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广义 QCD 中正则形式的 Ward 恒等式*

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摘 要

导出了高阶微商奇异拉氏量系统正则形式的 Ward 恒等式并将其应用于广义色动力学(QCD),得到了广义 QCD 中规范场和鬼场正规顶角间的某些新关系,它们有别于 BRS 不变性所导致的结果;还得到了广义 QCD 中的 PCAC 和 AVV 顶角的 Ward 恒等式。

关键词 高阶微商场论,约束系统的 Dirac 理论,广义 QCD, Ward 恒等式。

1 引 言

动力学系统用高阶微商拉氏量来描述,已经研究很久时间了。近来,对高阶微商杨-Mills 理论开展了讨论[1,2]。高阶微商理论与相对论性粒子动力学、引力理论、规范场论、Korteweg-de Vries (KdV) 方程、超对称和弦理论等问题直接有关[3]。在长距离情形下,杨-Mills 场的有效拉氏量含场的高阶微商[4]. Ward 恒等式在量子场论中占重要地位,它是理论可重整化的依据,在 QCD 的实际计算中,利用它可将高阶顶角的计算化为低阶顶角的计算。传统的 Ward 恒等式是用位形空间中的变量来表述的。当相空间中 Green 函数的生成泛函对动量的路径积分不能积出时,特别是约束 Hamilton 系统和高阶微商系统,当约束结构比较复杂时,要作出对动量的路径积分往往是很困难的,甚至是不可能的。此时就不能简单地用位形空间中的拉氏量(或有效拉氏量)来表出 Green 函数的生成泛函,从而给出位形空间中的 Ward 恒等式。在前文中,对一阶微商拉氏量系统,已讨论了相空间中的 Ward 恒等式[5]。这里继前文,研究高阶微商奇异拉氏量系统的正则形式 Ward 恒等式,并给出它在广义 QCD 中的初步应用。

本文首先从高阶微商奇异拉氏量系统在相空间中 Green 函数的生成泛函出发,导出了该系统正则形式的 Ward 恒等式,将此恒等式用于广义 QCD,得到了广义 QCD 中规范场和鬼场正规顶角间的一些新关系,这些关系有别于由 BRS 不变性所得到的结果。此外,还导出了广义 QCD 中的 PCAC 和 AVV 顶角的 Ward 恒等式。指出了在文献

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[6]中忽略了对约束的处理.

2 奇异拉氏量系统正则形式的 Ward 恒等式

设 $\phi^\alpha(x)$ $(\alpha=1,2,\cdots,n)$ 为场变量,场的运动由含高阶微商的拉氏量

$$L[\phi^\alpha_{(0)},\phi^\alpha_{(1)},\cdots,\phi^\alpha_{(N)}]=\int d^3x\mathscr{L}(\phi^\alpha,\phi^\alpha_{,\mu},\cdots,\phi_{,\mu(N)}) \tag{1}$$

来描述[7]. 用奇异拉氏量描述的系统(简称奇异系统),其 Hess 矩阵 $(H_{\alpha\beta})$ 退化,$\det|H_{\alpha\beta}|=\det\left|\frac{\delta^2L}{\delta\phi^\alpha_{(N)}\delta\phi^\beta_{(N)}}\right|=0$,此时正则变量 $\phi^\alpha_{(s)},\pi^{(s)}_\alpha$ 之间存在初级约束[7]

$$\Phi^0_a(\phi^\alpha_{(s)},\pi^{(s)}_\alpha)\approx 0\quad(a=1,2,\cdots,n-R), \tag{2}$$

其中 R 为 Hess 矩阵的秩. 按约束的自洽性条件,从初级约束可逐次求出各次级约束.全部约束分为第一类约束和第二类约束两类.

在约束 Hamilton 系统的路径积分量子化中,对每一个第一类约束,必须选取适当的规范条件,以限制理论中的规范自由度. 高阶微商奇异系统在相空间中 Green 函数的生成泛函为[8]

$$Z[J,K]=\int\mathscr{D}\phi^\alpha_{(s)}\mathscr{D}\pi^{(s)}_\alpha\delta(\Phi_l)\sqrt{\det\{\Phi_l,\Phi_m\}}$$
$$\cdot\exp\{i\int d^4x(\mathscr{L}^P+J^{(s)}_\alpha\phi^\alpha_{(s)}+K^\alpha_{(s)}\pi^{(s)}_\alpha)\}, \tag{3}$$

其中

$$\mathscr{L}^P=\pi^{(s)}_\alpha\phi^\alpha_{(s+1)}-\mathscr{H}^c, \tag{4}$$

$\mathscr{H}^c$ 为系统的正则哈氏量[7]. 对含第二类约束的系统,$\{\Phi_l\}$ 为所有第二类约束;对含第一类约束的系统,$\{\Phi_l\}$ 包括所有第一类约束和规范条件. {,}代表场的广义 Poisson 括号. $J^{(s)}_\alpha$ 和 $K^\alpha_{(s)}$ 分别为 $\phi^\alpha_{(s)}$ 和 $\pi^{(s)}_\alpha$ 的外源.

利用 Grassmann 变量 $C(x)$ 和 $\bar{C}(x)$ 的积分性质,有

$$\det\{\Phi_l(x),\Phi_m(y)\}=\int\mathscr{D}C_m(y)\mathscr{D}\bar{C}_l(x)$$
$$\times\exp\left[i\int d^4xd^4y\bar{C}_l(x)\{\Phi_l(x),\Phi_m(y)C_m(y)\right]. \tag{5}$$

由(5)式和 δ-函数的性质,可将(3)式写为

$$Z[J,K]=\int\mathscr{D}\phi^\alpha_{(s)}\mathscr{D}\pi^{(s)}_\alpha\mathscr{D}\lambda\mathscr{D}C\mathscr{D}\bar{C}$$
$$\cdot\exp\left\{i\int d^4x(\mathscr{L}^P_{\text{eff}}+J^{(s)}_\alpha\phi^\alpha_{(s)}+K^\alpha_{(s)}\pi^{(s)}_\alpha)\right\} \tag{6}$$

其中

$$\mathscr{L}^P_{\text{eff}}=\mathscr{L}^P+\lambda_l\Phi_l+\frac{1}{2}\int d^4y\bar{C}_l(x)\{\Phi_l(x),\Phi_m(y)\}C_m(y), \tag{7}$$

$\lambda_l(x)$ 为拉氏乘子.

考虑增广相空间中的无穷小变换

$$\begin{cases} x^{\mu'} = x^\mu + \Delta x^\mu = x^\mu + R^\mu_\sigma \varepsilon^\sigma(x), \\ \phi^{a\prime}_{(s)}(x') = \phi^a_{(s)}(x) + \Delta\phi^a_{(s)}(x) = \phi^a_{(s)}(x) + S^a_{(s)\sigma}\varepsilon^\sigma(x), \\ \pi^{(s)\prime}_a(x') = \pi^{(s)}_a(x) + \Delta\pi^{(s)}_a(x) = \pi^{(s)}_a(x) + T^{(s)}_{a\sigma}\varepsilon^\sigma(x), \end{cases} \tag{8}$$

其中 $\varepsilon^\sigma(x)$ $(\sigma = 1,2,\cdots,r)$ 为无穷小任意函数，它们和它们的各级微商在四维时空区域的边界上为零。$R^\mu_\sigma, S^a_{(s)\sigma}$ 和 $T^{(s)}_{a\sigma}$ 为线性微分算符[7]。在(8)式变换下，有[7]

$$\Delta I_{\rm eff} = \Delta\int \mathscr{L}^P_{\rm eff} d^4x = \int d^4x \left\{\frac{\delta I^P_{\rm eff}}{\delta\phi^a_{(s)}}\delta\phi^a_{(s)} + \frac{\delta I^P_{\rm eff}}{\delta\pi^{(s)}_a}\delta\pi^{(s)}_a + \partial_\mu[(\pi^{(s)}_a\phi^a_{(s+1)} - \mathscr{H}^c)\Delta x^\mu + \frac{d}{dt}(\pi^{(s)}_a\delta\phi^a_{(s)})\right\}, \tag{9}$$

其中

$$\delta\phi^a_{(s)} = \Delta\phi^a_{(s)} - \phi^a_{(s),\mu}\Delta x^\mu,\quad \delta\pi^{(s)}_a = \Delta\pi^{(s)}_a - \pi^{(s)}_{a,\mu}\Delta x^\mu, \tag{10}$$

$$\frac{\delta I^P_{\rm eff}}{\delta\phi^a_{(s)}} = -\dot{\pi}^{(s)}_a - \frac{\delta H^c_{\rm eff}}{\delta\phi^a_{(s)}},\quad \frac{\delta I^P_{\rm eff}}{\delta\pi^{(s)}_a} = \dot{\phi}^a_{(s)} - \frac{\delta H^c_{\rm eff}}{\delta\pi^{(s)}_a}, \tag{11}$$

$H^c_{\rm eff}$ 为 $\mathscr{L}^P_{\rm eff}$ 相应的正则哈氏量。设变换(8)的 Jacobi 行列式为 $J[\phi,\pi,\varepsilon]$。在(8)式变换下，生成泛函(6)式是不变的，表明 $\dfrac{\delta Z}{\delta\varepsilon^\sigma} = 0$，于是由(6)式和(9)式得正则形式的 Ward 恒等式：

$$\Bigg[J^0_\sigma + \tilde{S}^a_{(s)\sigma}\left(\frac{\delta I^P_{\rm eff}}{\delta\phi^a_{(s)}}\right) - \tilde{R}^\mu_\sigma\left(\phi^a_{(s),\mu}\frac{\delta I^P_{\rm eff}}{\delta\phi^a_{(s)}}\right) + \tilde{S}^a_{(a)\sigma}J^{(s)}_a - \tilde{R}^\mu_\sigma(\phi^a_{(s),\mu}J^{(s)}_a) + \tilde{T}^{(s)}_{a\sigma}\left(\frac{\delta I^P_{\rm eff}}{\delta\pi^{(s)}_a}\right) - \tilde{R}^\mu_\sigma\left(\pi^{(s)}_{a,\mu}\frac{\delta I^P_{\rm eff}}{\delta\pi^{(s)}_a}\right) + \tilde{T}^{(s)}_{a\sigma}K^a_{(s)} - \tilde{R}^\mu_\sigma(\pi^{(s)}_{a,\mu}K^a_{(s)})\Bigg]_{\substack{\phi^a_{(s)}\to\frac{1}{i}\frac{\delta}{\delta J^{(s)}_a} \\ \pi^{(s)}_a\to\frac{1}{i}\frac{\delta}{\delta K^a_{(s)}}}} Z[J,K] = 0, \tag{12}$$

其中 $J^0_\sigma = \left.\dfrac{\delta J[\phi,\pi,\varepsilon]}{\delta\varepsilon^\sigma}\right|_{\varepsilon^\sigma=0}$，$\tilde{R}^\mu_\sigma, \tilde{S}^a_{(s)\sigma}$ 和 $\tilde{T}^{(s)}_{a\sigma}$ 分别为 $R^\mu_\sigma, S^a_{(s)\sigma}$ 和 $T^{(s)}_{a\sigma}$ 的伴随算符[9]。在导出(12)式时，用了关系式 $J[\phi,\pi,0] = 1$。将(12)式对外源多次求泛函微商，然后让外源等于零，从而可得到多种形式的正则 Ward 恒等式。

3　广义 QCD 中规范场-鬼场固有顶角

广义 QCD 的拉氏量密度为[2,8]

$$\mathscr{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4K^2}D^a_{b\mu}G^b_{\lambda\nu}D^{a\mu}_c G^{c\lambda\nu} + i\bar{\psi}^\alpha\gamma^\mu\nabla^\alpha_{\beta\mu}\psi^\beta - m\bar{\psi}^\alpha\psi^\alpha, \tag{13}$$

其中

$$G^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + f^a_{bc}B^b_\mu B^c_\nu, \tag{14}$$

$$D^a_{b\mu} = \delta^a_b\partial_\mu + f^a_{cb}B^c_\mu, \tag{15}$$

$$\nabla^\alpha_{\beta\mu} = \delta^\alpha_\beta\partial_\mu - i(T_a)^\alpha_\beta B^a_\mu, \tag{16}$$

$T_a = \lambda_a/2$, λ_a 为 Gell-Mann 矩阵. 拉氏量(13)在下列规范变换下是不变的:

$$\begin{cases}\delta\psi^\alpha = -\mathrm{i}\varepsilon^a(x)(T_a)^\alpha_\beta\psi^\beta, \\ \delta A^a_\mu = \mathrm{D}^a_{b\mu}\varepsilon^b(x).\end{cases}$$

广义 QCD 的拉氏量是奇异的，系统在相空间存在固有约束. 设 $\pi_{a\mu}, \pi^{(1)}_{ai}, \pi_\psi$ 和 $\pi_{\bar\psi}$ 分别代表与 $B^{a\mu}$, $B^{ai}_{(1)} = \dot B^{ai}$, ψ 和 $\bar\psi$ 相应的正则共轭动量. 系统所含的约束为[8]

$$\Phi^{(1)}_{a1} = \pi_{a0} + \mathrm{D}^a_{bi}\pi^{(1)}_{ib} \approx 0, \tag{17}$$

$$\Phi^{(1)}_2 = \pi_\psi - \mathrm{i}\bar\psi\gamma_0 \approx 0, \tag{18}$$

$$\Phi^{(1)}_3 = \pi_{\bar\psi} \approx 0, \tag{19}$$

$$\Phi^{(2)}_a = \mathrm{D}^a_{bi}\pi_{bi} - \bar\psi^\alpha\gamma_0(T_a)^\alpha_\beta\psi^\beta - [\Phi^{(1)}_{2\alpha}(T_a)^\alpha_\beta\psi^\beta + \Phi^{(1)}_{3\alpha}(\tilde T_a)^\alpha_\beta\psi^\beta] - f^a_{bc}(B^{bi}_{(1)}\pi^{(1)}_{ci} + \pi_{c0}B^{b0}) \tag{20a}$$

而

$$(\tilde T_a)^\alpha_\beta = -\gamma^0(T_a)^\alpha_\beta\gamma^0 \tag{20b}$$

$\Phi^{(1)}_{a1}, \Phi^{(2)}_a$ 为第一类约束, $\Phi^{(1)}_2, \Phi^{(1)}_3$ 为第二类约束. 采用路径积分量子化，正则规范条件取为

$$\Phi^G_{a1} = \partial_i B^{ai} \approx 0, \tag{21}$$

$$\Phi^G_{a2} = \partial_i B^{ai}_{(1)} \approx 0. \tag{22}$$

全部约束和规范条件一起构成第二类约束. 此时 Green 函数的生成泛函为

$$Z[J] = \int \mathscr{D}B\mathscr{D}B_{(1)}\mathscr{D}\psi\mathscr{D}\bar\psi\mathscr{D}\pi\mathscr{D}\pi^{(1)}\mathscr{D}\pi_\psi\mathscr{D}\pi_{\bar\psi}\delta(\Phi)\sqrt{\det\{\Phi,\Phi\}} \cdot \exp\left\{\mathrm{i}\int \mathrm{d}^4x(\mathscr{L}^P + J^\mu_a B^a_\mu + \bar J\psi + \bar\psi J)\right\}, \tag{23}$$

其中

$$\mathscr{L}^P = \pi_{a\mu}\dot B^{a\mu} + \pi^{(1)}_{a\mu}\dot B^{a\mu}_{(1)} + \pi_\psi\dot\psi + \pi_{\bar\psi}\dot{\bar\psi} - \mathscr{H}^c, \tag{24}$$

$\mathscr{H}^c$ 为系统的正则哈氏量密度. Φ 代表全部约束条件和规范条件, $\Phi = (\Phi_l, \Phi^G_m)$. 这里仅对场量引入了外源. (23) 式又称为“坐标” Green 函数的生成泛函[8]. 在约束超曲面上,有

$$\det\{\Phi,\Phi\} = \det{}^4\mathrm{D}^a_{bi}\partial_i \tag{25}$$

引入鬼场 $C(x)$ 和 $\bar C(x)$, 由(25)式,可将(23)式写为

$$Z[J^\mu_a, \bar J, J, \bar\xi, \xi, \eta, \zeta] = \int \mathscr{D}B\mathscr{D}B_{(1)}\mathscr{D}\psi\mathscr{D}\bar\psi\mathscr{D}\pi\mathscr{D}\pi^{(1)}\mathscr{D}\pi_\psi\mathscr{D}\pi_{\bar\psi}\mathscr{D}\bar C\mathscr{D}C\mathscr{D}\lambda\mathscr{D}\mu \cdot \exp\left\{\mathrm{i}\int \mathrm{d}^4x(\mathscr{L}^P_{\mathrm{eff}} + J^\mu_a B^a_\mu + \bar J\psi + \bar\psi J + \bar\xi C + \bar C\xi + \eta^m\lambda_m + \zeta^n\mu_n)\right\}, \tag{26}$$

其中

$$\mathscr{L}^P_{\mathrm{eff}} = \mathscr{L}^P + \mathscr{L}_f + \mathscr{L}_{gh}, \tag{27}$$

$$\mathscr{L}_f = \lambda^a_1\Phi^{(1)}_{a1} + \lambda^2\Phi^{(1)}_2 + \lambda^3\Phi^{(1)}_3 + \lambda^a_2\Phi^{(2)}_a + \mu^a_1\Phi^G_{a1} + \mu^a_2\Phi^G_{a2}. \tag{28a}$$

$$\mathscr{L}_{gh} = 2\bar C_a(x)\mathrm{D}^a_{bi}\partial_i C(x). \tag{28b}$$

下面来寻找一变换,使 $\mathscr{L}^P$ 和 $\mathscr{L}_{gh}$ 保持不变. 由于在变换

$$\begin{cases}B^{a\prime}_\mu(x) = B^a_\mu(x) + \mathrm{D}^a_{\sigma\mu}\varepsilon^\sigma(x), \\ C^{a\prime}(x) = C^a(x) + \mathrm{i}g(T_\sigma)^a_b\varepsilon^\sigma(x)C^b(x).\end{cases} \tag{29}$$

下，$D^a_{b\mu}C^b$ 变为

$$D^{a'}_{b'\mu}C^{b'} = D^a_{b\mu}C^b + ig(T_\sigma)^a_b\varepsilon^\sigma(x)D^b_{c\mu}C^c. \tag{30}$$

因此，如果设 $\bar{C}^a(x)$ 作下列变换

$$\partial^\mu\bar{C}^{a'} = \partial^\mu\bar{C}^a - ig\bar{C}^b(T_\sigma)^b_a\varepsilon^\sigma(x), \tag{31}$$

那么 $\mathscr{L}^P$ 和 $\mathscr{L}_{gh}$ 在下列变换

$$\begin{cases} C^{a'}(x) = C^a(x) + ig(T_\sigma)^a_b\varepsilon^\sigma(x)C^b(x), & (32a)\\ \bar{C}^{a'}(x) = \bar{C}^a(x) - ig\bar{C}^b(T_\sigma)^b_a\varepsilon^\sigma + \frac{ig}{\Box}\partial_\mu[\bar{C}^b(T_\sigma)^b_a\partial^\mu\varepsilon^\sigma(x)], & (32b)\\ B^{a'}_\mu(x) = B^b_\mu(x) + D^a_{\sigma\mu}\varepsilon^\sigma(x), & (32c)\\ B^{a'}_{(1)\mu}(x) = B^a_{(1)\mu}(x) + \partial_0 D^a_{\sigma\mu}\varepsilon^\sigma(x), & (32d)\\ \pi^{\mu'}_a(x) = \pi^\mu_a(x) + f^a_{\sigma c}\pi^\mu_c\varepsilon^\sigma(x) + f^a_{bc}\pi^{(1)\mu}_c\varepsilon^\sigma(x), & (32f)\\ \pi^{(1)\mu'}_a(x) = \pi^{(1)\mu}_a(x) + f^a_{bc}\pi^{(1)\mu}_c(x)\varepsilon^\sigma(x), & (32g)\\ \psi'(x) = \psi(x) - igT_\sigma\varepsilon^\sigma(x)\psi(x) & (32h)\\ \bar{\psi}'(x) = \bar{\psi}(x) + ig\bar{\psi}(x)T_\sigma\varepsilon^\sigma(x) & (32i)\\ \pi'_\psi(x) = \pi_\psi(x) + ig\pi_\psi(x)T_\sigma\varepsilon^\sigma(x) & (32j)\\ \pi'_{\bar{\psi}}(x) = \pi_{\bar{\psi}}(x) & (32k)\end{cases}$$

下，是不变的．(32b) 式又可写为

$$\bar{C}^{a'}(x) = \bar{C}^a(x) - ig\bar{C}^b(x)(T_\sigma)^b_a\varepsilon^\sigma(x) + ig\int d^4y\Delta_0(x,y)\partial_\mu[\bar{C}^b(y)(T_\sigma)^b_a\partial^\mu\varepsilon^\sigma(y)], \tag{33}$$

其中

$$\Box\Delta_0(x,y) = i\delta^{(4)}(x-y). \tag{34}$$

设变换(32)的 Jacobi 行列式记为 $J_\varepsilon[\phi,\pi,\varepsilon]$，量 $J^0_\sigma = \left.\frac{\delta J_\varepsilon}{\delta\varepsilon}\right|_{\varepsilon^\sigma=0}$ 与场量无关[6]．在(32)式变换下，设 $\mathscr{L}_f$ 的变更为

$$\delta\mathscr{L}_f = F_\sigma(\phi,\pi,\lambda,\mu)\varepsilon_\sigma(x),$$

其中 F_σ 与场的正则变量和乘子场 $\lambda_m(x)$，$\mu_m(x)$ 有关，当乘子场 $\lambda_m = \mu_m = 0$ 时，$F_\sigma = 0$．生成泛函(26)式在(32)式变换下不变，此时 Ward 恒等式(12)化为

$$\begin{aligned} &J^0_\sigma + iF_\sigma - i\partial_\mu J^\mu_\sigma + gf^a_{\sigma c}J^\mu_a\frac{\delta}{\delta J^\mu_c} + ig\bar{J}T_\sigma\frac{\delta}{\delta\bar{J}} - igJT_\sigma\frac{\delta}{\delta J} + ig\bar{\xi}_a(T_\sigma)^a_b\frac{\delta}{\delta\bar{\xi}_b}\\ &- ig\xi_a(T_\sigma)^b_a\frac{\delta}{\delta\xi_b} + ig\partial^\mu\left[\partial_\mu\left(\xi_a\frac{1}{\Box}\right)(T_\sigma)^b_a\frac{\delta}{\delta\xi_b}\right]Z[J^\mu_a,\bar{J},J,\bar{\xi},\xi,\eta,\zeta] = 0. \end{aligned} \tag{35}$$

让 $Z[J^\mu_a,\bar{J},J,\bar{\xi},\xi,\eta,\zeta] = \exp\{iW[J^\mu_a,\bar{J},J,\bar{\xi},\xi,\eta,\zeta]\}$，利用泛函 Legendre 变换将 $W[J^\mu_a,\bar{J},J,\bar{\xi},\xi,\eta,\zeta]$ 换为 $\Gamma[B^a_\mu,\psi,\bar{\psi},C,\bar{C},\lambda,\mu]$，

$$\begin{aligned} \Gamma[B^a_\mu,\psi,\bar{\psi},C,\bar{C},\lambda,\mu] &= W[J^\mu_a,\bar{J},J,\bar{\xi},\xi,\eta,\zeta]\\ &- \int d^4x(J^\mu_a B^a_\mu + \bar{J}\psi + \bar{\psi}J + \bar{\xi}C + \bar{C}\xi + \eta^m\lambda_m + \zeta^n\mu_n), \end{aligned} \tag{36}$$

且

$$\frac{\delta W}{\delta J^\mu_a(x)} = B^a_\mu(x), \quad \frac{\delta\Gamma}{\delta B^a_\mu(x)} = -J^\mu_a(x), \tag{37a}$$

$$\frac{\delta W}{\delta J(x)}=\bar{\psi}(x),\quad \frac{\delta \Gamma}{\delta \bar{\psi}(x)}=-J(x), \tag{37b}$$

$$\frac{\delta W}{\delta \bar{J}(x)}=\psi(x),\quad \frac{\delta \Gamma}{\delta \psi(x)}=-\bar{J}(x), \tag{37c}$$

$$\frac{\delta W}{\delta \xi(x)}=\bar{C}(x),\quad \frac{\delta \Gamma}{\delta \bar{C}(x)}=-\xi(x), \tag{37d}$$

$$\frac{\delta W}{\delta \bar{\xi}(x)}=C(x),\quad \frac{\delta \Gamma}{\delta C(x)}=-\bar{\xi}(x), \tag{37e}$$

$$\frac{\delta W}{\delta \eta^m(x)}=\lambda_m(x),\quad \frac{\delta \Gamma}{\delta \lambda_m(x)}=-\eta^m(x), \tag{37f}$$

$$\frac{\delta W}{\delta \zeta^n(x)}=\mu_m(x),\quad \frac{\delta \Gamma}{\delta \mu_n(x)}=-\zeta^n(x). \tag{37g}$$

这样,(35)式又可化为

$$\begin{aligned} J_\sigma^0 &+ \mathrm{i}F_\sigma + \mathrm{i}\partial_\mu \frac{\delta \Gamma}{\delta B_\mu^\sigma} - \mathrm{i}g f_{\sigma c}^a B_\mu^c \frac{\delta \Gamma}{\delta B_\mu^a} - \mathrm{i}g T_\sigma \psi \frac{\delta \Gamma}{\delta \psi} + \mathrm{i}g T_\sigma \bar{\psi} \frac{\delta \Gamma}{\delta \bar{\psi}} \\ &- \mathrm{i}g C^a (T_\sigma)_b^a \frac{\delta \Gamma}{\delta C^b} + \mathrm{i}g \bar{C}^a (T_\sigma)_a^b \frac{\delta \Gamma}{\delta \bar{C}^b} - \mathrm{i}g \partial^\mu \left[\partial_\mu \left(\frac{\delta \Gamma}{\delta \bar{C}^a} \frac{1}{\Box}\right)(T_\sigma)_a^b \bar{C}^b\right] \\ &= 0. \end{aligned} \tag{38}$$

将(38)式关于 $\bar{C}^k(x_2)$ 和 $C^m(x_3)$ 求泛函微商，然后让所有场(包括乘子场)为零，$B_\mu^a = \psi = \bar{\psi} = C^a = \bar{C}^a = \lambda_m = \mu_n = 0$，于是得

$$\begin{aligned} \partial_{x_1}^\mu & \frac{\delta^3 \Gamma[0]}{\delta \bar{C}^k(x_2)\delta C^m(x_3)\delta B_\mu^\sigma(x_1)} - g(T_\sigma)_b^m \delta(x_1 - x_3) \frac{\delta^2 \Gamma[0]}{\delta \bar{C}^k(x_2)\delta C^b(x_1)} \\ &+ g(T_\sigma)_k^b \delta(x_1 - x_2) \frac{\delta^2 \Gamma[0]}{\delta \bar{C}^b(x_1)\delta C^m(x_3)} \\ &- g\partial^\mu \left[\partial_\mu \left(\frac{\delta^2 \Gamma[0]}{\delta \bar{C}^a(x_1)\delta C^m(x_3)} \frac{1}{\Box}\right)(T_\sigma)_a^k \delta(x_1 - x_2)\right] = 0. \end{aligned} \tag{39}$$

(39)式为(32)式下不变性相应的规范场-鬼场正规顶角的 Ward 恒等式. 将(38)式关于其他场求泛函微商，然后让场变量等于零，可得到场的传播子和正规顶角间更多的关系式。

(39)式与传统的 BRS 不变性导致的结果不同，BRS 变换中关于鬼场的变换是非线性的，这里的变换(32)式中关于鬼场的变换则是线性的(非定域). BRS 变换保证了有效拉氏量不变，这里的变换仅保证了 $\mathscr{L}^P$ 和 $\mathscr{L}_{gh}$ 的不变性，而 $\mathscr{L}_f$ 可以是非不变的. 导出规范场-鬼场正规顶角 Ward 恒等式(39)时，勿需作出对动量的积分，这也是和传统讨论不同的另一突出优点.

4 广义 QCD 中的 PCAC 和 AVV 顶角

考虑由费米场 $\psi(x)$ 构成的矢量流、轴矢流、标量流和赝标量流

$$V^a_\mu(x)=\bar{\psi}(x)\gamma_\mu T^a\psi(x),\quad A^a_\mu(x)=\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)$$
$$S^a(x)=\bar{\psi}(x)T^a\psi(x),\qquad P^a(x)=\mathrm{i}\bar{\psi}(x)\gamma_5T^a\psi(x) \tag{40}$$

引入与这些流相应的外源 $v^\mu_a(x),a^\mu_a(x),s_a(x)$ 和 $p_a(x)$，并构成与(26)式相应的扩展生成泛函

$$\begin{aligned}&Z[J^\mu_a,\bar{J},J,\bar{\xi},\xi,\eta,\zeta,v,a,s,p]\\&=\int\mathscr{D}B\mathscr{D}B_{(1)}\mathscr{D}\psi\mathscr{D}\bar{\psi}\mathscr{D}\pi\mathscr{D}\pi^{(1)}\mathscr{D}\pi_\psi\mathscr{D}\pi_{\bar{\psi}}\mathscr{D}C\mathscr{D}\bar{C}\mathscr{D}\lambda\mathscr{D}\mu\\&\quad\cdot\exp\Big\{\mathrm{i}\int\mathrm{d}^4x(\mathscr{L}^P_{\mathrm{eff}}+J^\mu_aB^a_\mu+\bar{J}\psi+\bar{\psi}J+\bar{\xi}C+\bar{C}\xi+\eta^m\lambda_m\\&\quad+\zeta^n\mu_n+v^\mu_aV^a_\mu+a^\mu_aA^a_\mu+s_aS^a+p_aP^a)\Big\}.\end{aligned}\tag{41}$$

在手征变换

$$\psi'(x)=(1+\mathrm{i}\varepsilon^\sigma(x)\gamma_5T_\sigma)\psi(x),\quad \pi'_\psi(x)=\pi_\psi(x)(1-\mathrm{i}\varepsilon^\sigma(x)\gamma_5T_\sigma),$$
$$\bar{\psi}'(x)=\bar{\psi}(x)(1+\mathrm{i}\varepsilon^\sigma(x)\gamma_5T^\sigma),\quad \pi'_{\bar{\psi}}(x)=(1-\mathrm{i}\varepsilon^\sigma(x)\gamma_5T_\sigma)\pi_{\bar{\psi}}(x) \tag{42}$$

下，

$$\delta I^P=\delta\int\mathrm{d}^4x\mathscr{L}^P=\int\mathrm{d}^4x\varepsilon^\sigma(x)(\partial^\mu A^\sigma_\mu-2mP^\sigma-gf^\sigma_{bc}A^b_\mu B^{c\mu}). \tag{43}$$

其中 $\varepsilon^\sigma(x)$ 是四维时空区域边界上为零的无穷小任意函数。变换(42)的 Jacobi 行列式为 1[6]。又因为 $\delta\Phi^{(1)}_3=-\mathrm{i}\varepsilon^\sigma(x)\gamma_5T_\sigma\Phi^{(1)}_3$，其他约束条件和规范条件在(42)式变换下不变。这样生成泛函(41)式在(42)式变换下的不变性，有

$$\begin{aligned}&\int\mathscr{D}B\mathscr{D}B_{(1)}\mathscr{D}\psi\mathscr{D}\bar{\psi}\mathscr{D}\pi\mathscr{D}\pi^{(1)}\mathscr{D}\pi_\psi\mathscr{D}\pi_{\bar{\psi}}\mathscr{D}C\mathscr{D}\bar{C}\mathscr{D}\lambda\mathscr{D}\mu[\partial^\mu A^\sigma_\mu-2mP^\sigma\\&\quad-gf^\sigma_{bc}A^b_\mu B^{c\mu}-\mathrm{i}\lambda_3\gamma_5T_\sigma\Phi^{(1)}_3+\mathrm{i}\bar{J}\gamma_5T_\sigma\psi+\mathrm{i}\bar{\psi}\gamma_5T_\sigma J+f^\sigma_{bc}v^b_\mu A^{c\mu}+f^\sigma_{bc}a^b_\mu V^{c\mu}\\&\quad-d^\sigma_{bc}s_bP^c+d^\sigma_{bc}p^bS^c]\exp\Big\{\mathrm{i}\int\mathrm{d}^4x\Big(\mathscr{L}^P_{\mathrm{eff}}+J^\mu_aB^a_\mu+\bar{J}\psi+\bar{\psi}J+\bar{\xi}C+\bar{C}\xi\\&\quad+\eta^m\lambda_m+\zeta^\mu\mu_n+v^\mu_aV^a_\mu+a^\mu_aA^a_\mu+s_aS^a+p_aP^a\Big)\Big\}=0.\end{aligned}\tag{44}$$

当所有外源为零时，在约束超曲面上，由(44)式得

$$\langle0|[\partial^\mu\hat{A}^a_\mu(x)-2m\hat{P}^a(x)-gf^a_{bc}\hat{A}^b_\mu(x)\hat{B}^{c\mu}(x)]|0\rangle=0. \tag{45}$$

此即广义 QCD 中轴矢流部分守恒（PCAC）的一种表达式。将(44)式关于 $v^b_\nu(y)$ 和 $v^c_\lambda(z)$ 求泛函微商，然后让所有外源为零，在约束超曲面上，有

$$\begin{aligned}&\langle0|T^*[(\partial^\mu\hat{A}^a_\mu(x)-2m\hat{P}^a(x)-gf_{de}^a\hat{A}^d_\mu(x)\hat{B}^{e\mu}(x))\hat{V}^b_\nu(y)\hat{V}^c_\lambda(z)]|0\rangle\\&=\mathrm{i}\delta^{(4)}(x-y)f^a_{be}\langle0|T[\hat{A}^e_\nu(x)\hat{V}^c_\lambda(y)]0\rangle+\mathrm{i}\delta^{(4)}(x-z)f^a_{ce}\langle0|T[\hat{A}^e_\lambda(x)\hat{V}^b_\nu(y)]|0\rangle.\end{aligned}\tag{46}$$

这就是 AVV 顶角的广义 PCAC 关系。相应的问题在文献[6]和[10]的讨论中，忽略了对约束的处理。这里给予了一个补充说明。

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Canonical Ward Identities in Generalized QCD

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Abstract

The canonical Ward identities for a system with singular higher-order Lagrangian are derived and some application to the generalized QCD are given. The new relations of the Ward identities for gauge ghost field proper vertices are obtained which differ from the usual Ward-Takáhashi identities arising from BRS invariance. The expressions for PCAC and generalized PCAC of AVV vertices are also obtained.

Key words field theories with higher-order derivatives, Dirac's theory of constrained system, generalized QCD, Ward identities.