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Equation of State in the F-L Model and a Physical Picture of Deconfinement Transition

Deng Shenghua and Li Jiarong

(Institute of Particle Physics, Huazhong Normal University, Wuhan, Hubei, China)

The equation of state in the F-L model is calculated at the finite temperature and density. It turns out from the analysis of the isotherms of pressure versus net baryon number density that in the mean-field approximation, the deconfinement phase transition in the F-L model is of the first order.

Key words: equation of state, soliton bag model, phase structure, deconfinement phase transition.

1. INTRODUCTION

After the pioneer work of Friedberg and Lee [1], the non-topological soliton bag model (the F-L model has been investigated in detail [2-4]. An intuitive physical explanation of color confinement was provided by soliton solutions at the zero temperature [1,2]. At the finite temperature and density, a physical picture of deconfining phase transition was given in our previous work [4]. But the phase structure and characteristic features of the transition should be further investigated.

It is well known that investigating the equation of state of a system at the finite temperature is an efficient approach to analyze the phase structure and characters of the phase transition. In particular, the equation of state of the strong interacting system is very important for exploring the phase transition from the hadron matter to QGP in high energy heavy-ion collisions. The aim of this paper is to investigate the equation of state in the F-L model at the finite temperature and density, and to

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analyze the phase structure and features of the deconfining phase transition.

2. MODEL AND EQUATION OF STATE

The Lagrangian density of the F-L model contains a quark field ψ and a scalar field σ. Its form reads

$$\mathscr{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - g\sigma)\psi + \frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - U(\sigma), \tag{1}$$

with

$$U(\sigma) = \frac{a}{2}\sigma^2 + \frac{b}{6}\sigma^3 + \frac{c}{24}\sigma^4 + B, \tag{2}$$

where B denotes the bag constant, a, b, c and g are constants which are generally obtained by fitting hadronic properties appropriately.

In the thermal equilibrium system described by Eq. (1), where the temperature is T and the chemical potential is μ, any physical observable should be given by its corresponding Gibbs average

$$\langle A \rangle = \frac{\mathrm{Tr}\{\exp[-\beta(H - \mu N)]A\}}{\mathrm{Tr}\{\exp[-\beta(H - \mu N)]\}} \tag{3}$$

where $\beta = 1/T$, H and N represent the Hamiltonian and particle number operators respectively. In general, it is too hard to evaluate the equation of state of a real system at the finite temperature exactly, but we can find it by making use of the mean-field approximation [5]. For this purpose, following Linde's technique [6], we shift σ by its Gibbs average valued $v \equiv <\sigma>$: and rewrite the Lagrangian density as follows

$$\sigma \to \sigma + v. \tag{4}$$

$$\mathscr{L}_{\mathrm{eff}} = -U(v) + \mathscr{L}_0 + \mathscr{L}_i, \tag{5a}$$

with

$$\mathscr{L}_0 = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m_{\psi})\psi + \frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - \frac{1}{2}m_{\sigma}^2\sigma^2, \tag{5b}$$

and

$$\mathscr{L}_i = -\bar{\psi}g\sigma\psi - \frac{1}{6}(b + cv)\sigma^3 - \frac{c}{24}\sigma^4. \tag{5c}$$

where m_{ψ} and m_{σ} are the effective masses of ψ and the thermal excited field σ, respectively,

$$m_{\psi} = gv, \tag{6}$$

and

$$m_{\sigma}^2 = a + bv + \frac{c}{2}v^2. \tag{7}$$

In addition, $\mathscr{L}_{\mathrm{eff}}$ contains some terms which are linear in σ, but these terms can be dropped because of their negligible contributions to $\mathscr{L}_{\mathrm{eff}}$ [5].

According to Eq. (5) and the grand canonical partition function

$$Z = \mathrm{Tr}\{\exp[-\beta(H - \mu N)]\} \tag{8}$$

all the standard thermodynamic properties can be determined with the aid of the functional technique. For example, the pressure of the system reads

$$P = \frac{1}{\beta}\frac{\partial \ln Z}{\partial V}. \tag{9}$$

Taking the chemical potential to be μ for fermion and zero for boson, in the mean-field approximation [5], we have

$$\begin{aligned}\ln Z &= \ln\left\{\int [\mathrm{d}\sigma][\mathrm{d}\bar{\psi}][\mathrm{d}\psi]\exp\left\{\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x\,[-U(v) + \mathscr{L}_0 + \bar{\psi}\mu\gamma_0\psi]\right\}\right\}\\ &= -VU(v)\beta - V\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left[\frac{1}{2}\beta\omega_\sigma + \ln(1 - \mathrm{e}^{-\beta\omega_\sigma})\right]\\ &+ V.\delta\int\frac{\mathrm{d}^3k}{(2\pi)^3}\{\beta\omega_\psi + \ln[1 + \mathrm{e}^{-\beta(\omega_\psi+\mu)}]\\ &+ \ln[1 + \mathrm{e}^{-\beta(\omega_\psi-\mu)}]\},\end{aligned} \tag{10}$$

where δ refers to the degenerate factor, δ = 2(spin) × 2(flavor) × 3(color), and

$$\omega_i = \sqrt{k^2 + m_i^2}, \quad i = \sigma, \psi. \tag{11}$$

Then, one can obtain the pressure of the system

$$\begin{aligned}P &= -U(v) + \frac{1}{3}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{k^2}{\omega_\sigma}\frac{1}{\mathrm{e}^{\beta\omega_\sigma} - 1}\\ &+ \frac{\delta}{3}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{k^2}{\omega_\psi}\left[\frac{1}{\mathrm{e}^{\beta(\omega_\psi+\mu)} + 1} + \frac{1}{\mathrm{e}^{\beta(\omega_\psi-\mu)} + 1}\right],\end{aligned} \tag{12}$$

and the energy density

$$\begin{aligned}\varepsilon &= \frac{T^2}{V}\frac{\partial \ln Z}{\partial T} + \frac{\mu T}{V}\frac{\partial \ln Z}{\partial \mu} = U(v) + \int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\omega_\sigma}{\mathrm{e}^{\beta\omega_\sigma} - 1}\\ &+ \delta\int\frac{\mathrm{d}^3k}{(2\pi)^3}\omega_\psi\left[\frac{1}{\mathrm{e}^{\beta(\omega_\psi+\mu)} + 1} + \frac{1}{\mathrm{e}^{\beta(\omega_\psi-\mu)} + 1}\right].\end{aligned} \tag{13}$$

In obtaining Eqs. (12) and (13), zero-point energy has been neglected. It is evident that, in Eqs. (12) and (13), the first term corresponds to the classical potential energy density, while the last two terms come from the contributions of the boson and fermion, respectively, which are equivalent to those provided by the ideal gases of quasi-particles whose effective masses are m_σ and m_ψ, respectively. Since both m_ψ and m_σ are v-dependent, the relationship between p and ϵ, namely, the equation of state of the system, can be obtained by removing v from Eqs. (12) and (13).

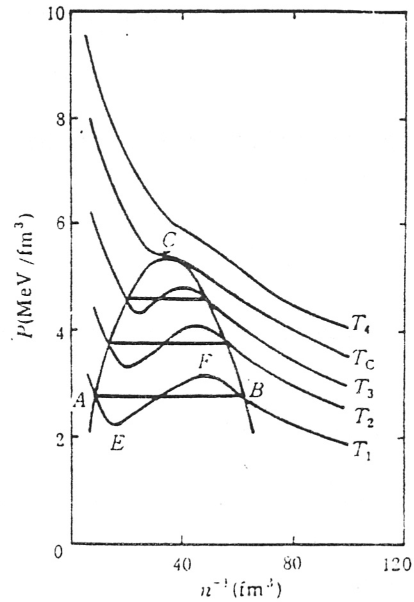

Fig. 1
The isotherms of pressure p versus net baryon number density n. $T_1 < T_2 < T_3 < T_c (= 115\text{Mev}) < T_4$.

3. PHASE STRUCTURE AND DECONFINEMENT

In our previous calculation, the effective potential and the soliton solutions of field equations in the F-L model [4] showed that the properties of soliton solutions were closely related to the vacuum structure. At lower temperatures and densities, there existed stable soliton solutions with the quarks being confined within soliton bags to form hadrons. As the temperature or density increased, solitons became unstable and there existed a metastable state of the system. When the temperature or density reached its critical value, the confinement was entirely removed with the disappearance of solitons. In order to analyze the phase structure and the specific features of the system in detail, it is worth to investigate the isotherms of p versus net baryon number density n determined by

$$n = \frac{T}{V}\frac{\partial \ln Z}{\partial \mu} = \delta \int \frac{d^3k}{(2\pi)^3}\left[\frac{1}{e^{\beta(\omega_{ll}+\mu)}+1} - \frac{1}{e^{\beta(\omega_{ll}-\mu)}+1}\right] \tag{14}$$

The isotherms for different temperatures are plotted in Fig. 1 where the values of model parameters are chosen to be [4]: $a = 17.70\text{fm}^{-2}$, $b = -1457.4\text{fm}^{-1}$, $c = 20000$ and $g = 12.16$, and the horizontal line is the Maxwell structure for phase equilibrium.

It can be seen from Fig. 1 that the isotherms are rather similar to the Van der Waals' isotherms for liquid-gas system. At a given temperature, for instance T_1, the isotherm has a minimum and a maximum. According to [4], we know that when $n < n_B$, there exist solitons, then the quarks are confined and only the hadron phase presents. When $n > n_A$, the solitons disappear, and the confinement is no longer effective, then only the QGP phase presents. When $n_B < n < n_A$, the equilibrium configuration is a mixture of the hadron phase and QGP phase. The points A and B, having the same value of μ, are two ends of the two-phase-coexistent region. The segments BF and AE on the isotherm correspond to the metastable hadron phase and QGP phase, respectively. The region between E and F on isotherm is a unstable region.

As T increase, the above-mentioned two points of the isotherms becomes closer and closer, and the coexistent region becomes smaller and smaller (curve ACB is the coexistent curve). At $T = T_c$, these two points go the same point, inflection point, namely the critical point C. So the deconfinement phase transition in the F-L model, being essentially of the Van der Waals type, is of the first order.

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