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Relative Information Entropy of Particle Production in High-Energy Induced Nuclear Reactions

EMU01 Collaboration

A new characteristic variable, relative entropy R, is suggested to analyze the multiplicity distributions of particles produced in 14.6, 60 and 200A GeV ^{16}O and 200A GeV ^{32}S induced nuclear reactions. It is found that R appears approximately energy independent in the present energy region. The saturation of the rapidity-window dependence of R shows that the dominant part of created entropy is concentrated in the central rapidity region. Experimental data are in agreement with the prediction based on the FRITIOF of Lund model.

Key words: high-energy induced nuclear reaction, particle production, relative information entropy.

1. INTRODUCTION

The production of a great number of hadrons with low transverse momentum in the final states of high energy collisions is the "soft process," which cannot be explained by the existing theory on strong interaction, Quantum Chromodynamics (QCD). It is suggested that researches on multiparticle production in high energy heavy ion induced nuclear reactions will provide signals on the phase transition of strong interacting matters and the formation of quark gluon plasma [1]. One of

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characteristic measurable variables for describing the final state of colliding system is the multiplicity of produced particles, and the detailed analyze of the properties of multiplicity distribution will enable us to explore the background of the reaction mechanism of the "soft" process in a wider energy range.

Recently some authors [2] discussed the multiplicity distributions in proton-proton (antiproton) collision (for short, denoted as pp) with the center of mass system (c.m.s.) energy $\sqrt{s}$ from several tens to several hundreds GeV in terms of the information entropy method. They suggested a scaled entropy S/Y_{max}, where entropy S and the maximum c.m.s. rapidity Y_{max} are defined as

$$S^{pp} = -\sum_{n=1}^{N} P^{pp}(n)\ln P^{pp}(n), \tag{1}$$

$$Y^{pp}_{max} = \ln\frac{\sqrt{s} - 2m_p}{m_\pi}, \tag{2}$$

in above expressions, m_p and m_π are masses of proton and pion, $P^{pp}(n)$ is the multiplicity n-distribution for produced particles (most of them are pions) and N is the maximum multiplicity. They gave the following experimental results: This kind of the scaled entropy seems to be independent of energy at high energy region whether in the whole phase space or in some limited rapidity windows $|y| \leq Y_c$, but depends on the scaled rapidity window

$$\xi = Y_c/Y_{max} \tag{3}$$

The quantity S/Y_{max} (the so-called entropy per unit rapidity) as defined above, however, does not have very clear physical meaning and can not be applied to investigate nucleus-nucleus (for short, denoted as BA) collisions. In addition, no appropriate criterion can be used to define Y^{BA}_{max} for different combinations of projectile and target nuclei as defined in Eq.(2) for pp collisions Faced with these difficulties, X. Cai *et al.* [3] proposed a new characteristic variable-relative entropy R, which not only has clear and definite physical meaning but also can be used to treat pp and BA collisions consistently. The purpose of this paper is to report experimental data on relative entropy of multiparticle production for oxygen and sulfur ions induced nuclear reactions at energies up to 200A GeV from the EMU01 Collaboration. In Sec. 2, the concept of relative entropy given in [3] will be briefly reviewed and the necessary discussion on nuclear collision geometry is made. The description for EMU01 experiment and Monte Carlo simulations (FRITIOF) is given in Sec. 3. Section 4 includes experimental data analyses and physical discussion. Section 5 presents conclusions.

2. RELATIVE ENTROPY FOR PARTICLE PRODUCTION

Let us consider a measure of uncertainty for a trial (event) with a finite set of N possible outcome (final states). According to the information theory [3,4], the information entropy is defined as

$$H = -\sum_{n=1}^{N} P(n)\log_I P(n), \tag{4}$$

where the base I of the logarithm determines the unit of entropy and $P(n)$ is the probability for the n-th outcome and satisfies the normalization condition

$$\sum_{n=1}^{N} P(n) = 1. \tag{5}$$

If a trial is certain, namely, its probability for a specific outcome is 1 and those for all other $N-1$ outcomes are 0, the entropy should be minimum, equal to zero. Otherwise, if a trial is completely uncertain, i.e., the probabilities for all N outcomes are equal, H should be maximum

$$H_{max} = \log_I N. \tag{6}$$

Obviously, the entropy as a measure of the uncertainty of a variable, has real meaning only for making comparison among the trials whose numbers of possible outcomes are equal. For trials with different outcome number, the values of their entropies are in fact incomparable with each other even by using the same base I. This is the main difficulty in the suggestion proposed by the authors of [2].

X. Cai *et al.* [3] suggested a new quantity of relative entropy defined as

$$R = \frac{H}{H_{max}} \tag{7}$$

which could be used as a measure of the uncertainty of trials. The difficulty appeared in [2] can be overcome by using this new quantity because the relative entropy for trials with different number of outcomes has the same range of value,

$$0 \leqslant R \leqslant 1, \tag{8}$$

and it is a dimensionless quantity independent of the base I.

This new quantity has a more important physical meaning for it can be used simultaneously to treat pp and BA collisions. An important label of a final state in high energy collisions is its multiplicity of produced particles. For given energy and given target and projectile the maximum multiplicity is exactly the number N of possible final states. For example, the number of possible final states (i.e., maximum multiplicity) for pp collision at c.m.s. energy $\sqrt{s}$ can in principle be estimated by

$$N^{pp} = \frac{\sqrt{s} - 2m_p}{m_\pi}. \tag{9}$$

For BA collisions (the nucleon numbers of the projectile and the target are B and A, respectively) whose corresponding nucleon-nucleon c.m.s. energy is the same as $\sqrt{s}$, the nucleus-nucleus reaction can be regarded as the linear composition of subprocess of participating nucleon-nucleon collisions by using the normal nuclear collision geometry picture, and the number of possible final states might be estimated by

$$N^{BA} = W(B,A,b)\frac{N^{pp}}{W(1,1,0)}. \tag{10}$$

In the above expression $W(B,A,b)$ is the participating nucleon number, a function of B, A and impact parameter b [5], i.e.,

$$W(B,A,b) = B + A\left[1 - \left(1 - \left(\frac{B}{A}\right)^{2/3}\right)^{3/2}\right]\lambda(b), \tag{11}$$

where $\lambda(b) = 1$ when $b = 0$ and $\lambda(b) = 0$ when $b = (B^{1/3} + A^{1/3})r_0$. r_0 is the radius of a nucleon. In this paper, we do not consider the nonlinear cascade effects. The maximum possible final states can be estimated by the value of λ for central collisions at $b = 0$. In terms of the above estimation of the

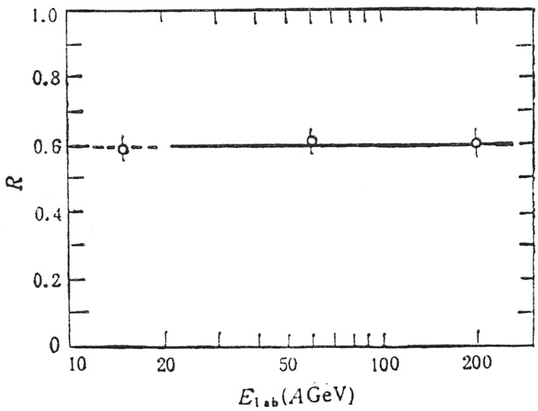

Fig. 1
Relative entropy in whole phase space in ^{16}O + Em interactions at 14.6, 60 and 200A GeV. The solid line is the result of FRITIOF simulation, o : data of ^{16}O + Em.

number of possible final states, the maximum entropy can be calculated; then, relative entropy of multiplicity distribution can be obtained for the considered experiments.

3. EMU01 EXPERIMENT AND FRITIOF SIMULATION

Emulsion detector technique for nuclear research has two features for its complete interaction-star picture and high spatial resolution so that it is specially suitable for the measurements of multiplicity and angular for produced particles in final state of high energy induced nuclear reactions. The EMU01 Collaboration exposed both emulsion stacks along the beam line and emulsion chambers perpendicular to the beam. The experiments have been performed with oxygen-ion beam at 14.6 A GeV at the

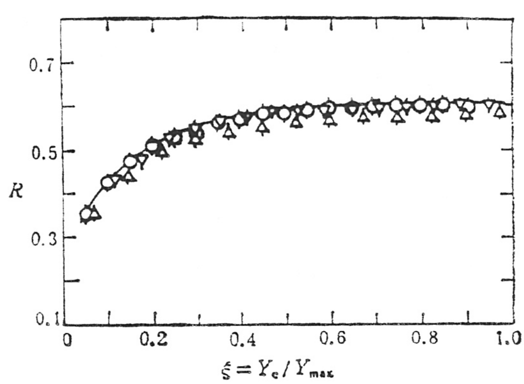

Fig. 2
Dependence of relative entropy on scaled rapidity window size in ^{16}O + Em reactions at 14.6, 60 and 200A GeV. The curve is the result of FRITIOF simulation, Δ: 14.6 A GeV; ∇: 60A GeV; o : 200A GeV.

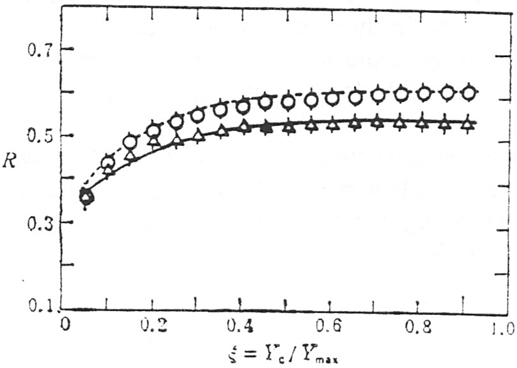

Fig. 3
Relative entropy in ^{16}O + Em and ^{32}S + Em reactions at 200A GeV; o : ^{16}O + Em; Δ: ^{32}S + Em; the dashed and the solid curves are FRITIOF results.

BNL/AGS in USA and with oxygen-ion beam at 60, 200A GeV and sulfur-ion beam at 200A GeV at the CERN/SPS in Europe. The measurement instrument for EMU01 experiments is a digital controlled semiautomatic microscope with solution to 1 μm for x, y, z coordinates, thus the precision for project angle to 3×10^{-2} mrad. The EMU01 experimental details have been described in previous publication [6].

The EMU01 experimental data reported in next section are compared with the results of Monte Carlo simulation of FRITIOF [7]. This simulation program package is based on the Lund model for high energy nucleus-nucleus collisions, a generalization of the Lund model for hadron-hadron scatterings. The main feature of the model is the formation of strings (longitudinal excitation) through the momentum exchange between partons in projectile and target hadrons, and the nonlinear cascade effect can be neglected due to the long formation time.

4. DATA ANALYZE AND PHYSICAL DISCUSSIONS

Unbiased data samples of 14.6, 60, 200A GeV oxygen-ion and 200A GeV sulfur-ion induced interactions in emulsion (^{16}O + Em and ^{32}S + Em) are used in our analyses, respectively.

In Fig. 1, relative entropies in the whole phase space for three energies are given. The solid line is the FRITIOF result in high energy Monte Carlo simulation, and the broken line is the extrapolation to low energy region. It is obvious that the relative entropy R of produced particles is almost a constant, independent of the energy and approximately equal to 0.6 in the whole phase space. Experimental data are in good agreement with FRITIOF model prediction.

Study on the variation of R with the size of the considered rapidity window also gives very interesting function between them. With the scaled rapidity window size ξ defined in Eq. (3), we draw $R(\xi) - \xi$ for ^{16}O + Em data at three energies in Fig. 2. Very clearly, the relative entropy increases sharply at first when the size of the symmetric c.m.s. rapidity window increase from a very small value and then slowly reaches the saturation when the window size approaches the whole phase space. In addition, the $R(\xi) - \xi$ relation shows very weak dependence on the energy. FRITIOF simulation for two higher energies (60, 200A GeV) gives results in firm agreement with experimental characteristic. Due to the high energy constrain of the original Lund model, it is not suitable to make FRITIOF simulation for low energy of 14.6A GeV.

Comparisons between the relations of $R(\xi) - \xi$ for 200A GeV ^{16}O + Em and ^{32}S + Em interactions are given in Fig. 3. With the same target nucleus FRITIOF simulation results are in agreement with experimental data for these two projectile nuclei.

In our earlier works [8], an approximate scale property has been shown from data of multiplicity distributions at 14.6, 60 and 200A GeV. In terms of a scaled multiplicity $z = n/<n>$ ($<n>$ is mean multiplicity) the scaled distribution $\psi(z) = <n>P(n)$ is approximately independent of energy.

Let the relation between the maximum multiplicity N and the mean multiplicity $<n>$ be $N = K(n)$, and define a new scaled variable $u = z/K$ and a corresponding scaled distribution function $\phi(u) = K\psi(z)$. The saturation values R_{sat} of relative entropy $R(\xi)$ in whole phase space can readily be derived from Eqs. (4), (6) and (7)

$$R_{sat} = 1 - \frac{\beta}{\log_l\langle n\rangle + \log_l K}, \tag{12}$$

where

$$\beta = \int_0^1 du\phi(u)\log_l\phi(u). \tag{13}$$

In information theory β is called the negative entropy of distribution $P(n)$. The phenomena that the scaled distribution $\psi(z)$ is independent of incident energy has been shown themselves in the measurement of relative entropy in present experiments with three energies. Experimentally $R < 1$, which means the negative entropy $\beta < \log_l <n> + \log_l K$. Although β remains a constant at present energy region, we cannot decide if the negative entropy would decrease at higher energies. This decrease already has a hint in the phenomena of KNO scaling variation from CERN/ISR energy region ($\sqrt{s}$ = 10 ~ 60 GeV) to CERN/SPS energy region ($\sqrt{s}$ = 540 ~ 900 GeV) in proton-proton (antiproton) collision experiments.

The saturation phenomena of relative entropy with the increase of rapidity window size, namely, the negative entropy is maximum in central rapidity region in nucleus-nucleus collisions, indicates that in high energy induced nuclear reactions, the dominant part of entropy is created in central rapidity region, and the entropy production can be neglected in fragmentation regions. The experimental fact of entropy concentration in central rapidity region is consistent with some model predictions based on chaos and coherent production mechanisms [10].

5. CONCLUSIONS

In this paper, we have introduced a new characteristic quantity, relative entropy R, to study the multiplicity in Oxygen and Sulphur induced nuclear reactions in the energy region from 14.6 to 200A GeV. The following phenomena have been observed in our experiments.

1) The value of R is insensitive to energy in the whole rapidity space as well as in different sizes of c.m.s. symmetric rapidity windows in present energy region.

2) $R(\xi)$ increases with increasing ξ. It increases rapidly in small ξ region and slowly in large ξ region and then approaches saturation.

3) R decreases with heavier projectile nucleus for the same target at the same energy.

All of the above experimental facts agree with the prediction give by the FRITIOF simulation.

REFERENCES

[1] X. Cai, *Progress in Physics* (in Chinese), **8** (1988), p. 186; H. Satz, CERN preprint, CERN-TH 6216/91, Oct. 1991; H. R. Schmidt and J. Schukraft, GSI preprint, GSI-92-19 March 1992.

[2] V. Simak, M. Sumbera and I. Zborovsky, *Phys. Rev.*, **B206** (1988), p. 159.

[3] Z. M. Zhou and X. Cai, *Chinese Phys. Lett.*, **6** (1989), p. 317; X. Cai, D. C. Zhou, J. C. Zhou and Z. M. Zhou, Proc. of Intern. Workshop on Correlations and Multiparticle production, Marburg, Germany, May 14-16, 1990, Edited by M. Plümer, S. Raha and R. M. Weiner, World Scientific.

[4] L. Brillouin, *Science and Information Theory,* Academic, New York, 1962; A. Wehrl, *Rev. Mod. Phys.*, **50** (1978), p. 221.

[5] I. Otterlund, *Nucl. Phys.*, **A418** (1984), p. 87c.

[6] EMU01 Collaboration: M. I. Adamovich *et al.*, *Phys. Rev. Lett.*, **62** (1989), p. 2801; X. Cai *et al.*, EMU01 Collaboration, Proceedings of the 16th International Conference on Nuclear Tracks in Solids, Sept. 7-11, 1992, Beijing, China.

[7] B. Nilsson-Almqvist and E. Stenlund, *Comp. Phys. Comm.*, **43** (1987), p. 373; references therein.

[8] EMU01 Collaboration: M. I. Adamovich *et al.*, *Phys. Lett.*, **B223** (1989), p. 262.

[9] UA5 Collaboration: G. J. Alner *et al.*, *Phys. Rep.*, **154** (1987), p. 247.

[10] P. A. Carruthers *et al.*, *Phys. Lett.*, **B212** (1988), p. 369; R. M. Weiner, *Z. Phys.*, **C38** (1988), p. 396; L. Z. Yu, D. C. Zhou, X. Cai and Z. M. Zhou, *Phys. Rev.*, **D43** (1991), p. 2856.

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