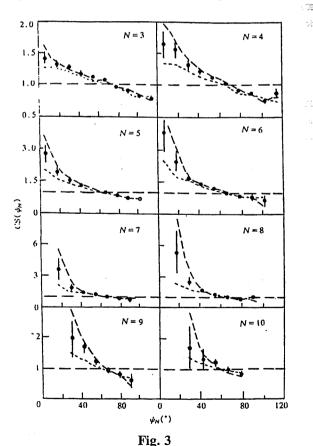
-group correlations than its distribution in the whole region of  $\psi_N$ . With the increase of N value, the difference between  $CS(\psi_N)$  and 1 increases rapidly. Therefore, the correlation strength among particle groups can be tested quantitatively by means of  $|CS(\Psi_N) - 1|$ . The distribution of  $CS(\psi_N)$  contains the accumulative effect of correlations of all orders up to N particle groups. Affected by the statistics of experimental events, N = 10 is the maximum allowed in the experimental sample analysis. The error of the  $CS(\psi_N)$  value for N = 9, 10 is over 80% in the region of  $0 \le \psi_N \le \pi/6$ . Therefore, in Fig. 3, we removed the calculated points in the region of  $0 \le \psi_N \le \pi/6$  when analyzing  $CS(\psi_N)$  for N = 9, 10. Next, we will infer the "collectivity" of collective flow for experimental events by comparison with Monte Carlo results.

## 5. ANALYSIS OF THE "COLLECTIVITY" IN EXPERIMENTAL EVENTS

In order to reveal how many particles participate in the collective directed motion in the collision events, in accordance with the approach by Jiang *et al.* [11], we define the "collectivity" parameter  $\alpha$  as

$$\alpha = (M_1 / M) \times 100\%, \tag{7}$$

where M is the multiplicity in the event and  $M_1$  is the number of particles participating in the collective



The correlation function for many particle groups  $CS(\psi_N)$  (N=3-10). Solid circles show experimental events with error bars, the Monte Carlo events of simulated collective flow as shown by the dotted line ( $\alpha=75\%$ ) and dashed line ( $\alpha=95\%$ ).

directed motion. Collectivity of collective flow is phenomenologically estimated through comparison between the values of correlation function in the N (N=2-10) particle groups for Monte Carlo simulation and the analytic results of experimental data. The steps are described in the following sections.

## 5.1. Producing Monte Carlo events with non-collective flow

- (1) Cascade events are generated at impact parameter b = 0 fm and then selected by the cascade model [19]. There is no collective flow because the effect of the nuclear equation of state is not included in these events.
- (2) The above-mentioned polar cut performed on experimental events is applied to these cascade events. It has been verified that the distribution of multiplicity M, inclusive transverse momentum, and the rapidity after the cut conforms with the experimental events.
- (3) Corresponding to experimental events, the impact parameter b can be randomly redefined for cascade events in the range of b between 0 and 5.0 fm. A cascade sample with statistics five times that of the experimental sample is generated.

# 5.2. Producing Monte Carlo events with "collectivity" being $\alpha$

 $M_1$  particles are randomly selected from the cascade events of multiplicity M' and then a component of collective directed motion  $f_0 p_i^{\text{flow}}/\alpha$  is added to the projection of the transverse momentum on the reaction plane [11] for each fragment, where  $f_0$  is a free parameter that controls the strength of the flow, i is the order sign of the fragment and  $i \in [4, 23]$ ,  $\alpha = M_1/M'$ . The parameters will be described in the following. In this paper,  $p_i^{\text{flow}}$  is expressed as [20].

$$p_i^{\text{flow}} = A_i B_i u_i | \cos(\varphi_i)|, \tag{8}$$

where  $A_i$  is the *i*-th particle mass number.  $B_i$  is defined as

$$B_{\rm i} = \sin^{2/3}(b / b_{\rm max}),$$
 (9)

where b is the impact parameter in the event,  $b_{\text{max}} = R_{\text{p}} + R_{\text{T}}$ .  $R_{\text{p}}$  and  $R_{\text{T}}$  are the radii of the projectile and target, respectively,  $u_i$  is the ratio of i-th fragment rapidity to projectile rapidity in the center-mass,  $\varphi_i$  is the angle of the transverse momentum vector relative to the reaction plane. Equation (8) phenomenologically describes the dependence of transverse collective flow on fragment mass, rapidity, and azimuthal angle relative to the reaction plane, and impact parameter. This description is certainly not unique, but it maintains momentum and energy conservation for each event, and it is approximately consistent with the available experimental data [11, 20].

The approach to determine  $f_0$  is to calculate the distribution of the projection average value of the transverse momentum of each nucleus in the reaction plane as a function of the rapidity  $\langle p_{\perp}^x \rangle$  or the distribution of  $CS(\psi_2)$  for the events of simulated collective flow, corresponding to the  $\langle p_{\perp}^x \rangle$  or  $CS(\psi_2)$  in the experimental events, thus to determine  $f_0$  value. In this paper,  $f_0 = 200$  MeV.

# 5.3. Inferring the collectivity of collective flow in the experimental events

Adjusting the parameter  $\alpha$  and, in the range of N=3-10, make the distribution of  $CS(\psi_N)$  for the simulated events consistent with that for the experimental events in the range of present experiment precision. The dashed lines and dotted lines in Figs. 2 and 3 show the calculated results. The errors of dashed lines and dotted lines are approximately half of that of experimental points. Therefore, from

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Figs. 2 and 3, we conclude that the value of  $\alpha$  is the inference value of "collectivity" of collective flow in the experimental events. Analysis shows that the range of  $\alpha$  value is between 75 and 95%.

On the other hand, adopting the methods introduced in Refs. [11, 16], we compare N-particle azimuthal correlation function  $C(\psi_N)$  and N-particle transverse correlation function  $F(V_N)$  of Monte Carlo simulation with that of experimental data. The results show that  $\alpha$  is not less 75% within the range of estimated error.

#### 6. CONCLUSIONS

The correlations exist not only among particles but among particle groups in relativistic heavy-ion collisions in the final state. On the basis of papers [3, 14], a concept of particle-group correlations is proposed in this paper. In accordance with the new concept, a new method of collective flow measurement is constructed. Using the new method, the particle-group correlations arising from collective flow are studied with  $4\pi$  data for 2.1A GeV Ne + NaF collisions at the Bevalac streamer chamber. Through comparison with the Monte Carlo results, the collectivity of collective flow of the experimental events in the final state is inferred in the range from 75 to 95%. Because "collectivity" information reflects the essential characteristic of collective flow, it has become a topic of significance in the study on collective flow to seek the sensitive parameter to infer "collectivity" and to improve the test accuracy. The results in this paper provide an efficient approach to test quantitatively the "strength" and "collectivity" (more constraint with the same data statistics) of collective flow so that the essence of collective flow is to be studied in the future.

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