

# 中微子与极化核子的深度非弹散射过程

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## 摘 要

本文利用文献[1]给出的具有一定自旋取向的层子的分布函数,计算了中微子及反中微子与极化核子深度非弹散射过程的微分截面与不对称度。

部份子模型中关于层子是自旋为 1/2 的费米子的假设已得到实验证实。因而,可以在部份子模型中引入具有一定自旋取向的层子的分布函数<sup>[2]</sup>,并用它们去分析许多极化散射过程。

本文讨论中微子及反中微子与极化核子的深度非弹散射过程,这些过程的微分截面与具有一定自旋取向的层子的分布函数之间有很简单的关系。文中用我们在[1]中得到的分布函数,给出具体的计算结果,并对此结果进行了讨论。

中微子或反中微子与极化核子的深度非弹散射过程  $\nu(\bar{\nu}) + N \rightarrow \mu^-(\mu^+) + \dots$  的微分截面可以写成

$$\frac{d\sigma^r}{dE'dQ} = \frac{G^2 E_\mu}{8\pi^2 E_\nu E_N} \tau_{\mu\nu} W_{\mu\nu}^r \quad (1)$$

其中  $r$  表示核子的 helicity,

$$\tau_{\mu\nu} = -2 \left\{ k_\mu k'_\nu + k_\nu k'_\mu + \frac{q^2}{2} \delta_{\mu\nu} + k'_\rho k_\sigma \varepsilon_{\mu\nu\rho\sigma} \right\}, \quad (2)$$

$k, k'$  分别为初态中微子和末态  $\mu$  介子的四动量,

$$q^2 = (k - k')^2, \quad W_{\mu\nu}^r = \frac{1}{2\pi} \int d^4y e^{-iqy} \langle P, r | J_\mu^+(y) J_\nu(0) | P, r \rangle E_N, \quad (3)$$

由洛伦兹协变性,并注意到在高能情况下轻子质量可以忽略,所以可略去与  $q_\mu, q_\nu$  成正比的项,从而得到  $W_{\mu\nu}^r$  的一般形式为:

$$\begin{aligned} W_{\mu\nu}^r(P, q) = & \delta_{\mu\nu} W_1 + \frac{1}{m_N^2} P_\mu P_\nu W_2 + \frac{1}{m_N^3} \varepsilon_{\mu\nu\rho\sigma} P_\rho q_\sigma W_3 \\ & + \frac{i}{m_N} \varepsilon_{\mu\nu\rho\sigma} q_\rho \bar{u}_r(P) \gamma_5 \gamma_\sigma u_r(P) G_1 + \frac{i}{m_N^3} \varepsilon_{\mu\nu\rho\sigma} P_\rho q_\sigma \bar{u}_r(P) \gamma_5 \hat{q} u_r(P) G_2 \\ & + \frac{i}{m_N} \varepsilon_{\mu\nu\rho\sigma} P_\rho \bar{u}_r(P) \gamma_5 \gamma_\sigma u_r(P) G_3 + \frac{i}{m_N} \delta_{\mu\nu} \bar{u}_r(P) \gamma_5 \hat{q} u_r(P) G_4 \\ & + \frac{i}{m_N^3} P_\mu P_\nu \bar{u}_r(P) \gamma_5 \hat{q} u_r(P) G_5 + \frac{i}{m_N} P_\mu \bar{u}_r(P) \gamma_5 \gamma_\nu u_r(P) G_6 + \frac{i}{m_N} P_\nu \bar{u}_r(P) \gamma_5 \gamma_\mu u_r(P) G_7. \quad (4) \end{aligned}$$

本文 1978 年 12 月 31 日收到。

利用

$$W_{ij}^{r*} = W_{ji}^r, \quad (i, j = 1, 2, 3) \quad (5)$$

可以知道,  $W_i (i = 1, 2, 3)$  和  $G_i (i = 1, 2, 3, 4, 5, 6, 7)$  都是实函数, 它们是  $q^2, P \cdot q$  的洛伦兹不变的函数.

用(3)、(4)式, 在实验室系中得微分截面为

$$\begin{aligned} \frac{d\sigma_{\uparrow}^{p, \bar{v}}}{dx dy} - \frac{d\sigma_{\downarrow}^{p, \bar{v}}}{dx dy} = & \frac{2G^2 E_{\nu}^3 y}{\pi m_N} \left\{ \pm \frac{m_N}{E_{\nu}} xy \left( 2 - y - \frac{m_N}{E_{\nu}} xy \right) G_1 \right. \\ & \pm xy(2-y) \left( y + \frac{m_N}{E_{\nu}} xy \right) G_2 \pm \frac{m_N^2}{E_{\nu}^2} xy G_3 - \frac{m_N}{E_{\nu}} xy \left( y + \frac{m_N}{E_{\nu}} xy \right) G_4 \\ & - \left( 1 - y - \frac{m_N}{2E_{\nu}} xy \right) \left( y + \frac{m_N}{E_{\nu}} xy \right) G_5 + \frac{m_N}{E_{\nu}} \left( 1 - y - \frac{m_N}{2E_{\nu}} xy \right) \\ & \cdot (G_6 + G_7) \left. \right\}. \quad (6) \end{aligned}$$

对上式中的“±”号, 中微子取“+”号, 反中微子取“-”号.  $d\sigma_{\uparrow}$  和  $d\sigma_{\downarrow}$  分别表示核子的 helicity 为  $\pm 1/2$  时的微分截面.

按照部份子模型, 有

$$\begin{aligned} W_{\mu\nu}^r = & \frac{1}{2\pi} \int e^{-iyq} \sum_{i,s} \{ f_i^r(x), \langle p^i, s | J_{\mu}^+(y) J_{\nu}(0) | p^i, s \rangle \\ & + f_i^r(x), \langle p^i, s | J_{\mu}^+(y) J_{\nu}(0) | p^i, s \rangle E_N d^4y dx \}. \quad (7) \end{aligned}$$

其中  $f_i^r(x)$ , 表示质子 helicity 为  $r$  时, helicity 为  $s$  的第  $i$  种层子和反层子的分布函数.

$$\begin{aligned} J_{\mu}(x) = & \cos\theta \bar{\Psi}_u(x) \gamma_{\mu} (1 + \gamma_5) \Psi_d(x) + \sin\theta \bar{\Psi}_u(x) \gamma_{\mu} (1 + \gamma_5) \Psi_s(x) \\ & + \cos\theta \bar{\Psi}_c(x) \gamma_{\mu} (1 + \gamma_5) \Psi_s(x) - \sin\theta \bar{\Psi}_c(x) \gamma_{\mu} (1 + \gamma_5) \Psi_d(x), \quad (8) \end{aligned}$$

$$J_{\mu}^+(x) = (-1)^{\delta_{\mu 4} + 1} J_{\mu}(x). \quad (9)$$

经推导可得

$$\begin{aligned} \frac{\nu}{m_N} G_1 = & - \sum_{i,s} g_i^{L^2} (-1)^{\frac{1}{2}+s} \{ f_i^s(x) + f_i^i(x) \}, \quad G_2 = 0, \\ \frac{\nu}{m_N} G_3 = & - \sum_{i,s} g_i^{L^2} (-1)^{\frac{1}{2}+s} x \{ f_i^s(x) + f_i^i(x) \}, \\ \frac{\nu}{m_N} G_4 = & \sum_{i,s} g_i^{L^2} (-1)^{\frac{1}{2}+s} \{ f_i^s(x) - f_i^i(x) \}, \quad G_5 = 0, \\ \frac{\nu}{m_N} G_6 = & \frac{\nu}{m_N} G_7 = - \sum_{i,s} g_i^{L^2} (-1)^{\frac{1}{2}+s} x \{ f_i^s(x) - f_i^i(x) \}, \quad (10) \end{aligned}$$

上式中  $f_i^s(x)$  和  $f_i^i(x)$  分别表示 helicity 为  $1/2$  的质子中 helicity 为  $s$  的层子和反层子的分布函数.  $g_i^{L^2}$  的定义见下表.

在推导(10)式时用了部份子模型的结果  $f_{\pm\frac{1}{2}}^{i,i}(x)_{-\frac{1}{2}} = f_{\mp\frac{1}{2}}^{i,i}(x)$ , 等式左边表示 helicity 为  $-1/2$  的质子中 helicity 为  $\pm 1/2$  的层子或反层子的分布函数.

忽略  $\Delta s = 1, \Delta c = 1$  的流, 对中微子过程有

$$\frac{\nu}{m_N} G_1 = f_{\frac{1}{2}}^d(x) - f_{-\frac{1}{2}}^d(x) + f_{\frac{1}{2}}^u(x) - f_{-\frac{1}{2}}^u(x), \quad G_2 = 0,$$

	$g_i^{L^1}$ (象数激发)	$g_i^{L^2}$ (象数未激发)
中微子过程 $i = d$	1	$\cos^2\theta$
$i = s$	1	$\sin^2\theta$
$i = \bar{u}$	1	1
反中微子过程 $i = u$	1	1
$i = \bar{s}$	1	$\sin^2\theta$
$i = \bar{d}$	1	$\cos^2\theta$

$$\begin{aligned} \frac{\nu}{m_N} G_3 &= \{f_{\frac{1}{2}}^d(x) - f_{-\frac{1}{2}}^d(x) + f_{\frac{1}{2}}^{\bar{u}}(x) - f_{-\frac{1}{2}}^{\bar{u}}(x)\}x, \\ \frac{\nu}{m_N} G_4 &= -\{f_{\frac{1}{2}}^d(x) - f_{-\frac{1}{2}}^d(x) - f_{\frac{1}{2}}^{\bar{u}}(x) + f_{-\frac{1}{2}}^{\bar{u}}(x)\}, \quad G_5 = 0, \\ \frac{\nu}{m_N} G_6 &= \frac{\nu}{m_N} G_7 = \{f_{\frac{1}{2}}^d(x) - f_{-\frac{1}{2}}^d(x) - f_{\frac{1}{2}}^{\bar{u}}(x) + f_{-\frac{1}{2}}^{\bar{u}}(x)\} \end{aligned} \quad (11)$$

对反中微子过程有

$$\begin{aligned} \frac{\nu}{m_N} G_1 &= f_{\frac{1}{2}}^u(x) - f_{-\frac{1}{2}}^u(x) + f_{\frac{1}{2}}^{\bar{d}}(x) - f_{-\frac{1}{2}}^{\bar{d}}(x), \quad G_2 = 0, \\ \frac{\nu}{m_N} G_3 &= \{f_{\frac{1}{2}}^u(x) - f_{-\frac{1}{2}}^u(x) + f_{\frac{1}{2}}^{\bar{d}}(x) - f_{-\frac{1}{2}}^{\bar{d}}(x)\} \cdot x, \\ \frac{\nu}{m_N} G_4 &= -\{f_{\frac{1}{2}}^u(x) - f_{-\frac{1}{2}}^u(x) - f_{\frac{1}{2}}^{\bar{d}}(x) + f_{-\frac{1}{2}}^{\bar{d}}(x)\}, \quad G_5 = 0, \\ \frac{\nu}{m_N} G_6 &= \frac{\nu}{m_N} G_7 = \{f_{\frac{1}{2}}^u(x) - f_{-\frac{1}{2}}^u(x) - f_{\frac{1}{2}}^{\bar{d}}(x) + f_{-\frac{1}{2}}^{\bar{d}}(x)\} \cdot x. \end{aligned} \quad (12)$$

将(11)式和(12)式代入(6)式,得

$$\frac{d\sigma^{\nu\uparrow}}{dxdy} - \frac{d\sigma^{\nu\downarrow}}{dxdy} = \frac{4G^2 m_N E_\nu}{\pi} x \{f_{\frac{1}{2}}^d(x) - f_{-\frac{1}{2}}^d(x) - (1-y)^2 [f_{\frac{1}{2}}^{\bar{u}}(x) - f_{-\frac{1}{2}}^{\bar{u}}(x)]\} \quad (13)$$

$$\frac{d\sigma^{\bar{\nu}\uparrow}}{dxdy} - \frac{d\sigma^{\bar{\nu}\downarrow}}{dxdy} = \frac{4G^2 m_N E_\nu}{\pi} x \{(1-y)^2 [f_{\frac{1}{2}}^u(x) - f_{-\frac{1}{2}}^u(x)] - f_{\frac{1}{2}}^{\bar{d}}(x) + f_{-\frac{1}{2}}^{\bar{d}}(x)\} \quad (14)$$

其中我们已忽略了  $O\left(\frac{m_N}{E_\nu}\right)$  量级的项。这里

$$x = \frac{q^2}{2m_N \nu}, \quad y = \frac{\nu}{E_\nu}, \quad \nu = \frac{-(P \cdot q)}{m_N}. \quad (15)$$

在部份子模型中,一般认为层子海除带能量和动量外,只带真空量子数,因此有

$$f_{\pm\frac{1}{2}}^i(x)_{\text{海}} = f_{\mp\frac{1}{2}}^i(x)_{\text{海}}, \quad f_{\pm\frac{1}{2}}^i(x)_{\text{海}} = f_{\mp\frac{1}{2}}^i(x)_{\text{海}}. \quad (16)$$

用(16)式后,中微子和反中微子的微分截面简化为

$$\frac{d\sigma^{\nu\uparrow}}{dxdy} - \frac{d\sigma^{\nu\downarrow}}{dxdy} = \frac{4G^2 m_N E_\nu}{\pi} x \{f_{\frac{1}{2}}^d(x) - f_{-\frac{1}{2}}^d(x)\}, \quad (17)$$

$$\frac{d\sigma^{\bar{\nu}\uparrow}}{dxdy} - \frac{d\sigma^{\bar{\nu}\downarrow}}{dxdy} = \frac{4G^2 m_N E_\nu}{\pi} x (1-y)^2 \{f_{\frac{1}{2}}^u(x) - f_{-\frac{1}{2}}^u(x)\}. \quad (18)$$

从(17)、(18)两式可以看到,中微子的微分截面与 $y$ 无关,而反中微子的微分截面与 $(1-y)^2$ 成正比.由此可以由实验结果来检验假设(16)的正确性.

将(17)、(18)两式作替代 $u \leftrightarrow d$ ,即得中微子及反中微子与极化中子靶深度非弹散射的微分截面.由此可导出中微子和反中微子与极化核子靶散射的微分截面

$$\frac{d\sigma^{\nu\uparrow}}{dx dy} - \frac{d\sigma^{\nu\downarrow}}{dx dy} = \frac{2G^2 m_N E_\nu}{\pi} x \{f_{\frac{1}{2}}^u(x) + f_{\frac{1}{2}}^d(x) - f_{-\frac{1}{2}}^u(x) - f_{-\frac{1}{2}}^d(x)\}, \quad (19)$$

$$\frac{d\sigma^{\bar{\nu}\uparrow}}{dx dy} - \frac{d\sigma^{\bar{\nu}\downarrow}}{dx dy} = \frac{2G^2 m_N E_\nu}{\pi} x(1-y)^2 \{f_{\frac{1}{2}}^u(x) + f_{\frac{1}{2}}^d(x) - f_{-\frac{1}{2}}^u(x) - f_{-\frac{1}{2}}^d(x)\}. \quad (20)$$

由(19)、(20)式,可得

$$\left(\frac{d\sigma^{\bar{\nu}\uparrow}}{dx dy} - \frac{d\sigma^{\bar{\nu}\downarrow}}{dx dy}\right) / \left(\frac{d\sigma^{\nu\uparrow}}{dx dy} - \frac{d\sigma^{\nu\downarrow}}{dx dy}\right) = (1-y)^2, \quad (21)$$

$$\left(\frac{d\sigma^{\bar{\nu}\uparrow}}{dx} - \frac{d\sigma^{\bar{\nu}\downarrow}}{dx}\right) / \left(\frac{d\sigma^{\nu\uparrow}}{dx} - \frac{d\sigma^{\nu\downarrow}}{dx}\right) = \frac{1}{3}, \quad (22)$$

$$(\sigma^{\bar{\nu}\uparrow} - \sigma^{\bar{\nu}\downarrow}) / (\sigma^{\nu\uparrow} - \sigma^{\nu\downarrow}) = \frac{1}{3}, \quad (23)$$

因为这三个表达式是在取(16)式的假定下得到的,因而实验对上面三式的检验,就可知道(16)式的正确性.

定义不对称度

$$A(x, y) = \left(\frac{d\sigma^{\uparrow}}{dx dy} - \frac{d\sigma^{\downarrow}}{dx dy}\right) / \left(\frac{d\sigma^{\uparrow}}{dx dy} + \frac{d\sigma^{\downarrow}}{dx dy}\right), \quad (24)$$

在忽略 $\Delta s = 1, \Delta c = 1$ 的弱流的情况下,对于质子靶有

$$\begin{aligned} \frac{d\sigma^{\nu\uparrow}}{dx dy} + \frac{d\sigma^{\nu\downarrow}}{dx dy} &= \frac{4G^2 m_N E_\nu}{\pi} x \{f_{\frac{1}{2}}^d(x) + f_{-\frac{1}{2}}^d(x) \\ &+ (1-y)^2 [f_{\frac{1}{2}}^u(x) + f_{-\frac{1}{2}}^u(x)]\}, \end{aligned} \quad (25)$$

$$\frac{d\sigma^{\bar{\nu}\uparrow}}{dx dy} + \frac{d\sigma^{\bar{\nu}\downarrow}}{dx dy} = \frac{4G^2 m_N E_\nu}{\pi} x \{(1-y)^2 [f_{\frac{1}{2}}^u(x) + f_{-\frac{1}{2}}^u(x)] + [f_{\frac{1}{2}}^d(x) + f_{-\frac{1}{2}}^d(x)]\}. \quad (26)$$

对于核子靶有

$$\frac{d\sigma^{\nu\uparrow}}{dx dy} + \frac{d\sigma^{\nu\downarrow}}{dx dy} = \frac{2G^2 m_N E_\nu}{\pi} \{Q(x) + \bar{Q}(x)(1-y)^2\}, \quad (27)$$

$$\frac{d\sigma^{\bar{\nu}\uparrow}}{dx dy} + \frac{d\sigma^{\bar{\nu}\downarrow}}{dx dy} = \frac{2G^2 m_N E_\nu}{\pi} \{Q(x)(1-y)^2 + \bar{Q}(x)\}. \quad (28)$$

其中

$$\begin{aligned} Q(x) &= x \{f_{\frac{1}{2}}^u(x) + f_{-\frac{1}{2}}^u(x) + f_{\frac{1}{2}}^d(x) + f_{-\frac{1}{2}}^d(x)\}, \\ \bar{Q}(x) &= x \{f_{\frac{1}{2}}^u(x) + f_{-\frac{1}{2}}^u(x) + f_{\frac{1}{2}}^d(x) + f_{-\frac{1}{2}}^d(x)\}. \end{aligned} \quad (29)$$

用[1]中我们求得的具有一定自旋取向的价层子的分布函数和在该文中已用过的海层子的分布函数 $0.2(1-x)^2/x$ ,就可得到过程 $\nu(\bar{\nu}) + N \rightarrow \mu^-(\mu^+) + \dots$ 的微分截面、总截面和不对称度.结果表明:

1. 中微子及反中微子和极化质子靶散射的微分截面都在 $x = 0.3$ 附近达到最大,其值为

$$\frac{m_N^3}{E_\nu} \left( \frac{d\sigma^{\nu\downarrow}}{dxdy} - \frac{d\sigma^{\nu\uparrow}}{dxdy} \right)_{x=0.3} = 1.5 \times 10^{-11},$$

$$\frac{1}{(1-y)^2} \frac{m_N^3}{E_\nu} \left( \frac{d\sigma^{\bar{\nu}\uparrow}}{dxdy} - \frac{d\sigma^{\bar{\nu}\downarrow}}{dxdy} \right)_{x=0.3} = 0.6 \times 10^{-10}.$$

总截面为

$$\sigma^{\nu\downarrow} - \sigma^{\nu\uparrow} = 0.63 \times 10^{-11} E_\nu / m_N^3, \quad \sigma^{\bar{\nu}\uparrow} - \sigma^{\bar{\nu}\downarrow} = 0.85 \times 10^{-11} E_\nu / m_N^3.$$

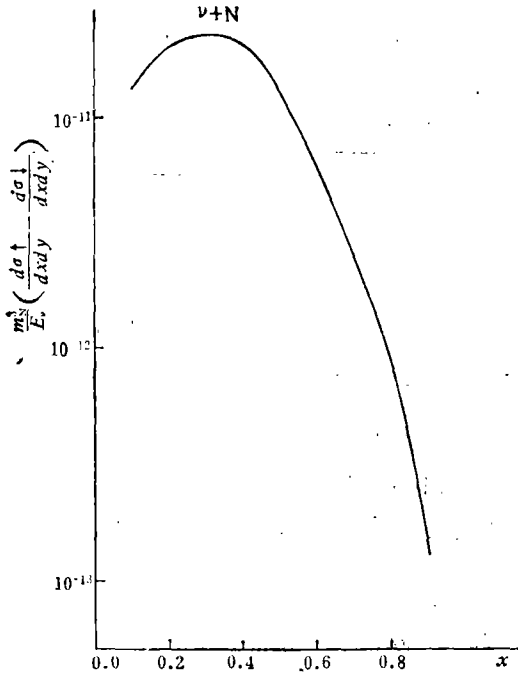


图 1

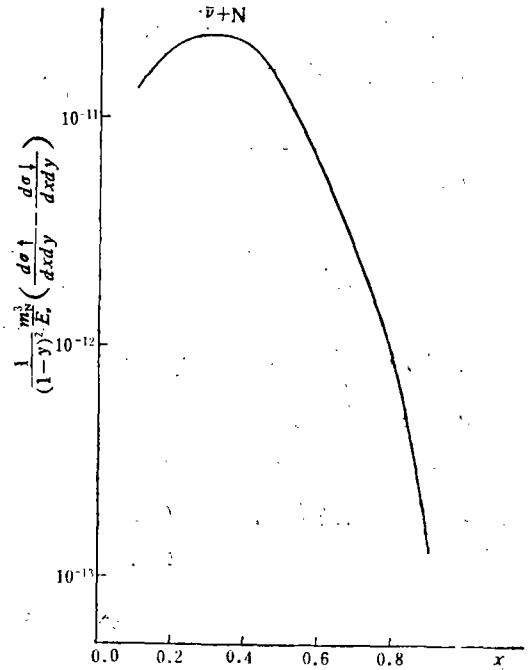


图 2

2. 中微子及反中微子和极化核子靶  $N$  ( $N = \frac{1}{2}(p+n)$ ) 散射的微分截面 (见图 1、图 2) 也在  $x = 0.3$  附近达到最大, 其值为

$$\frac{m_N^3}{E_\nu} \left( \frac{d\sigma^{\nu\downarrow}}{dxdy} - \frac{d\sigma^{\nu\uparrow}}{dxdy} \right)_{x=0.3} = 0.22 \times 10^{-10},$$

$$\frac{1}{(1-y)^2} \frac{m_N^3}{E_\nu} \left( \frac{d\sigma^{\bar{\nu}\uparrow}}{dxdy} - \frac{d\sigma^{\bar{\nu}\downarrow}}{dxdy} \right)_{x=0.3} = 0.22 \times 10^{-10}.$$

总截面为

$$\sigma^{\nu\downarrow} - \sigma^{\nu\uparrow} = 0.97 \times 10^{-11} E_\nu / m_N^3, \quad \sigma^{\bar{\nu}\uparrow} - \sigma^{\bar{\nu}\downarrow} = 0.32 \times 10^{-11} E_\nu / m_N^3.$$

3. 中微子及反中微子和极化核子靶的不对称度分别见图 3 和图 4.

4. 不对称度  $A = (\sigma^\uparrow - \sigma^\downarrow) / (\sigma^\uparrow + \sigma^\downarrow)$  为

$$A(\nu + p \rightarrow \mu^- + X) = -0.33, \quad A(\nu + N \rightarrow \mu^- + X) = 0.37,$$

$$A(\bar{\nu} + p \rightarrow \mu^+ + X) = 0.60, \quad A(\bar{\nu} + N \rightarrow \mu^+ + X) = 0.27. \quad (30)$$

从 (30) 式给出的结果看到, 中微子和极化质子靶的不对称度  $A$  是负的, 而所讨论的其它散射过程的不对称度是正的. 这有待于实验作出检验.

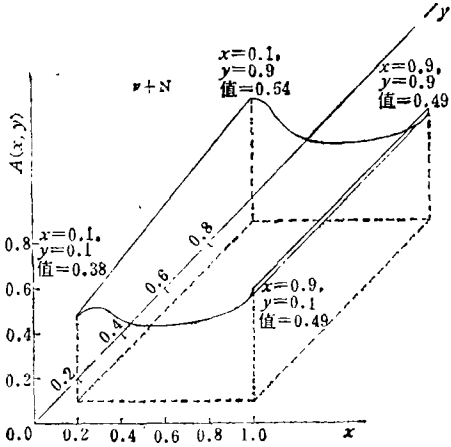


图 3

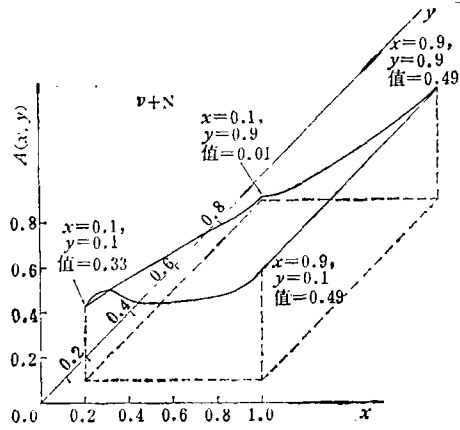


图 4

中微子与极化核子散射实验对研究具有一定自旋取向的层子的分布函数有特殊的意义,从(17)–(19)式可以看到,从中微子、反中微子与极化质子靶的深度非弹散射实验可以得到分布函数  $f_{\frac{1}{2}}^d(x) - f_{-\frac{1}{2}}^d(x)$ ,  $f_{\frac{1}{2}}^u(x) - f_{-\frac{1}{2}}^u(x)$ . 而从中微子与极化核子的深度非弹散射实验可以得到  $f_{\frac{1}{2}}^u(x) + f_{\frac{1}{2}}^d(x) - f_{-\frac{1}{2}}^u(x) - f_{-\frac{1}{2}}^d(x)$ .

另外,在极化部份子模型的讨论中,一般都作假定(16),但是这个假定是否完全合理,还需要实验的直接检验. 因为(21)–(23)式是在此假定下得到的,所以实验对这三式的检验就是对假定(16)的直接检验.

### 参 考 文 献

- [1] 李炳安、沈齐兴、郁宏、张美曼,高能物理与核物理,3(1979),723.  
 [2] R. P. Feynman, "Photon-Hadron Interaction", 1972 Advanced Book Program, Massachusetts.

## DEEP INELASTIC SCATTERING PROCESSES OF THE NEUTRINOS ON POLARIZED NUCLEONS

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### ABSTRACT

By using the distribution functions<sup>[1]</sup> of the valence stratoms with definite helicities, the differential cross sections and asymmetries of the deep inelastic scattering processes of neutrinos or antineutrinos on polarized nucleons are computed.'