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Transport Theory Approach to Parton Multiple Scattering in Nuclei---Monte Carlo Simulation*

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Transport theory approach to the parton multiple hard scattering in nucleus and the corresponding Monte Carlo techniques are developed. The difficulty of Q^2 dependence is overcome by biased sampling method. As an example, the ratio of π^+ production in p (E_p = 400 GeV) + W reaction to p + p reaction is calculated. The calculation result agrees well with the experimental data.

Since the current theory of hadronic matter has prospected the possible existence of new forms of nuclear matter and the corresponding liquid-gas and confine-deconfine phase transitions, a great deal of experimental and theoretical research work has been produced.

One of the issues that people are interested in is the behavior of the transverse momentum of fast particles (mainly pions) produced in relativistic heavy ion collisions. Some people think that the possible signal of phase transition would be exhibited there. In our opinion, the transport theory and the corresponding Monte Carlo (M-C) technique should be a powerful tool for such kind of studies.

As the first step we have studied the transport of partons in nucleus based on parton-parton hard scattering by M-C techniques. The puzzles in dealing with parton multiple scattering in nucleus were mainly the Q^2 dependence of parton structure function, fragmentation function and hard scattering differential cross section $d\sigma/dt$. They are overcome by a special biased sampling technique in this paper.

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scattering within nucleus, which is the main mechanism for single hadron production. The target nucleus is assumed as a sphere of radius $R = r_0 A^{1/3}$ (r_0 = 1.14 fm) and uniform density $\rho = 3/(4\pi r_0^3)$.

The invariant inclusive cross section of single hadron production by parton multiple hard scattering in nucleus can be expressed as

$$E_F \frac{d\sigma(E_I, E_F)}{d^3P_F} = \sum_{n=1}^{\infty} \left[E_F \frac{d\sigma(E_I, E_F)}{d^3P_F} \right]_n,$$

$$\left[E_F \frac{d\sigma(E_I, E_F)}{d^3P_F} \right]_n = \sum_{i,j} \int dx d^2P_t \frac{dz}{z} d^2q_t G_{I/i}(x, P_t, Q_I^2) \times H_n(i, P; j, P') F_{j/F}(z, q_t, Q_F^2). \tag{1}$$

where $G_{I/i}$ refers to the structure function [2], $F_{j/F}$ stands for the fragmentation function [3], $H_n(i,p;j,P')$ is the invariant inclusive cross section of n sequential parton-nucleus collisions. H_n is constructed by the differential cross section of parton-nucleon collision

$$\sigma(i, P; j, P') = \frac{1}{\pi} \sum_k \int dx_k d^2P_{tk} G_{N/k}(x_k, P_{tk}, Q^2) \times \frac{d\sigma}{d\hat{t}}(ik \to jX) \delta\left(1 + \frac{\hat{t}}{\hat{s}} + \frac{\hat{u}}{\hat{s}}\right), \tag{2}$$

and the corresponding total inelastic cross section

$$\sigma(i, P) = \frac{1}{2} \sum_j \int \frac{d^3P'}{E'} \sigma(i, P; j, P'), \tag{3}$$

as following

$$H_n(i, P; j, P') = \sum_{1,\cdots,n-1} \rho^n \int \frac{d^3P_1}{E_1} \cdots \int \frac{d^3P_{n-1}}{E_{n-1}} \sigma(i, P; j, P_1) \cdots \sigma(n-1, P_{n-1}; j, P') \times D(i, j; P_1, \cdots, P_n), \tag{4}$$

$$D(i, j; P_1, \cdots P_n) = \int d^2b \int_{z_0}^{z_e} dz_1 \cdots \int_{z_{n-1}}^{z_e} dz_n \exp\left[-\rho \sum_{k=0}^{n} \sigma(k, P_k)(z_{k+1} - z_k) \right]. \tag{5}$$

Here the same notation is used for the order number of collisions and the label of partons. Z_0, Z_i and Z_e are the entry, the i-th collision and the exit points along the straight line trajectory in the target, respectively. In Eq.(2) $d\sigma/d\hat{t}$ stands for the parton-parton differential cross section[4]. Following Ref.[1] we regularize them by adding 1 GeV to $\hat{s}$, $\hat{t}$ and $\hat{u}$ in denominators. A M-C simulation has been developed here instead of expanding the exponential in D of Eq.(5) and cutting-off at the term of the order of $A^{4/3}$ as in Ref.[1].

In statistical physics, the observed value of a physical variable A_{obs} is given by ensemble average, e.g., in one dimensional case,

$$A_{obs} = \langle A \rangle = \int_{-\infty}^{\infty} A(x) f(x)\, dx, \quad \int_{-\infty}^{\infty} f(x)\, dx = 1. \tag{6}$$

For calculating A_{obs} what one does in M-C simulation is to sample phase point x according to the density distribution function $f(x)$ and estimate A_{obs} as follows

$$A_{obs}^{M-C} = \frac{1}{I}\sum_{i=1}^{I} A(x_i). \tag{7}$$

where I stands for the total number of samples generated.

In case of unknown or too complicated density distribution function, one can first sample x from certain approximate density distribution function $f_a(x)$ and a bias factor $f(x_i)/f_a(x_i)$ should then be introduced into the expression of A_{obs}^{M-C} (where x_i refers to the i-th sample).

The M-C simulation of the parton transport in nucleus based on parton-parton hard scattering can then be designed as the following procedures:

1. Randomly select the impact parameter b out of the geometric cross section of the target nucleus.

2. Determine whether the incident parton is quark (u or d) or gluon. This is done by first sampling from a proper density distribution function $f_1(Q_0)$ (Q^2 = 5 GeV^2/c^2), and then, introducing a corresponding bias factor

$$B_1 = f_1(Q)/f_1(Q_0). \tag{8}$$

where $f_1(Q)$ is composed of the normalization factor of the structure function in the following way

$$f_1(Q) = N_{I/i}^G(Q^2) \Big/ \sum_k N_{I/k}^G(Q^2). \tag{9}$$

3. Sample the transverse momentum of the incident parton p_t from the transverse momentum part of the structure function

$$f(P_t)dP_t = \frac{b_t^2}{2\pi}\exp(-b_tP_t)dP_t \quad \left(b_t = \frac{2}{\langle P_t\rangle} = 2.05\ C/\mathrm{GeV}\right), \tag{10}$$

As for the longitudinal part, $p_{//}$, first, sample from the effective structure function [5]

$$G_{I/i}(x) = \frac{\alpha e^{-\alpha x}}{e^{-0.05x} - e^{-\alpha x}}, \quad (\alpha = 2.0) \tag{11}$$

and then introduce the corresponding bias factor

$$B_2 = G_{I/i}(x, Q^2)/G_{I/i}(x). \tag{12}$$

where $G_{I/i}(x,Q^2)$ stands for the longitudinal part of the structure function.

4.Sample the position of the n-th parton-nucleon collision within nucleus from the density distribution function

$$\rho\sigma(n-1, P_{n-1})\exp[-\rho\sigma(n-1, p_n-1)(z_n - z_{n-1})]/ \{1 - \exp[-\rho\sigma(n-1, p_n-1)(z_e - z_{n-1})]\} \tag{A}$$

5. Determine if the parton after the n-th collision is a quark or a gluon randomly from the density distribution function

$$\sigma(n-1, P_{n-1}; n)/\sigma(n-1, P_{n-1}). \tag{13}$$

where

$$\sigma(i, P; j) = \frac{1}{2}\int \frac{d^3P'}{E'}\sigma(i, P; j, P'). \tag{14}$$

6. The momentum of the parton after the n-th collision should originally be sampled from the corresponding parton-nucleus differential cross section $\sigma(n\text{-}1, p_{n-1}; n, p_n)$. Since σ is an extremely complicated three dimensional function, we first sample p_{nt} from the density distribution (10) and then sample $p_{n//}$ by the following approximate longitudinal momentum density distribution

$$\begin{aligned}\sigma_a(n-1, P_{n-1}; n, P_n) &= \sigma(n-1, P_{n-1}; n, P_n)|_{P_{nt}=\langle P_t\rangle, \varphi_n=0}/\sigma_a(n-1, P_{n-1}),\\ \sigma_a(n-1, P_{n-1}) &= \frac{1}{2}\sum_n \int dP_{n//}\sigma(n-1, P_{n-1}; n, P_n)|_{P_{nt}=\langle P_t\rangle, \varphi_n=0}.\end{aligned} \tag{15}$$

Here the summation is only concerned with the various types n of partons. The sequential bias factor is needed to be introduced, that is,

$$B_3^{(n)} = \frac{[\sigma(n-1, P_{n-1}; n, P_n)/\sigma(n-1, P_{n-1})]}{[\sigma_a(n-1, P_{n-1}; n, P_n)/\sigma_a(n-1, P_{n-1})]b_t^2\exp(-b_tP_t)/(2\pi)}. \tag{16}$$

7. The contribution of the n-th collision to the secondary hadron production in hadron-nucleus collision can be estimated as follows

$$\begin{aligned}\left[E_F\frac{d\sigma(P_I, P_F)}{d^3P_F}\right]_n = \left(\sum_i N_{I/i}^G\right)\pi R^2 \prod_{\alpha=1}^{n-1}\{1-\exp[-\rho\sigma(\alpha-1, P_{\alpha-1})(z_e - z_{\alpha-1})]\}\\ \times \exp[-\rho\sigma(n, p')(z_e - z_n)]B_1B_2\prod_{\alpha=1}^{n}\frac{\sigma(\alpha-1, P_{\alpha-1})}{\sigma(\alpha-1, P_{\alpha-1}; \alpha)}B_3^{(\alpha)}\\ \times \frac{N_{n/F}^F}{P_{n//}}[P_{Ft}b_t^2\exp(-b_tP_{Ft})]F_{n/F}\left(\frac{P_{F//}}{P_{n//}}, Q_F^2\right).\end{aligned} \tag{17}$$

where $N_{n/F}^F$ refers to the normalization factor of the fragmentation function (from parton n to hadron F).

The exponent $\alpha(p_t)$ for the ratio of the invariant inclusive cross section of proton-nucleus collision to proton-proton collision can be calculated by

$$E_F\frac{d\sigma^{PA}(P_I, P_F)}{d^3P_F} = A^{\alpha(P_t)}E_F\frac{d\sigma^{PP}(P_I, P_F)}{d^3P_F}. \tag{18}$$

Fig.1 shows the results of $\alpha(p_t)$ for the ratio of the cross section of $P + W \rightarrow \pi^+ + X$ reaction at incident energy 400 GeV and 90° in nucleon-nucleon CMS to that of $P + P \rightarrow \pi^+ + x$ reaction. The error bars stand for the data taken from Ref.[7], while solid and dotted lines refer to the results of this paper ($n \leq 3$) with and without considering the EMC effect, respectively. As EMC data are unavailable for target

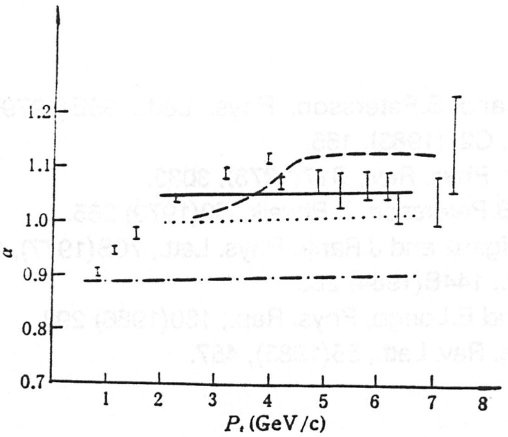

FIGURE.1 The exponent $\alpha(p_t)$ for ratio of invariant inclusive cross section of proton-nucleus (W, A = 184) to proton-proton collisions verses pt for the production of π^+ at 90° in nucleon-nucleon CMS at incident energy E_{lab} = 400 GeV. Solid line: results of this paper with EMC. Dotted line: without EMC. Dash-dotted line:results without EMC and constraint of hard scattering in transport processes. Dashed line:results of Ref.[1]. Data are taken from Ref.[7].

nucleus W(A = 184), an effective EMC modification [6] is deduced by fitting muon scattering data of F_2(Fe)/F_2(D), i.e.

$$f_{EMC}(x) = 1.179 - 0.5085x, \quad (19)$$

is used here. The dash-dotted line is the results without considering EMC effect and the constraint of hard scattering ($Q^2 > 4$ GeV^2/C^2) in transport processes. The dashed line represents the results in Ref.[1], where contributions from n 2 and the shadowing effect of secondary scattering were neglected.

From Fig.1 one sees that our results with EMC effect taken into consideration agree quite well with the experimental data. It turns out that the EMC effect is still important even at such high incident energy.

Although extending the hard scattering treatment to the soft scattering region (p_t < 2 GeV/c) is not reasonable, however, by comparing the dotted and dash-dotted lines with the experimental data one may possibly come to the conclusion that data in the region p_t < 2 GeV/c can be hopefully explained by some kind of combinations of the hard and soft scattering.

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